

Properties of Logarithms

Special Logarithm Values

Let b be positive numbers such that $b \neq 1$.

$$\log_b 1 = 0$$

$$\log_b b = 1$$

Inverse Properties

Let b be positive numbers such that $b \neq 1$.

$$\log_b b^n = n$$

$$b^{\log_b n} = n$$

Properties of Logarithms

Let b , u , and v be positive numbers such that $b \neq 1$.

Examples:

Product Property $\log_b uv = \log_b u + \log_b v$

Quotient Property $\log_b \frac{u}{v} = \log_b u - \log_b v$

Power Property $\log_b u^n = n \log_b u$

Example 1:

Use $\log_a 5 \approx 0.732$ and $\log_a 11 \approx 1.091$ to approximate the following.

a. $\log_a \frac{5}{11} = \log_a 5 - \log_a 11 = .732 - 1.091 = \boxed{-.359}$

b. $\log_a 55 = \log_a 5 + \log_a 11 = .732 + 1.091 = \boxed{1.823}$

c. $\log_a 25 = \log_a 5 + \log_a 5 = .732 + .732 = \boxed{1.464}$

$\log_a 5^2 = 2 \log_a 5 = 2(.732) = \boxed{1.464}$

Example 2:

Expand each logarithm. Assume x is positive.

a. $\log_2 \frac{x}{2}$

$$\log_2 x - \log_2 2$$
$$\boxed{\log_2 x - 1}$$

b. $\log_3 9x$

$$\log_3 9 + \log_3 x$$
$$\log_3 3^2 + \log_3 x$$
$$\boxed{2 + \log_3 x}$$

c. $\log_5 2x^6$

$$\boxed{\log_5 2 + 6 \log_5 x}$$

Example 3:

Condense each logarithm.

a. $\log_4 2 + \log_4 8$

$$\log_4 2(8)$$
$$\boxed{\log_4 16}$$

b. $2\log_3 5 + \frac{1}{2}\log_3 x$

$$\log_3 5^2 \sqrt{x}$$
$$\boxed{\log_3 25\sqrt{x}}$$

c. $2\log_3 7 - 5\log_3 x$

$$\log_3 \frac{7^2}{x^5} = \boxed{\log_3 \frac{49}{x^5}}$$

Log Properties WS

Name Key

Use a property of logarithms to evaluate the expression.

$$1. \log_3(3 \cdot 9) \quad \log_3 3 + \log_3 9 =$$

$$1 + 2 = \boxed{3}$$

$$2. \log_2 4^5$$

$$\log_2 2^{2 \cdot 5} = 2 \cdot 5 = \boxed{10}$$

$$3. \log_3 \frac{1}{3} \quad \log_3 1 - \log_3 3 = \boxed{-1}$$

$$0 - 1 = -1$$

$$4. \log_5 \left(\frac{1}{5}\right)^3 = \log_5 \left(\frac{1}{125}\right) = \log_5 1 - \log_5 125$$

$$= 0 - 3 = \boxed{-3}$$

Use the change of

$$5. \log_2 3$$

$$6. \log_2 49$$

SKIP

Use the value of the expression.

$$7. \log_2 147$$

$$8. \log_2 441$$

$$\frac{\log 147}{\log 2}$$

$$\frac{\log 441}{\log 2}$$

Expand the expression.

$$9. \log_2 9x = \log_2 9 + \log_2 x$$

$$10. \log 4x^5 = \log 4 + 5 \log x$$

$$11. \log_4 \frac{4}{3}$$

$$\log_4 4 - \log_4 3$$

$$12. \ln 3xy^3 = \ln 3 + \ln x + 3 \ln y$$

$$13. \log_8 64x^2 = \log_8 64 + 2 \log_8 x$$

$$14. \log \sqrt{x}$$

$$\log x^{1/2} = \frac{1}{2} \log x$$

Condense the expression.

$$15. 2 \log x + \log 5$$

$$\log 5x^2$$

$$18. 10 \log x + 2 \log 10$$

$$\log 100x^{10}$$

$$16. 4 \log_{16} 12 - 4 \log_{16} 2$$

$$\log_{16} \frac{12^4}{2^4} = \log_{16} \frac{20736}{16}$$

$$19. 2(\log_6 15 - \log_6 5) + \frac{1}{2} \log_6 \frac{1}{25}$$

$$2 \log_6 \frac{15}{5} + \left(\log_6 \frac{1}{25}\right)^{1/2} =$$

$$17. 7 \log_4 2 + 5 \log_4 x + 3 \log_4 y$$

$$\log_4 2^7 x^5 y^3$$

$$20. \frac{1}{4} \log_5 81 - \left(2 \log_5 6 - \frac{1}{2} \log_5 4\right)$$

$$\log_5 \frac{81^{1/4}}{6^2 4^{1/2}} = \log_5 \frac{3}{12(2)} =$$

$$= \log_5 \frac{3}{24}$$

$$= \boxed{\log_5 \frac{1}{8}}$$

MATH 3
LOG PROPERTIES—EXPANDING AND CONDENSING

NAME Key

#1-9 Expand each of the following using the properties of logs.

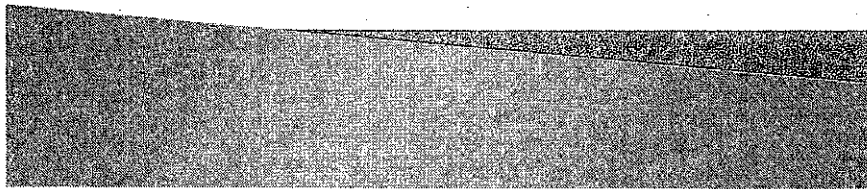
1. $\log_3 6x$ $\log_3 6 + \log_3 x$	2. $\log_6 x^5$ $5 \log_6 x$	3. $\log_5 \frac{7}{3}$ $\log_5 7 - \log_5 3$
4. $\log 5x^4y^2$ $\log 5 + 4 \log x + 2 \log y$	5. $\log_3 \sqrt{x}$ $\frac{1}{2} \log_3 x$	6. $\ln \frac{2}{xy}$ $\ln 2 - (\ln x + \ln y)$
7. $\log \frac{3y^4}{4x^3}$ $(\log 3 + 4 \log y) -$ $(\log 4 + \frac{1}{3} \log x)$	8. $\log_3 \sqrt[5]{p}$ $\frac{1}{5} \log_3 5 + \frac{1}{2} \log_3 p$	9. $\log_4 \frac{9(x+1)}{(x+2)}$ $\log_4 9 + \log_4 (x+1) - \log_4 (x+2)$

#10-18. Condense the following using the properties of logs.

10. $\log_2 3 + \log_2 x$ $\log_2 3x$	11. $\log_5 8 - \log_5 3$ $\log_5 \frac{8}{3}$	12. $5 \log_7 r$ $\log_7 r^5$
13. $2 \ln 3 - \ln 5$ $\ln \frac{3^2}{5}$	14. $\log 20 + 2 \log y + \log z$ $\log 20y^2z$	15. $\log_3 2 + \frac{1}{2} \log_3 y$ $\log_3 2\sqrt{y}$
16. $\log_2 x + \log_2 y - \log_2 3$ $\log_2 \frac{xy}{3}$	17. $\log_4 (x+1) - \log_4 7 - 2 \log_4 y$ $\log_4 \frac{(x+1)}{7y^2}$	18. $\ln 5 - \ln 3 - \ln x - \ln y$ $\ln \frac{5}{3xy}$

Solving Log Equations

Using Log Properties



1. Solve and check.

$$\log_4(x+3) = 2$$

$$x+3 = 16$$

$$\begin{array}{r} x+3 = 16 \\ -3 \quad -3 \\ \hline \end{array}$$

$$\boxed{x=13}$$



2. Solve and check.

$$\log_4(x+3) = \log_4(8x+17)$$

$$x+3 = 8x+17$$

$$-\frac{7x}{-7} = \frac{14}{-7}$$

$$x = -\frac{14}{7}$$

3. Solve and check.

$$\log_2 x + \log_2(x-7) = \log_2 8$$

$$\log_2 x(x-7) = \log_2 8$$

$$x(x-7) = 8$$

$$x^2 - 7x - 8 = 0$$

$$(x-8)(x+1) = 0$$

$$x=8 \quad x=-1$$

4. Solve and check.

$$\log(2x+3) = 1 + \log(x-3)$$

$$\log_{10}(2x+3) = \log_{10} 10 + \log_{10}(x-3)$$

$$\log(2x+3) = \log 10(x-3)$$

$$2x+3 = 10(x-3)$$

$$2x+3 = 10x-30$$

$$-8x+3 = -30$$

$$\frac{-8x}{-8} = \frac{-33}{-8}$$

$$x = \frac{33}{8}$$

I. Evaluate each expression

1. $n^{\log_n 3}$ <u>3</u>	2. $14^{\log_{14} 6}$ <u>6</u>
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II. Use $\log_{10} 5 = 0.6990$ and $\log_{10} 7 = 0.8451$ to evaluate each expression.

3. $\log_{10} 35$ $\log_{10} 5 + \log_{10} 7$ $.699 + .8451 = \boxed{1.5441}$	4. $\log_{10} \frac{7}{5}$ $\log 7 - \log 5$ $.8451 - .699 = \boxed{.1461}$
5. $\log_{10} 25$ $\log 5 + \log 5$ $.699 + .699 = \boxed{1.398}$	6. $\log_{10} 490 = \log_{10} 10 \cdot 7^2$ $= 1 + \log_{10} 7 + \log_{10} 7$ $1 + .8451 + .8451 = \boxed{2.6902}$
7. $\log_{10} \left(1 \frac{3}{7}\right) = \log_{10} \frac{10}{7}$ $= \log_{10} 10 - \log_{10} 7 = 1 - .8451 = \boxed{.1549}$	8. $\log_{10} 0.05 = \frac{5}{100}$ $\log_{10} 5 - \log_{10} 10^2 = \log_{10} 5 - 2$ $= .6990 - 2 = \boxed{-1.301}$

III. Solve each equation.

9. $\log_9 x + \log_9 9 = \log_9 54$ $\log_9 9x = \log_9 54$ $9x = 54$ <u>$x = 6$</u>	10. $\log_8 48 - \log_8 w = \log_8 4$ $\log_8 \frac{48}{w} = \log_8 4$ $\frac{48}{w} = 4$ $48 = 4w$ <u>$w = 12$</u>
11. $\log_7 n = \frac{2}{3} \log_7 8$ $\log_7 n = \log_7 8^{2/3}$ <u>$n = 4$</u>	12. $\log_3 y = \frac{1}{4} \log_3 16 + \frac{1}{3} \log_3 64$ $\log_3 y = \log_3 16^{1/4} (64)^{1/3}$ $y = 2(4)$ <u>$y = 8$</u>
13. $\log_9 (3u+14) - \log_9 5 = \log_9 2u$ $\frac{3u+14}{5} = \log_9 2u$ $5 \cdot \frac{3u+14}{5} = 2u(5)$ $14 = 7u$ <u>$u = 2$</u>	14. $\log_7 x + \log_7 x = \log_7 12$ $\log_7 x^2 = \log_7 12$ $\sqrt{x^2} = \sqrt{12}$ <u>$x = \pm 2\sqrt{3}$</u>
15. $4 \log_2 x + \log_2 5 = \log_2 405$ $\log_2 x^4 (5) = \log_2 405$ $x^4 = 81$ <u>$x = 3$</u>	16. $\log_6 (2x-5) + x = \log_6 (7x+10)$ $\log_6 (2x-5)6 = \log_6 (7x+10)$ $12x - 30 = 7x + 10$ <u>$x = 8$</u>
17. $\log_{16} (9x+5) - \log_{16} (x^2-1) = \frac{1}{2}$ $\log_{16} \frac{9x+5}{x^2-1} = \frac{1}{2}$ $16^{1/2} = \frac{9x+5}{x^2-1}$ $4 = \frac{9x+5}{x^2-1}$ $4(x^2-1) = 9x+5$ $4x^2 - 4 = 9x + 5$ $4x^2 - 9x - 9 = 0$ $x = \frac{9 \pm \sqrt{81 - 4(4)(-9)}}{8}$ <u>$x = \frac{9 \pm \sqrt{-63}}{8}$</u>	18. $\log_8 (n-3) + \log_8 (n+4) = 1$ $\log_8 (n-3)(n+4) = 1$ $8 = (n-3)(n+4)$ $8 = n^2 + n - 12$ $0 = n^2 + n - 20$ $0 = (n+5)(n-4)$ <u>$n = 4$</u>

$(x^2-1)4 = 9x+5$
 $4x^2 - 4 = 9x + 5$
 $4x^2 - 9x - 9 = 0$
 $x = \frac{9 \pm \sqrt{81 - 4(4)(-9)}}{8}$
 $x = \frac{9 \pm \sqrt{-63}}{8}$

Log Functions & Log Equations

[Key]

1) $\log_2 2 + 2 \log_2 x + 3 \log_2 y$

2) $\log_5 10 + \log_5 x - 2 \log_5 y$

3) $\log_3 \frac{xy}{z^3}$

4) $\log_4 \frac{8}{x^2 y}$

4) $4^{3/2} = x$
 $x = 8$

5) $y^{-4} = 16$
 $y = 1/2$

6) $a^{-3} = 1/8$
 $a = 2$

7) $7^{-1/2} = n$
 $n = 1/49$

8) $\sqrt{5}^{4/3} = y$

9) $(x^{1/6})^6 = (\sqrt[3]{9})^6$
 $x = 9^{4/3} = 9^2 = 81$

$\ominus \sqrt[3]{\sqrt{5}^4}$
 $y = \sqrt{5} \sqrt[3]{\sqrt{5}}$

10) $3x + 7 = 7x + 4$

11) $8x + 20 = x + 6$

$\frac{-4x}{-4} = \frac{-3}{-4}$

$\frac{7x}{7} = \frac{-14}{7}$

$x = 3/4$

$x = -2$

12) $9x - 1 = 4x - 16$

13) $x - 9 = 3x - 13$

$5x = -15$

$x = -3$

$-2x = -4$

$x = 2$

I. Expand each logarithmic expression

1. $\log_2 2x^2y^3$ $\log_2 2 + 2\log_2 x + 3\log_2 y$	2. $\log_5 \frac{10x}{y^2} = \log_5 10 + \log_5 x - 2\log_5 y$
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II. Condense each logarithmic expression

3. $\log_3 x + \log_3 y - 3\log_3 z$ $\log_3 \frac{xy}{z^3}$	4. $3\log_4 2 - (2\log_4 x + \log_4 y)$
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III. Solve each equation.

4. $\log_4 x = \frac{3}{2}$	5. $\log_y 16 = -4$
6. $\log_a \frac{1}{8} = -3$	7. $\log_7 n = -\frac{1}{2}$
8. $\log_{\sqrt{5}} y = \frac{4}{3}$	9. $\log_x \sqrt[3]{9} = \frac{1}{6}$
10. $\log_4 (3x+7) = \log_4 (7x+4)$	11. $\log_7 (8x+20) = \log_7 (x+6)$
12. $\log_3 (9x-1) = \log_3 (4x-16)$	13. $\log_{12} (x-9) = \log_{12} (3x-13)$
14. $\log_5 (x^2 - 30) = \log_5 6$	15. $\log_4 (x^2 + 6) = \log_4 5x$
16. $\log x + \log 6 = 1$	17. $\log_2 x = \frac{1}{2} \log_2 81$
18. $\log_3 14 + \log_3 x = \log_3 42$	19. $\log_4 (y-1) = \log_4 20 - 2$
20. $\log_4 (1-2x) = \log_4 (x+10)$	21. $\log_7 (m+1) + \log_7 (m-5) = 1$
22. $\log_5 7 + \frac{1}{2} \log_5 4 = \log_5 x$	23. $\log_8 (y+1) - \log_8 y = \log_8 4$
24. $\log_6 (x-3) + \log_6 (x+2) = 2$	

SOLVING EXPONENTIAL EQUATIONS ...

... with un-like bases

1. Solve: $5^x = 25$

$$\log_5 5^x = \log_5 25$$

* take \log_5 of each side

$$x = \log_5 25$$

$$x = \log_5 5^2$$

$$\boxed{x=2}$$

$$\text{or } 5^x = 5^2$$
$$\boxed{x=2}$$

2. Solve: $3^x = 25$

$$\ln 3^x = \ln 25$$

2.9

$$x \frac{\ln 3}{\ln 3} = \frac{\ln 25}{\ln 3}$$

$$x \approx 2.93$$

3. Solve: $e^x = 26$

$$\ln e^x = \ln 26$$

$$x = \ln 26$$

$$x \approx 3.26$$

4. Solve: $2(3)^{2t-5} - 4 = 11$

$$2(3)^{2t-5} - 4 = 11$$

$$\quad \quad \quad +4 \quad +4$$

$$\frac{2(3)^{2t-5}}{2} = \frac{15}{2}$$

$$3^{2t-5} = \frac{15}{2}$$

$$\ln 3^{2t-5} = \ln 7.5$$

$$\frac{(2t-5)\ln 3}{\ln 3} = \frac{\ln 7.5}{\ln 3}$$

$$2t-5 = 1.83$$

$$\quad +5 \quad +5$$

$$2t = 6.83$$

$$t \approx 3.42$$

5. Solve: $e^{2x} - 3e^x + 2 = 0$

$$(e^x - 1)(e^x - 2) = 0$$

$$e^x - 1 = 0$$

$$e^x - 2 = 0$$

$$\ln e^x = \ln 1$$

$$\ln e^x = \ln 2$$

$$x = \ln 1$$

$$x = \ln 2$$

$$x = 0$$

$$x = .69$$

6. Solve: $5^x = 7^{x-1}$

$$\frac{x \log 5}{\log 5} = \frac{(x-1) \log 7}{\log 5}$$

$$x = (x-1)(1.21)$$

$$x = 1.21x - 1.21$$

$$\frac{-0.21x}{-0.21} = \frac{-1.21}{-0.21}$$

$$x \approx 5.76$$

Solve each equation. Round your answer to three decimal places.

<p>1. $13^x = 7$</p> $\frac{x \ln 13}{\ln 13} = \frac{\ln 7}{\ln 13}$ $\boxed{x \approx .76}$	<p>2. $4^{5x} = 9$</p> $\frac{5x \ln 4}{\ln 4} = \frac{\ln 9}{\ln 4}$ $\frac{5x}{5} = \frac{1.58}{5}$ $\boxed{x = .32}$
<p>3. $6^{x+2} + 1 = 3$</p> $6^{x+2} = 2$ $\frac{(x+2) \ln 6}{\ln 6} = \frac{\ln 2}{\ln 6}$ $\frac{x+2}{-2} = \frac{.38}{-2}$ $\boxed{x = -1.61}$	<p>4. $1.5 = 2.7^{2x-3} - 4$</p> $5.5 = 2.7^{2x-3}$ $\frac{\ln 5.5}{\ln 2.7} = \frac{(2x-3) \ln 2.7}{\ln 2.7}$ $\frac{1.72}{+3} = \frac{2x-3}{+3}$ $4.72 = 2x$ $\boxed{x = 2.36}$
<p>4. $5^{-4x} = 3.21$</p> $\frac{-4x \ln 5}{\ln 5} = \frac{\ln 3.21}{\ln 5}$ $\frac{-4x}{-4} = \frac{.172}{-4}$ $\boxed{x \approx -.18}$	<p>5. $8^{\frac{x}{5}} = 3$</p> $\frac{x}{5} \frac{\ln 8}{\ln 8} = \frac{\ln 3}{\ln 8}$ $\frac{x}{5} = .53 (5)$ $\boxed{x \approx 2.64}$
<p>7. $4^{2x} = 9^{x-1}$</p> $\frac{2x \ln 4}{\ln 4} = \frac{(x-1) \ln 9}{\ln 4}$ $2x = (x-1)(1.59)$ $2x = 1.59x - 1.59$ $\frac{.41x}{.41} = \frac{-1.59}{.41}$ $\boxed{x \approx 3.88}$	<p>8. $7^{3x} = 12^{x+2}$</p> $\frac{3x \ln 7}{\ln 7} = \frac{(x+2) \ln 12}{\ln 7}$ $3x = (x+2)(1.28)$ $3x = 1.28x + 2.56$ $\frac{1.72x}{1.72} = \frac{2.56}{1.72}$ $\boxed{x \approx 1.49}$

9. $e^x = 5.1$

10. $e^{x-4} + 1 = 5$

11. $5e^{-7x} = 23$

12. $200e^{x+5} - 3 = 60$

ANSWERS: 1. .759 2. .317 3. -1.613 4. 2.358 5. -.181 6. 2.642
6. -3.819 8. 1.482 9. 1.629 10. 5.386 11. -.218 12. -6.155

Exp and Log Equations WS

Name Key

Solve the equation.

1. $10^{x-3} = 100^{4x-5}$
 $10^{x-3} = 10^{2(4x-5)}$
 $x-3 = 8x-10$
 $-7x = -7$
 $x = 1$

3. $8^{5x} = 16^{3x+4}$
 $2^{3(5x)} = 2^{4(3x+4)}$
 $15x = 12x + 16$
 $3x = 16$
 $x = 16/3$

5. $.25^x - .5 = 2$
 $.25^x = 2.5$
 $x \ln .25 = \ln 2.5$
 $x = -.66$

7. $10^{-12x} + 6 = 100$
 $10^{-12x} = 94$
 $-12x \ln 10 = \ln 94$
 $-12x = \frac{\ln 94}{-12}$
 $x \approx -.164$

2. $3^{x-7} = 27^{2x}$
 $3^{x-7} = 3^{3(2x)}$
 $x-7 = 6x$
 $-7 = 5x$
 $x = -7/5$

4. $e^{-x} = 6$
 $-\ln e^{-x} = \ln 6$
 $x = -\ln 6 \approx -1.79$

6. $7^{2x} + 3 = 8$
 $7^{2x} = 5$
 $2x \ln 7 = \ln 5$
 $\frac{2x}{2} = \frac{\ln 5}{2}$
 $x \approx .41$

8. $-16 + 0.2(3)^x = 35$
 $0.2(3)^x = 51$
 $\frac{2(3)^x}{2} = \frac{51}{2}$
 $3^x = 25.5$
 $x \ln 3 = \ln 25.5$
 $\frac{x \ln 3}{\ln 3} = \frac{\ln 25.5}{\ln 3}$
 $x \approx 5.04$

Solve the equation. Check for extraneous solutions.

9. $\ln(4x+1) = \ln(2x+5)$
 $4x+1 = 2x+5$
 $2x = 4$
 $x = 2$

10. $\log_2 x = -1$
 $2^{-1} = x$
 $x = 1/2$

11. $4 \log_3 x = 28$
 $\log_3 x = 7$
 $3^7 = x$
 $x = 2187$

12. $1 - 2 \ln x = -4$
 $-2 \ln x = -5$
 $\ln x = 2.5$
 $x \approx 12.18$

13. $\ln x + \ln(x-2) = 1$

14. $\log_5(11-6x) = \log_5(1-x)$
 $11-6x = 1-x$
 $10 = 5x$
 $x = 2$ \emptyset

15. $15 + 2 \log_2 x = 31$
 $2 \log_2 x = 16$
 $\log_2 x = 8$
 $2^8 = x$
 $x = 256$

16. $6.5 \log_5 3x = 20$
 $\log_5 3x = 3.08$
 $5^{3.08} = 3x$
 $\frac{3x}{3} = \frac{142.18}{3}$
 $x \approx 47.16$

Answers: 1) 1 2) -7/5 3) 16/3 4) -1.79 5) -0.66 6) 0.41 7) -0.164 8) 5.04
 9) 2 10) 1/2 11) 2187 12) 12.18 13) $1 + \sqrt{1+e}$ 14) \emptyset 15) 256 16) 47.16