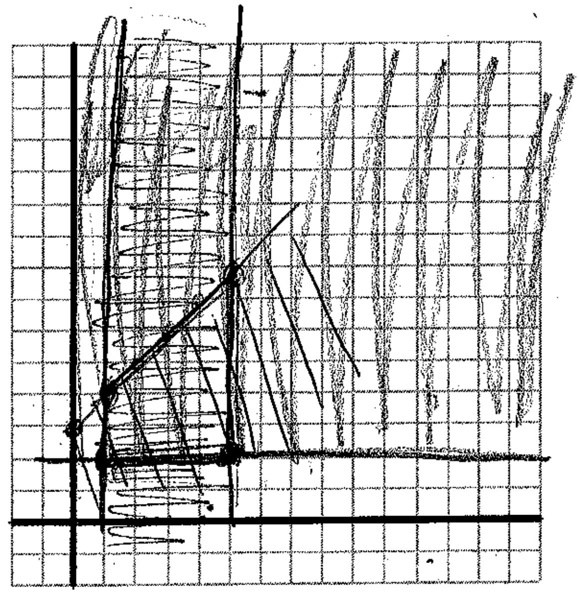
Intro to Linear Programming Notes

Find the maximum and minimum values of the objective function subject to the given constraints.

1. Objective Function:
   $F(x, y) = 3x - 2y$

   Constraints:
   $y \geq 2$
   $1 \leq x \leq 5$
   $y \leq x + 3$



$\leq, \geq$ ————

$<, >$ - - - - -

$(1,2) = 3(1) - 2(2) = -1$

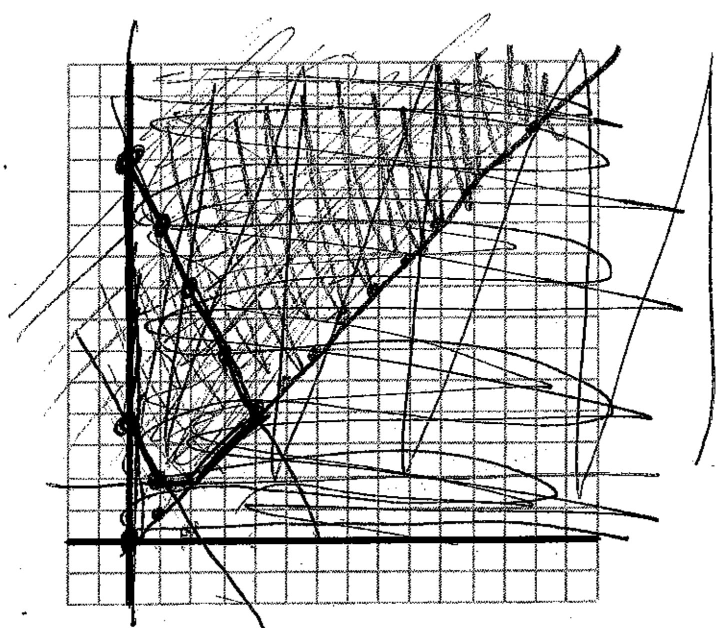
Min $\boxed{(1,4)} = 3(1) - 2(4) = -5$

Max $\boxed{(5,2)} = 3(5) - 2(2) = 11$

$\boxed{(5,8)} = 3(5) - 2(8) = -1$

2. Objective Function:
   $C = 10x + 3y$

   Constraints:
   $x \geq 0$
   $y \geq 2$
   $-x + y \geq 0$
   $2x + y \geq 4$
   $2x + y \leq 12$



$y \geq x + 0$

$y \geq -2x + 4$

$y \leq -2x + 12$

$(1,2)$ $(0,12)$

$(2,2)$ $\boxed{(0,4) = 12}$

$\boxed{(4,4) = 52}$

Intro to Linear Programming Notes

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   Constraints:
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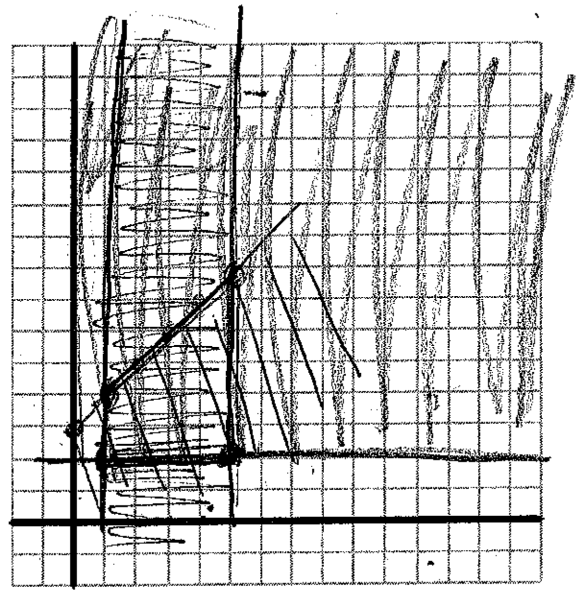

$\leq, \geq$ ———

$<, >$ - - - - -

$(1,2) = 3(1) - 2(2) = -1$

Min $\boxed{(1,4)} = 3(1) - 2(4) = -5$

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   Constraints:
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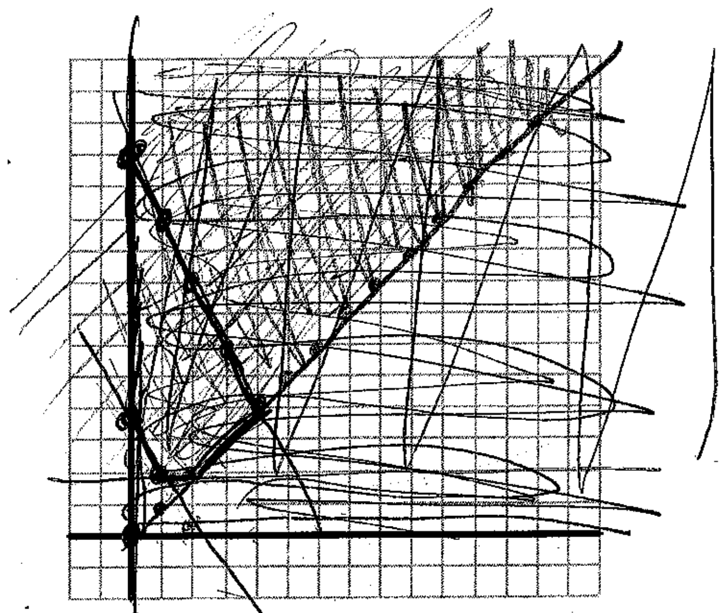
   $-x + y \geq 0$
   $2x + y \geq 4$
   $2x + y \leq 12$

$y \geq x + 0$

$y \geq -2x + 4$

$y \leq -2x + 12$



$(1,2)$ $(0,12)$

$(2,2)$ $\boxed{(0,4) = 12}$

$\boxed{(4,4) = 52}$

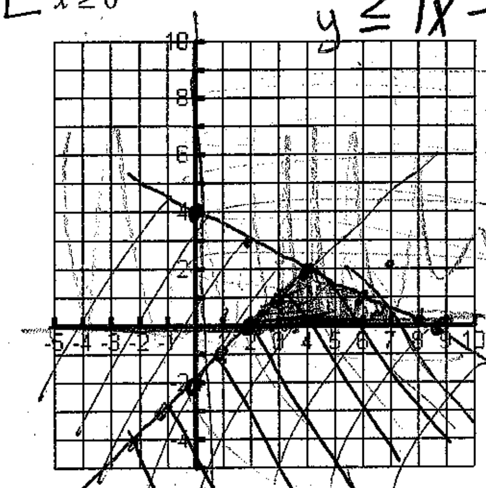
Math 3
Linear Programming 1

Name ______________________

Find the minimum and maximum values of the objective function subject to the given constraints.

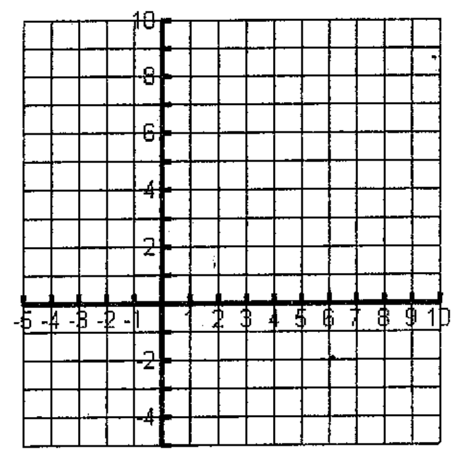
1. **Objective function**:
$C = x + 3y$
Constraints:
$x + 2y \le 8$
$x - y \ge 2$
$y \ge 0$
$x \ge 0$

$\frac{2y}{2} \le \frac{-x}{2} + \frac{8}{2}$
$y \le -\frac{1}{2}x + 4$
$-y \ge -x + 2$
$y \le 1x - 2$

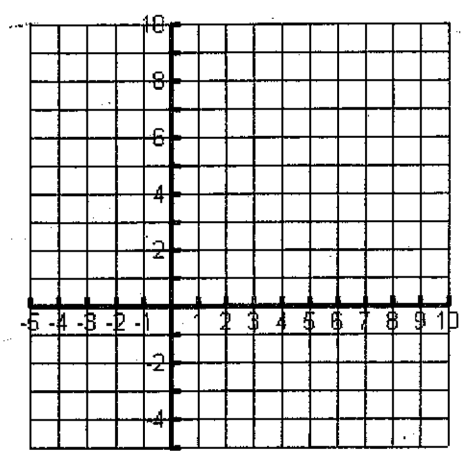


min $(2,0)$ $\boxed{2}$
max $(4,2)$ $\boxed{10}$
$(9,0)$ $\boxed{9}$

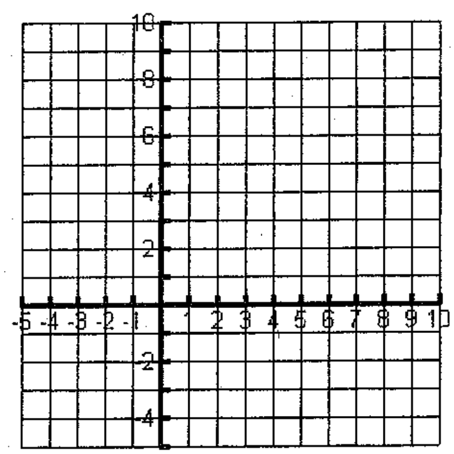
2. Objective function:
$C = 3x + 2y$
Constraints:
$x \ge 0; y \ge 0$
$x + y \le 4$
$x - y \ge -2$



3. **Objective function**:
$C = 5x - 2y$
Constraints:
$x \ge 0$
$y \ge 0$
$2x + y \le 8$
$x + 3y \le 9$



4. **Objective function**:
$C = 3x - y$
Constraints:
$y \le 4$
$x + y \ge 2$
$2x - y \le 4$
$-x + y \le 2$



Informal Algebra 2
Linear Programming 1

Name Key

Find the minimum and maximum values of the objective function subject to the given constraints.

1. **Objective function:** Max of 10 @ (4,2)

   $C = x + 3y$ Min of 2 @ (2,0)

   Constraints:

   $x + 2y \le 8$

   $x - y \ge 2$

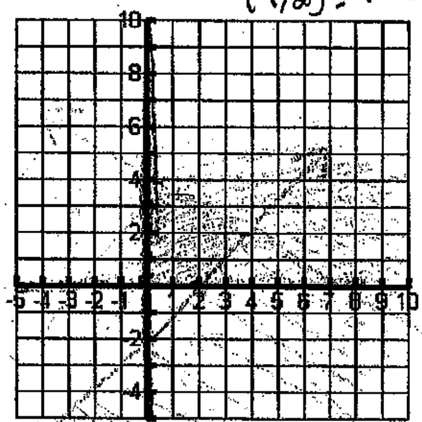
   $y \ge 0$

   $x \ge 0$

   $(2,0) = 2 + 0 = 2$

   $(8,0) = 8 + 0 = 8$

   $(4,2) = 4 + 3 \cdot 2 = 10$



2. **Objective function:**

   $C = 3x + 2y$

   Constraints:

   $x \ge 0; y \ge 0$

   $x + y \le 4$

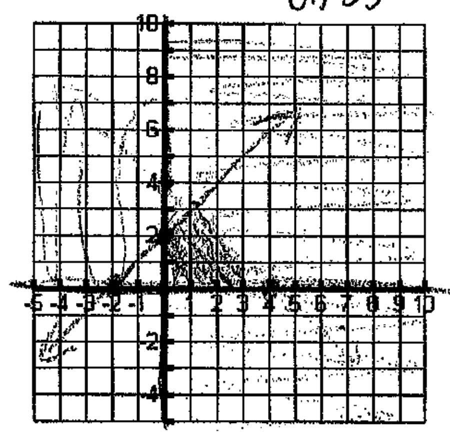
   $x - y \ge -2$

   $(0,2)$ $3 \cdot 0 + 2 \cdot 2 = 4$

   $(0,0)$ $= 0$ Min

   $(4,0)$ $3 \cdot 4 + 2 \cdot 0 = 12$ Max

   $(1,3)$ $3 \cdot 1 + 2 \cdot 3 = 9$



3. **Objective function:**

   $C = 5x - 2y$

   Constraints:

   $x \ge 0$

   $y \ge 0$

   $2x + y \le 8$ $y \le -2x + 8$

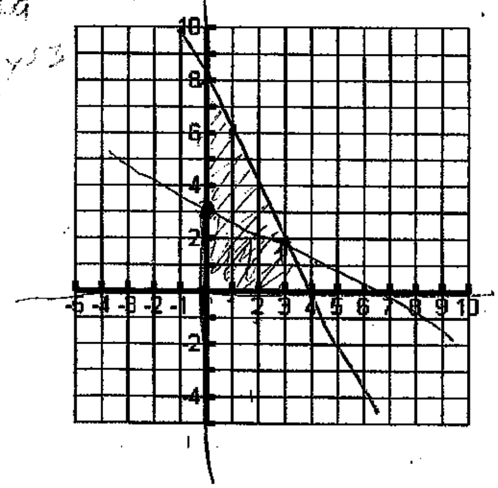
   $x + 3y \le 9$

   $3y \le -x + 9$

   $y \le -\frac{1}{3}x + 3$

   $(0,0)$ $(3,0)$ 15

   $(0,3)$ $(3,2)$ 9



4. **Objective function:**

   $C = 3x - y$

   Constraints:

   $y \le 4$

   $x + y \ge 2$ $y \ge -x + 2$

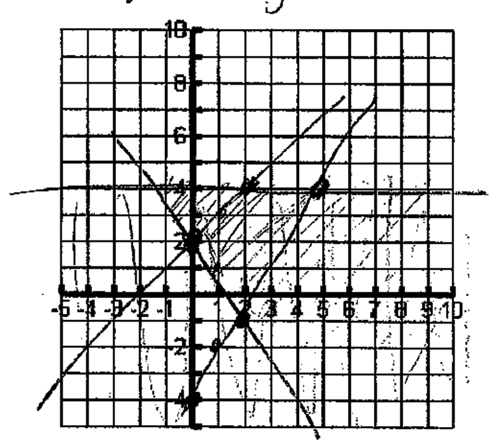
   $2x - y \le 4$ $-y \le -2x + 4$ $y \ge 2x - 4$

   $-x + y \le 2$ $y \le x + 2$

   $(0,2)$ $(2,4)$

   $(2,-1)$

   $(5,4)$



2

LINEAR PROGRAMMI NG APPLICATIONS WS NAME________________

On the provided answer sheet outline, include the following for each problem below.
(a) Define your variables
(b) Write the objective function
(c) Write the constraints (system of inequalities)
(d) Graph the constraints on graph paper
(e) Identify the ordered pairs that represent the vertices of polygon in your graph
(f) Evaluate the objective function using each of your vertices to determine the maximum/minimum value
(g) Answer the question using a complete sentence

1. A travel agent is arranging a ski trip for a local club. The agent can provide for at most 10 people to go on the trip. The trip cannot be finalized unless there are at least 4 men and 3 women committed to going. The agent makes a profit of $12.25 for each woman and $15.40 for each man she books. How many men and how many women need to go for the agent to make the maximum possible profit? What will be the agent's profit?

2. A carpentry shop makes end tables and coffee tables. Each week the shop must complete at least 9 end tables and at least 13 coffee tables to be shipped to furniture stores. The shop can produce at most 30 end tables and coffee tables combined each week. The shop sells end tables for $120 and coffee tables for $150. How many of each type of table should be produced each week to maximize the store's profit?

x = end tables $x \geq 9$ $x+y \leq 30$
y = coffee tables $y \geq 13$ $\boxed{120x + 150y = A}$

3. Farm Lady Frances wants to raise pigs and goats on a little section of her land. She can handle no more than 16 animals total and no more than 12 goats. It will cost Frances $5 to raise each pig and $2 to raise each goat and she has only $50 that can be used for this purpose. If Farm Lady Frances can make a profit of $7.50 per goat and $4.75 per pig, how many of each should she raise to maximize her profit? What will be her maximum profit?

x = pigs $x + y \leq 16$ $5x + 2y \leq 50$
y = goats $y \leq 12$ $\boxed{7.50y + 4.75x = A}$

4. A company produces mopeds and bicycles. It must produce at least 10 mopeds each month; however, the equipment the company owns cannot produce more than 60 mopeds each month. The maximum number of bicycles that can be produced using the company's equipment is 120. The production of mopeds and bicycles combined cannot exceed 160. The profit on a moped is $134 and the profit on a bicycle is $20. How many of each should the company manufacture each month to maximize their profit?

x = mopeds $x \geq 10$ $x \leq 60$ $x+y \leq 160$
y = bicycles $y \leq 120$

Objective: $134x + 20y$

3

5. Mrs. Woods' Biscuit Factory makes two types of biscuits--Biscuit Jumbos and Mini Bite Biscuits. The oven at the factory can bake at most 200 biscuits each day. Each Jumbo requires 2 ounces of flour and each Mini Bite requires 1 ounce of flour. There are 300 ounces of flour available daily for the biscuit production. Mrs. Woods makes a profit of 10 cents on each Biscuit Jumbo and 8 cents on each Mini Bite Biscuit. How many of each type should the factory produce daily to earn the greatest profit?

6. A parking area at a concert venue covers 600 square meters. Each car requires 6 square meters of space and each bus requires 30 square meters of space. No more than 60 vehicles can be parked in the area. The parking fee for each car is $2.50 and the fee for parking a bus is $7.50. How many cars and how many buses should be parked in the area to maximize the lot's income?

7. Your midterm exam in English consists of short answer as well as essay questions. Each short answer question is worth 5 points and each essay question is worth 15 points. You may choose to answer up to 20 questions of any type. It takes 2 minutes to answer each short answer question and 12 minutes to answer each essay question. You have one hour to complete the test. Assuming that you answer all of the questions that you attempt correctly, how many of each type should you choose to answer to earn the highest test score?

ANSWERS:

1. 7 men, 3 women; profit $144.55
2. 9 end tables, 21 coffee tables
3. 4 pigs, 12 goats; profit $109
4. 60 mopeds, 100 bicycles
5. 100 Biscuit Jumbos, 100 Mini Bite Biscuits
6. 50 cars, 10 buses
7. 18 short answer questions, 2 essay questions

Applications of Linear Programming Name______________________

1. x = men

y = women

Objective Function: 15.40x + 12.25y

Constraints: $x + y \le 10$

$x \ge 4$

$y \ge 3$

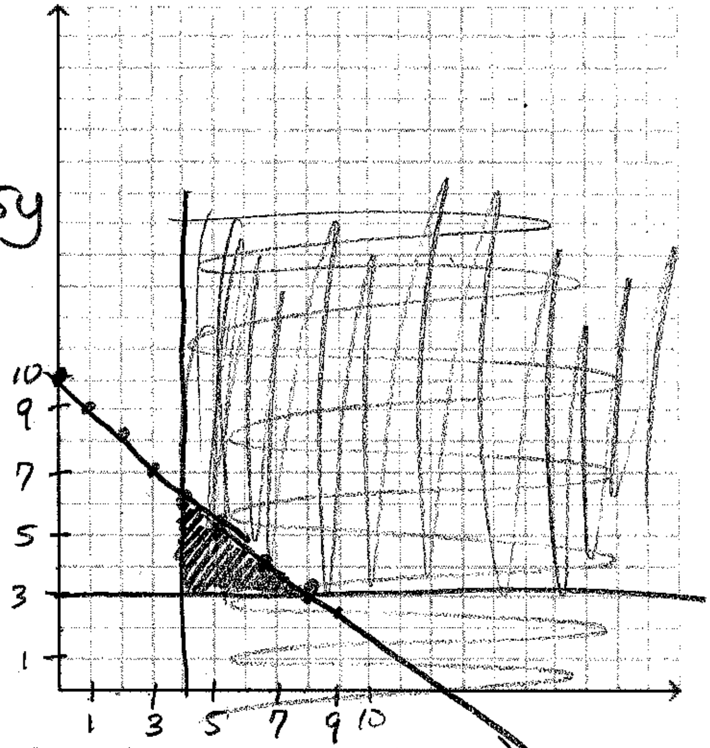
$y \le -x + 10$

How many of Men? 7

How many of Women? 3

Max profit? \$144.55

Concluding sentence:



(4,3) = 98.35 (7,3) = 144.55

(4,6) = 135.10

2. x = end tables

y = coffee tables

Objective Function: 120x + 150y

Constraints: $x \ge 9$

$y \ge 13$

$x + y \le 30$

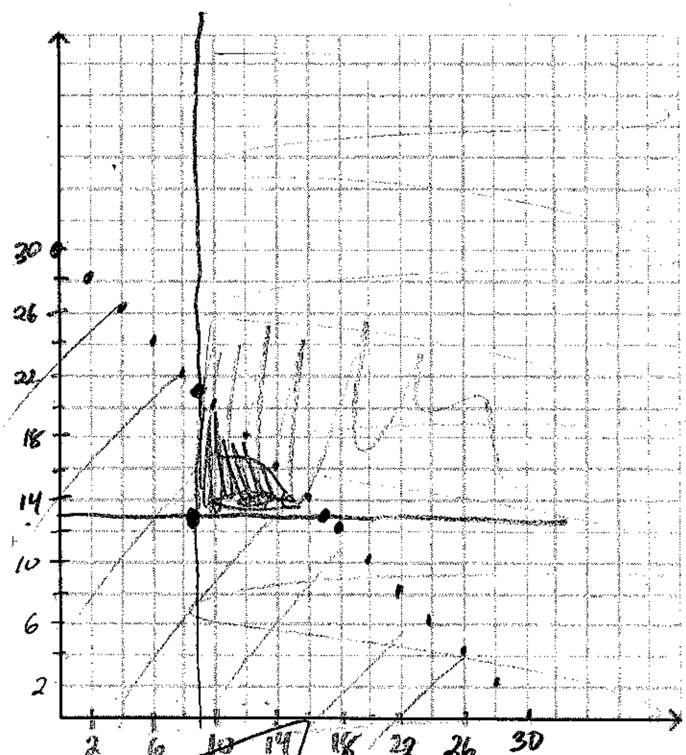
$y \le -x + 30$

How many end tables? 9

How many coffee tables? 21

Maximum profit? \$4230

Concluding sentence:



$\boxed{(9,21)}$

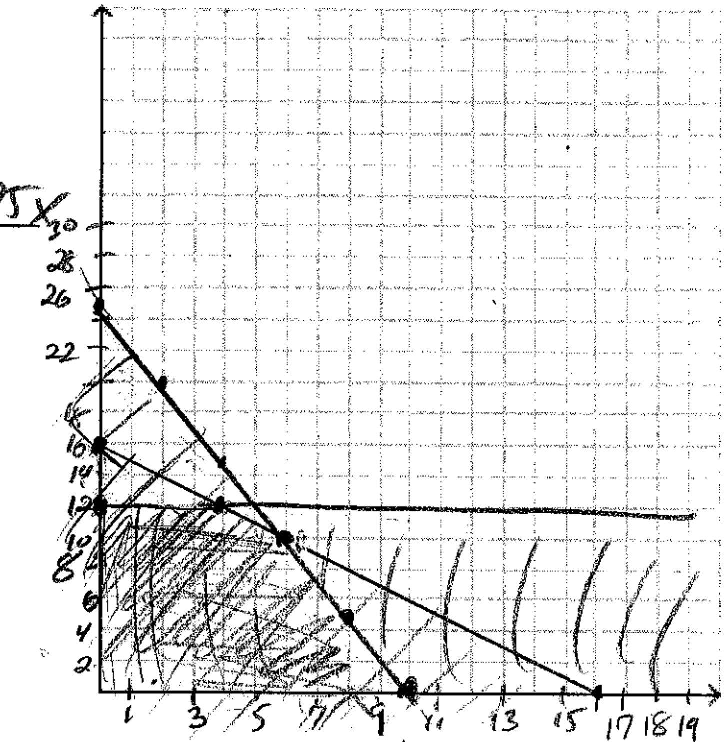
(9,13)

(17,13)

3. x = Pigs

y = goats

Objective Function: $7.50y + 4.75x$



Constraints: $x + y \le 16$

$\boxed{y \le -\frac{5}{2}x + 25}$ $y \le 12$

$\boxed{y \le -x + 16}$ $5x + 2y \le 50$

$2y \le 5x + 50$

How many pigs? 4

How many goats? 12

Max Profit? 109

Concluding sentence:

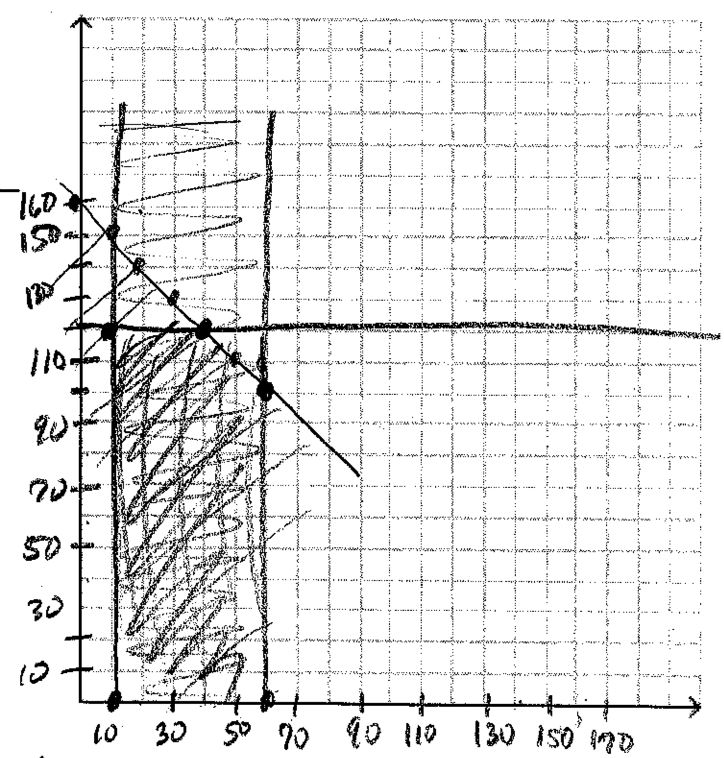
(0,12) (6,10)

$\boxed{(4,12)}$ (10,0)

4. x = mopeds

y = bicycles

Objective Function: $134x + 20y$



Constraints: $x \ge 10$

$x \le 60$

$y \le -x + 160$ $x + y \le 160$

$y \le 120$

How many mopeds? 60

How many bikes? 100

Max profit? \$10,040

Concluding sentence:

(10,0) (60, 100) (10,120)

(60,0) (40, 120)

5. x = Jumbo

y = mini

Objective Function: $.10x + .08y$

Constraints: $x + y \le 200$

$2x + 1y \le 300$

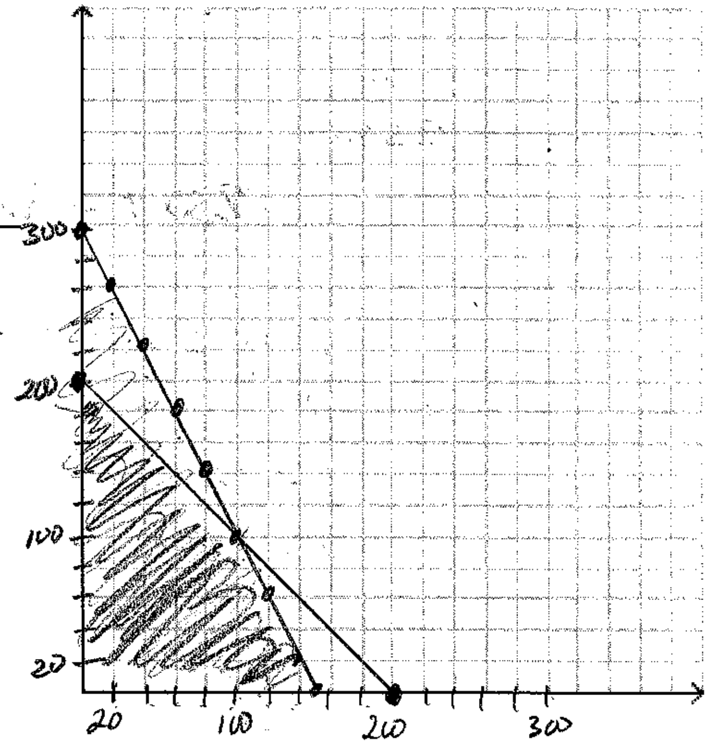
$y \le -2x + 300$

How many jumbos? 100

How many minis? 100

Max Profit? \$18

Concluding sentence:



(0, 200) (150, 0)

(100, 100)

6. x = car

y = bus

Objective Function: $2.50x + 7.50y$

Constraints: $6x + 30y \le 600$

$x + y \le 60$

$y \le -x + 60$

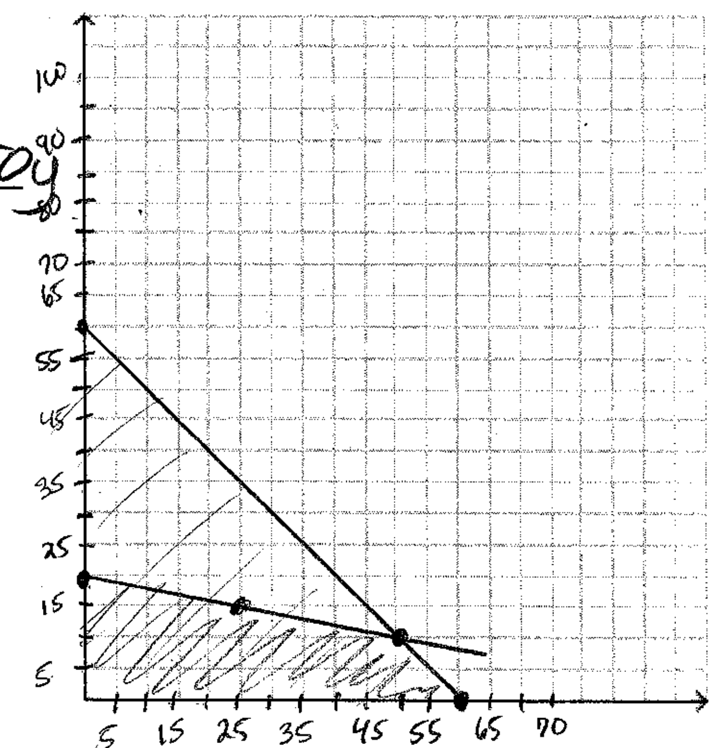
$y \le \frac{1}{5}x + 20$

How many cars? 50

How many buses? 10

Max profit? \$200

Concluding sentence:



(0, 20) (25, 15) = 175

(0, 60) (50, 10) = 200

7. x = Short answer

y = essay

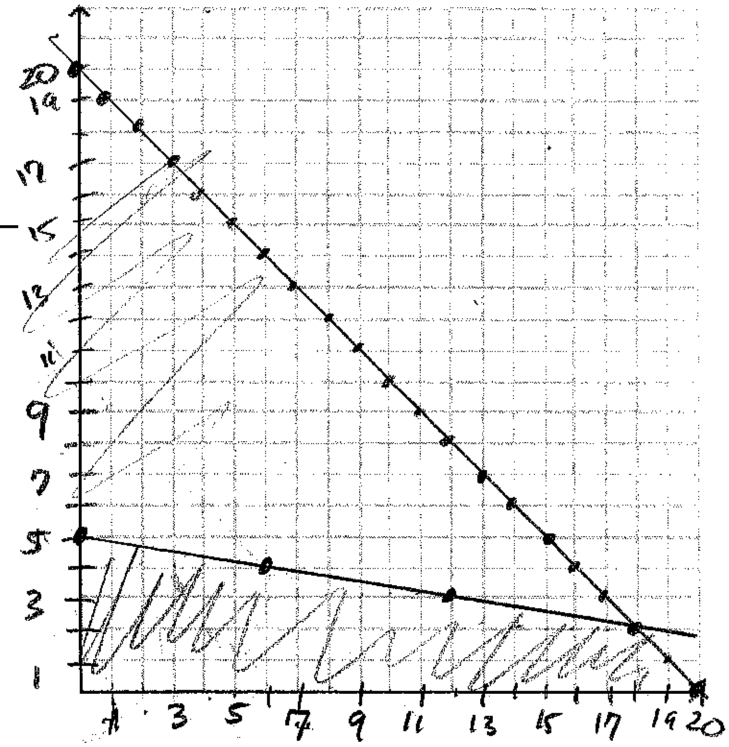
Objective Function: $5x + 15y$

Constraints: $x + y \leq 20$

$2x + 12y \leq 60$

$y \leq -x + 20$

$y \leq -\frac{1}{6}x + 5$



How many short answer? 18

How many essay? 2

Highest score? 120

Concluding sentence:

$(0,5)$

$(18,2)$

$(20,0)$

$$\frac{12y}{12} \leq \frac{-2x}{12} + \frac{60}{12}$$

$$y \leq -\frac{1x}{6} + 5$$

Advanced Algebra
Linear Programming – CFA

Name ____________________

1. Find the minimum and maximum values of the function subject to the given constraints. (10 points)

Objective function:
$C = 2x + 5y$

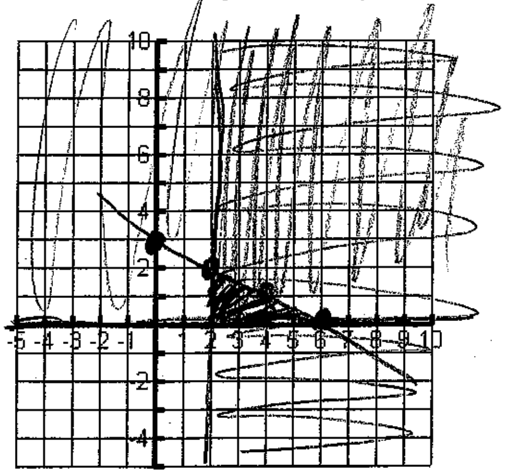
Constraints:
$x \ge 2$

$y \ge 0$

$4x + 8y \le 24$

$\frac{8y}{8} \le \frac{-4x}{8} + \frac{24}{8}$

$y \le -\frac{1}{2}x + 3$



Min
$\boxed{(2,0)} \rightarrow 4$

$(6,0) \rightarrow 12$

$\boxed{(2,2)} \rightarrow 14$
Max

2. For the following application, define your variables, write the objective function and the constraints ONLY. No need to graph, etc. this time! (20 points)

John works two jobs – one at McD's and one at Wendy's. He works at least 10 hours at McD's and at least 12 hours at Wendy's. He does not want to work more than 60 hours per week. John makes \$6.50 per hour at McD's and \$6.45 per hour at Wendy's. How many hours should he work at each job to maximize his paycheck?

a. Define the variables:

x = McD's

y = Wendys

b. Objective function:

P(x, y) = $6.50x + 6.45y$

c. Constraints:

$x \ge 10$

$y \ge 12$

$x + y \le 60$

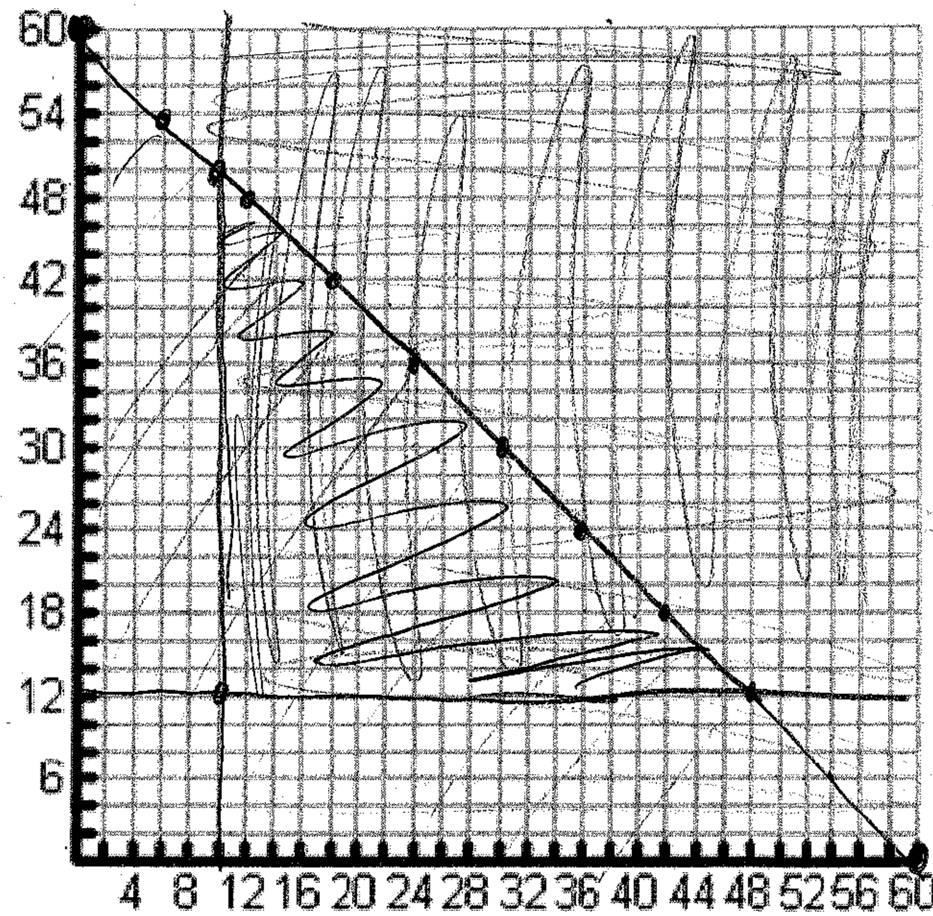
$y \le -x + 60$

d. Function Values:

P(10, 12) = 142.40

P(48, 12) = 389.40

P(10, 48) = 374.60



e. Write your answer in the form of a complete sentence!

John should work 48 hours at McD's + 12 hours at wendys, to make the most amount of Money of \$384:40 per week. 5

Advanced Algebra **Name** ______________________

Test 3 – Linear Programming – Review

Linear Programming Review:

1. Find the minimum value and maximum value of C subject to the given constraints.

$(0,6) \to -18$

$(0,2) \to -6$

$(2,4) \to -8$

$C(x, y) = 2x - 3y$

Constraint:

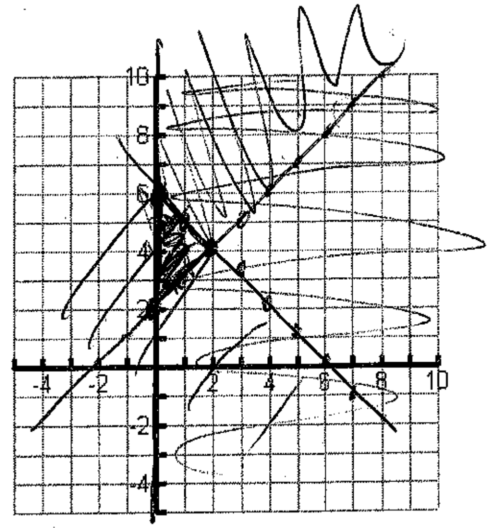
$x \geq 0$

$-x + y \geq 2$ $\quad y \geq x + 2$

$x + y \leq 6$ $\quad y \leq -x + 6$

min $\to (0,6)$

max $\to (0,2)$



2. Find the minimum value and maximum value of C subject to the given constraints.

$C(x, y) = x - 4y$

$(0,0) \to 0$

$\boxed{(4,0) \to 4 \text{ max}}$

$\boxed{(0,5) \to -20 \text{ min}}$

Constraint:

$x \geq 0$

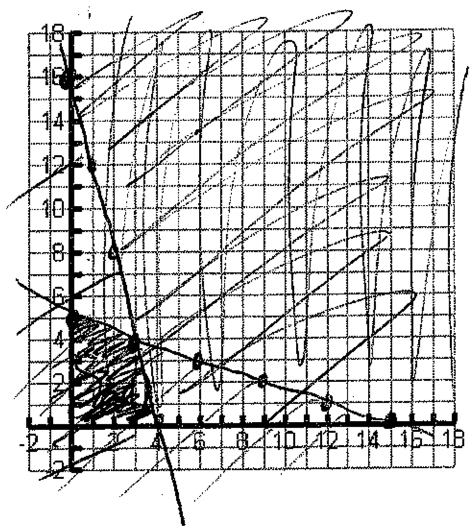
$y \geq 0$

$x + 3y \leq 15$

$4x + y \leq 16$ $\quad \to y \leq -4x + 16$

$3y \leq -x + 15$

$y \leq -\frac{1}{3}x + 5$



3. Find the minimum value and maximum value of C subject to the given constraints.

$C(x, y) = 2x + 5y$

$\boxed{\text{min } (-5,4) \to 10}$

Constraint:

$(5,4) \to 30$

$x \leq 5$

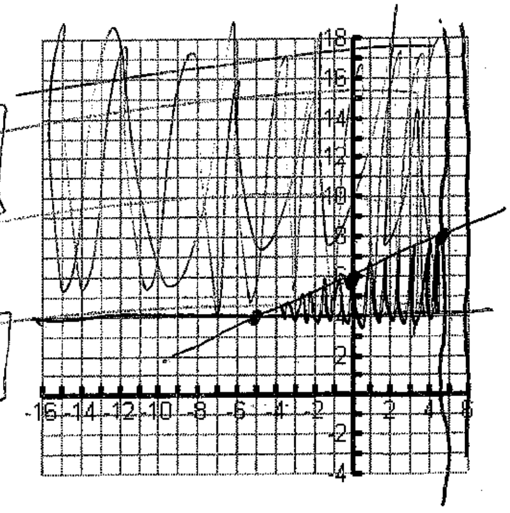
$y \geq 4$

$-2x + 5y \leq 30$

$\boxed{\text{max } (5,8) \to 50}$

$\frac{5y}{5} \leq \frac{2x}{5} + \frac{30}{5}$

$y \leq \frac{2x}{5} + 6$



6

4. Toy wagons are made to sell at a craft fair. It takes 4 hours to make a small wagon and 6 hours to make a large wagon. The craft booth owner has no more than 60 hours available to make wagons and wants to have at least 6 small wagons to sell. The owner will make a profit of $12 for a small wagon and $20 for a large wagon. How many of each size should be made to maximize profit?

x = # of Small

y = # of large

Objective Function:

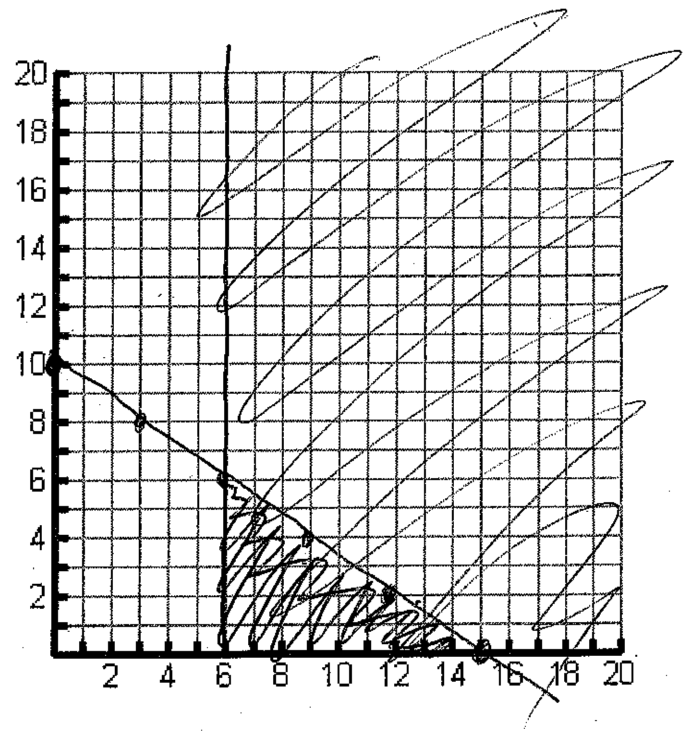
P(x, y) = $12x + 20y$

Constraints:

a) $4x + 6y \le 60$

b) $x \ge 6$

c) ______



Vertices / Evaluate the Function:

P(6, 0) = 72

P(6, 6) = 192

P(15, 0) = 180

$$\frac{6y}{6} \le \frac{-4x}{6} + \frac{60}{6}$$

$$y \le -\frac{2}{3}x + 10$$

How many small wagons? 6

How many large wagons? 6

Max profit? 192

Concluding sentence:

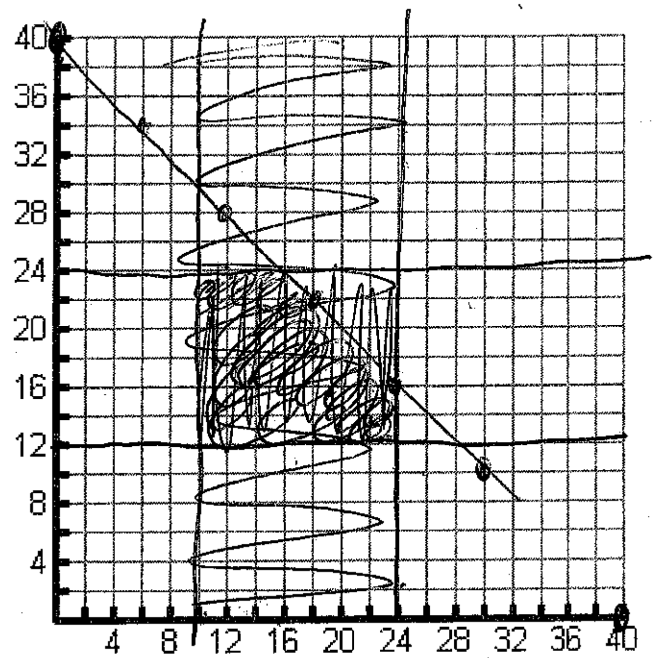
The owner should make & sell 6 small & 6 large wagons to maximize a profit of $192.

7

5. John works two jobs – one at McD's and one at Wendy's. He works at least 10 hours at McD's and at least 12 hours at Wendy's but no more than 24 hours at either. He does not want to work more than 40 hours per week. John makes $6.50 per hour at McD's and $6.45 per hour at Wendy's. How many hours should he work at each job to maximize his paycheck?

x = # of McD's

y = # of Wendys



Objective Function:

P(x, y) = $6.50x + 6.45y$

Constraints:

a) $10 \le x \le 24$

b) $12 \le y \le 24$

c) $x + y \le 40$

Vertices / Evaluate the Function:

P(10,12) = $142.40

P(24,12) = 233.40

— P(24,16) = 259.20

P(16,24) = 258.50

P(10,24) = 219.80

How many hours at McD's? 24

How many hours at Wendy's? 16

Max paycheck? 259.20

Concluding sentence:

John should work 24 hrs @ McD's + 16 at Wendy's to make a max of $259.20

8