Intro to Logarithms

Definition of Logarithm with Base *b*

The **logarithm of *y* with base *b*** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{iff} \quad b^x = y$$

$b > 0$, $y > 0$, $b \neq 1$

$\log_b y$ is read as " log base b of y"

Example 1:
Write the logarithmic equation in exponential form.

a. $\log_3 9 = 2$

$3^2 = 9$

b. $\log_8 1 = 0$

$8^0 = 1$

c. $\log_5\left(\frac{1}{25}\right) = -2$

$5^{-2} = \frac{1}{25}$

Special Logarithmic Values

$\log_b 1 = 0$ $\qquad$ $\log_b b = 1$

Example 2:
Evaluate the expression.

a. $\log_4 64$

$\log_4 4^3 = \boxed{3}$

b. $\log_2 0.125$

$\log_2 2^{-3}$

$\boxed{x = -3}$

c. $\log_{\frac{1}{4}} 256$

$\log_{\frac{1}{4}} 4^4 = \log_{\frac{1}{4}} \left(\frac{1}{4}\right)^{-4}$

$= \boxed{-4}$

d. $\log_{32} 2 = x$

$32^x = 2^1$

$2^{5x} = 2^1$

$5x = 1$

$\boxed{x = 1/5}$

Example 3:
Solve for x.

e. $\log_{\frac{1}{2}} x = -3$

$\frac{1}{2}^{-3} = x$

$\boxed{x = 8}$

b. $\log_x \sqrt{8} = \frac{1}{2}$

$x^{1/2} = \sqrt{8}$

$x^{1/2} = 8^{1/2}$

$\boxed{x = 8}$

Natural Logarithm: $\ln x = \log_e x$

Common Logarithm: $\log x = \log_{10} x$

Example 4:
Evaluate using a calculator.

a. $\log 7$

$\simeq .85$

b. $\ln 0.25$

$\simeq -1.39$

Student Name: ______________________ Score:

Logarithmic Form

Write the following exponents in logarithmic form:

Exponent Form	Logarithmic Form
$2^5 = 32$	$\log_2 32 = 5$
$3^3 = 27$	$\log_3 27 = 3$
$5^3 = 125$	$\log_5 125 = 3$
$2^{-4} = \frac{1}{16}$	$\log_2 1/16 = -4$
$4^3 = 64$	$\log_4 64 = 3$
$3^2 = 9$	$\log_3 9 = 2$
$7^{-2} = \frac{1}{49}$	$\log_7 1/49 = -2$

Student Name: ____________________ Score:

Exponent Form

Write the following logarithmic form into exponent form:

Logarithmic Form	Exponent Form
$\log_3 9 = 2$	$3^2 = 9$
$\log_4 64 = 3$	$4^3 = 64$
$\log_5 25 = 2$	$5^2 = 25$
$\log_2 128 = 7$	$2^7 = 128$
$\log_3\left(\frac{1}{27}\right) = -3$	$3^{-3} = \frac{1}{27}$
$\log_6 36 = 2$	$6^2 = 36$
$\log_5\left(\frac{1}{625}\right) = -4$	$5^{-4} = \frac{1}{625}$

Student Name: ____________________ Score:

Answers

Logarithmic Form	Exponent Form
$\log_3 9 = 2$	$3^2 = 9$
$\log_4 64 = 3$	$4^3 = 64$
$\log_5 25 = 2$	$5^2 = 25$
$\log_2 128 = 7$	$2^7 = 128$
$\log_3\left(\frac{1}{27}\right) = -3$	$3^{-3} = \frac{1}{27}$
$\log_6 36 = 2$	$6^2 = 36$
$\log_5\left(\frac{1}{625}\right) = -4$	$5^{-4} = \frac{1}{625}$

Logarithms WS   Name ______________________

I. Change the expression to logarithmic form.

1. $3^4 = 81$ $\log_3 81 = 4$	2. $10^3 = 1000$ $\log_{10} 1000 = 3$
3. $2^x = 32$ $\log_2 32 = x$	4. $2^3 = x$ $\log_2 x = 3$

II. Change the expression to exponential form.

5. $\log_4 16 = 2$ $4^2 = 16$	6. $\log_5 125 = 3$ $5^3 = 125$
7. $\log_3 9 = 2$ $3^2 = 9$	8. $\log_6 6 = 1$ $6^1 = 6$

III. Evaluate

9. $\log_8 2 = x$ $8^x = 2$ $\boxed{x = 1/3}$	10. $\log_7 1 = x$ $7^x = 1$ $\boxed{x = 0}$	11. $\log 0.01 = x$ $10^x = .01$ $\boxed{x = -2}$
12. $\log_3 \frac{1}{81} = x$ $3^x = \frac{1}{81}$ $3^x = 3^{-4}$ $\boxed{x = -4}$	13. $\log_{\frac{1}{2}} 8$ $\frac{1}{2}^x = 8$ $2^{-x} = 2^3$ $\boxed{x = -3}$	14. $\log_4 2 = x$ $4^x = 2$ $2^{2x} = 2^1$ $2x = 1$ $\boxed{x = 1/2}$
15. $\cancel{\log_m m^3} = x$ $m^x = m^3$ $\boxed{x = 3}$	16. $\log_{27} 9 = x$ $27^x = 9^1$ $3^{3x} = 3^2$ $3x = 2$ $\boxed{x = 2/3}$	17. $\log_3 243 = x$ $3^x = 243$ $3^x = 3^5$ $\boxed{x = 5}$
18. $\log_{\frac{1}{16}} \frac{1}{8}$ $\frac{1}{16}^x = \frac{1}{8}$ $2^{-4x} = 2^{-3}$ $-4x = -3$ $\boxed{x = 3/4}$	19. $\log \sqrt{1000}$ $\boxed{1.5}$	20. $5^{\log_5 14}$ $= \boxed{14}$
21. $\cancel{\log_3 3^4}$ $\boxed{4}$	22. $\log_{15} 1 = x$ $15^x = 1$ $\boxed{x = 0}$	23. $\log_2 \frac{1}{16} = x$ $2^x = \frac{1}{16}$ $2^x = 2^{-4}$ $\boxed{x = -4}$
24. $\log_{\frac{1}{3}} 27$ $\frac{1}{3}^x = 27$ $3^{-x} = 3^3$ $-x = 3$ $\boxed{x = -3}$	25. $\log_9 9 = x$ $9^x = 9$ $\boxed{x = 1}$	26. $\log_8 4 = x$ $8^x = 4$ $2^{3x} = 2^2$ $\frac{3x}{3} = \frac{2}{3}$ $\boxed{x = 2/3}$

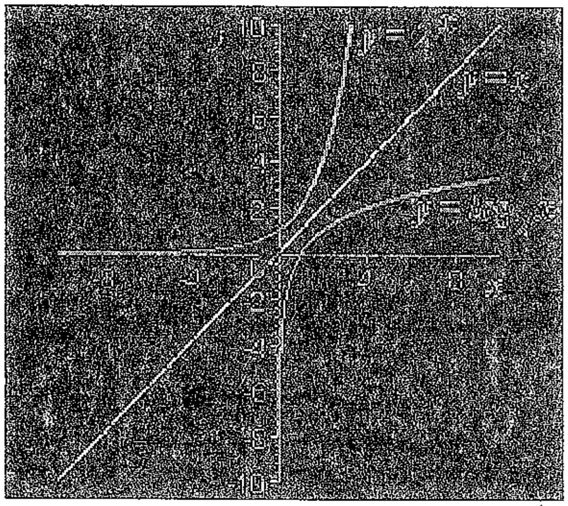
81
$\times 3$
3

IV. Solve

27. $\log_{\frac{1}{2}} 16 = x$ $\frac{1}{2}^x = 16$ $2^{-x} = 2^4$ $-x = 4$ $\boxed{x = -4}$	28. $\log_5 x = -2$ $5^{-2} = x$ $x = \frac{1}{25}$
29. $\log_m \frac{1}{27} = -3$ $m^{-3} = \frac{1}{27}$ $m^{-3} = 3^{-3}$ $\boxed{m = 3}$	30. $\log_x \sqrt[3]{7} = \frac{1}{3}$ $x^{\frac{1}{3}} = 7^{1/3}$ $\boxed{x = 7}$
31. $\log_{\frac{1}{2}} x = -6$ $\frac{1}{2}^{-6} = x$ $x = 64$	32. $\log_{64} 8 = x$ $64^x = 8$ $2^{6x} = 2^3$ $6x = 3$ $\boxed{x = 1/2}$

Odd Answers: 9) 1/3 11) –2 13) –3 15) 3 17) 5 19) 3/2 21) 4 23) –4 25) 1 27) –4 29) 3 31) 64

Finding Inverses of Exponential and Log Functions Notes



* The inverse of an exponential function is a log function.

* The inverse of a log function is an exponential function.

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

We can convert forms following the rule:

$$y=b^x \text{ ----------> } \log_b y=x$$

&

$$\log_b y=x \text{ ----------> } y=b^x$$

To find the inverse of either of these functions, simply convert forms, then switch the 'x' and 'y'.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

Example 1:

Find the inverse of the following.

$y=5^x$

switch x & y

$x=5^y$

put into log form

$\boxed{\log_5 x = y}$

Example 2:

Find the inverse of the following.

$y=5^{x-2}+1$

$\log_5$ $\log_5$

switch x & y

$x=5^{y-2}+1$

-1 -1

$(x-1)=5^{y-2}$

take $\log_5$ each side

$\log_5(x-1)=\log_5 5^{y-2}$

$\log_5(x-1)=y-2$

$\boxed{y=\log_5(x-1)+2}$

$x=5^{y-2}+1$

$\log_5(x-1)=\log_5 5^{y-2}$

$\log_5(x-1)=y-2$

$\log_5(x-1)+2=y$

Example 3:

Find the inverse of the following.

$y=\log_5 x-3$

switch x & y → $x=\log_5 y - 3$
$+3 \qquad +3$

add 3 to both sides → $(x+3)=\log_5 y$

Put in exponential form $\boxed{5^{x+3}=y}$

Finding Inverses of Exponential Functions WS
GPS Advanced Algebra

Name Key

For each of the following, find the inverse.

1. $f(x)=3^x$ $y=3^x$ $\log_3 \quad \log_3$ $\boxed{\log_3 x = y}$	2. $f(x)=3^x+4$ $y=3^x+4$ $x=3^y+4$ $-4 \quad -4$ $x-4=3^y$ $\log_3 \quad \log_3$ $\boxed{y=\log_3(x-4)}$
3. $f(x)=3^{x-2}$ $y=3^{x-2}$ $x=3^{y-2}$ $\log_3 \quad \log_3$ $\log_3 x = y-2$ $+2 \quad +2$ $\boxed{y=\log_3 x+2}$	4. $f(x)=3^{x+3}-5$ $x=3^{y+3}-5$ $(x+5)=3^{y+3}$ $\log_3 \quad \log_3$ $y+3=\log_3(x+5)$ $-3 \quad -3$ $\boxed{y=\log_3(x+5)-3}$
5. $g(x)=\left(\frac{1}{2}\right)^x+8$ $x=\frac{1}{2}^y+8$ $-8 \quad -8$ $x-8=\frac{1}{2}^y$ $\log_{1/2} \quad \log_{1/2}$ $\boxed{y=\log_{1/2}(x-8)}$	6. $f(x)=e^{x+3}$ $x=e^{y+3}$ $\log_e \quad \log_e$ $\log_e x = y+3$ $-3 \quad -3$ $\boxed{y=\log_e x-3}$
7. $f(x)=e^x-4$ $x=e^y-4$ $+4 \quad +4$ $x+4=e^y$ $\log_e \quad \log_e$ $\boxed{\log_e(x+4)=y}$	8. $h(x)=e^{x+2}-3$ $x=e^{y+2}-3$ $+3 \quad +3$ $x+3=e^{y+2}$ $\log_e \quad \log_e$ $\log_e(x+3)=y+2$ $-2 \quad -2$ $\boxed{y=\log_e(x+3)-2}$