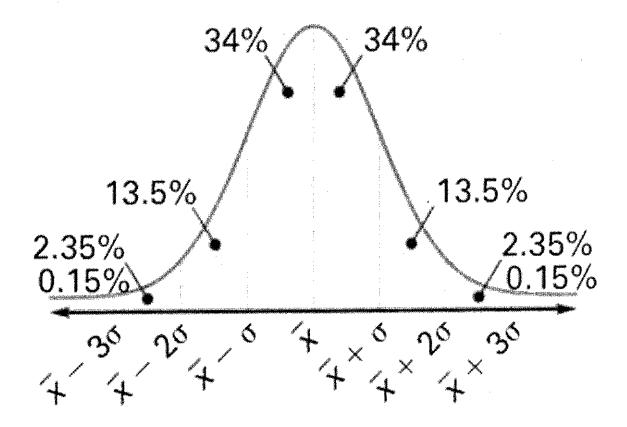
# **Normal Distribution and Confidence Intervals**

Def'n:NORMAL DISTRIBUTION is one type of distribution of data, which describes how data clusters around the mean. The graph of a Normal Distribution is bell-shaped and symmetric, with the peak at the mean.



The EMPIRICAL RULE (also known as the 68 - 95 - 99.7 rule) applies to all Normal Distributions. It dictates the following:

- 68% of all observations fall within one standard deviation of the mean
- 95% of all observations fall within two standard deviations of the mean
- 99.7% of all observations fall within three standard deviations of the mean

#### Ex #1

A normal distribution has a mean of 27 and a standard deviation of 5. Find the probability that a randomly selected x-value from the distribution is in the interval of 17 and 37.

#### Ex #2

A normal distribution has a mean of x and a standard deviation of  $\sigma$ . Find the probability that a randomly selected x-value from the distribution is in the interval  $x \le x - 2\sigma$ .

#### Ex #3

The distribution of heights of young women aged 18 to 24 is approximately normal with a mean ( x ) of 64.5 inches and a

a) Draw a Normal Distribution curve to represent this data, clearly showing the application of the empirical rule.

standard deviation ( $\sigma$ ) of 2.5 inches.

- b) What percent of woman are taller than 69.5 inches?
- c) Between what heights do the middle 95% of women fall?
- d) What percent of woman are shorter than 62 inches?
- e) A height of 67 inches corresponds to what percentile of adult female American heights?

- 1) For a normally distributed set of data with mean 74 and standard deviation 8, find the following probabilities.
  - a.  $P(58 \le x \le 74)$
  - b.  $P(66 \le x \le 90)$
  - c.  $P(x \ge 90)$
  - d.  $P(x \le 82)$
- 2) A normal distribution has a mean of 25 and a standard deviation of 5.

Find the percent of values are ...

- a. between 20 and 30
- b. between 10 and 25
- c. at least 20
- d. at most 30
- e. What values make up the middle 95%?
- 3) The weights of 1800 fish in a lake are normally distributed with a mean of 3 kg and a standard deviation of 0.6 kg.
  - a. About how many of the fish weigh 2.4 kg or more?
  - b. About how many of the fish weigh less than 1.8 kg?
  - c. About how many of the fish weigh between 2.4 kg and 4.2 kg?
  - d. About how many of the fish weigh between 1.8 kg and 4.8 kg?
- 4) A forester sampled 27 trees in a wooded area and found that the mean diameter of the trees is 15.4 inches with a standard deviation of 3.7 inches. Suppose that this sample of trees provides an accurate description of the entire forest and that the trees are normally distributed.
  - a. What is the range of diameters for the middle 95% of the trees in the forest?
  - b. What percent of the trees in the forest should be less than 8 inches in diameter?
  - c. What is the probability that a selected tree will be between 11.7 and 15.4 inches in diameter?
  - d. There are approximately 1540 trees in the forest. About <u>how many</u> trees are over 19.1 inches in diameter?

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## Empirical Rule Worksheet

- 1. Given an approximately normal distribution what percentage of all values are less than 1 standard deviation from the mean?
- 2. Given an approximately normal distribution what percentage of all values are less than 2 standard deviations from the mean?
- 3. Given an approximately normal distribution what percentage of all values are less than 3 standard deviations from the mean?
- 4. Given an approximately normal distribution with a mean of 175 and a standard deviation of 37.
  - a) Draw a normal curve and label 1, 2, and 3 standard deviations on both sides on the mean.

- b) What percent of values are within the interval (138, 212)?
- c) What percent of values are within the interval (101, 249)?
- d) What percent of values are within the interval (64, 286)?
- e) What percent of values outside the interval (138, 212)?
- f) What percent of values are outside the interval (101, 249)?
- g) What percent of values are outside the interval (64, 286)?

Name:	Date:
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- 5. Given an approximately normal distribution with a mean of 121 and a standard deviation of 40.
  - a) Draw a normal curve and label 1, 2, and 3 standard deviations on both sides on the mean.

- b) What interval contains 68% of all values?
- c) What interval contains 95% of all values?
- d) What interval contains 99.7% of all values?
- e) What percent of values are above 201?
- f) What percent of values are below 81?
- 6. Given an approximately normal distribution with a mean of 159 and a standard deviation of 70.
  - a) What percent of values are within the interval (89, 299)?
  - b) What percent of values are within the interval (19, 159)?
  - c) What interval contains 99.7% of all values?
  - d) What percent of values are above 229?
  - e) What percent of values are outside the interval (19, 229)?

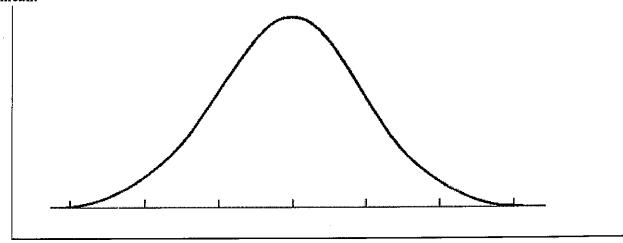
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## - Empirical Rule and Normal Distribution

In a normal distribution, what percent of the values lie:

1. below the mean?

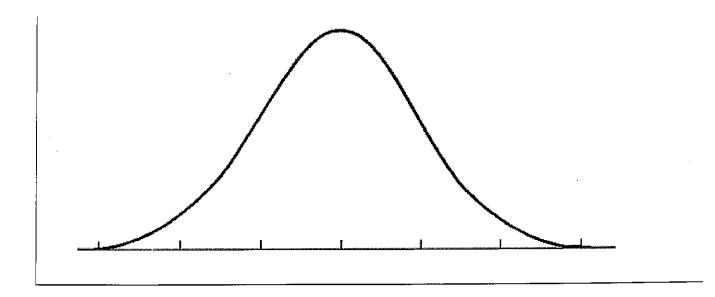
- 2. above the mean?
- 3. within one standard deviation of the mean?
- 4. within two standard deviations of the mean?
- 5. within three standard deviations of the mean?
- 6. 2000 freshmen at State University took a biology test. The scores were distributed normally with a mean of 70 and a standard deviation of 5. Label the mean and three standard deviations from the mean.



Answer the following questions based on the data:

- a) What percentage of scores are between scores 65 and 75?
- b) What percentage of scores are between scores 60 and 70?
- c) What percentage of scores are between scores 60 and 85?
- d) What percentage of scores is less than a score of 55?
- e) What percentage of scores is greater than a score of 80?
- f) Approximately how many biology students scored between 60 and 70?
- g) Approximately how many biology students scored between 55 and 60?

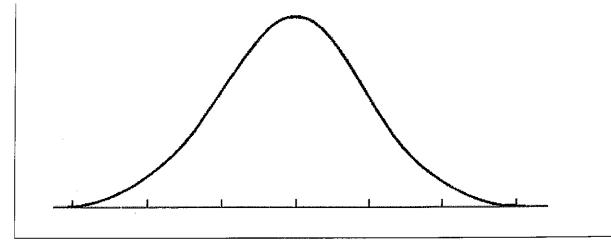
7. 500 juniors at Central High School took the ACT last year. The scores were distributed normally with a mean of 24 and a standard deviation of 4. Label the mean and three standard deviations from the mean.



## Answer the following questions based on the data:

- a) What percentage of scores are between scores 20 and 28?
- b) What percentage of scores are between scores 16 and 32?
- c) What percentage of scores are between scores 16 and 28?
- d) What percentage of scores is less than a score of 12?
- e) What percentage of scores is greater than a score of 24?
- f) Approximately how many juniors scored between 24 and 28?
- g) Approximately how many juniors scored between 20 and 28?
- h) Approximately how many juniors scored between 24 and 32?
- i) Approximately how many juniors scored between 16 and 20?
- j) Approximately how many juniors scored higher than 32?

8. 500 freshmen at Schaumburg High School took an algebra test. The scores were distributed normally with a mean of 75 and a standard deviation of 7. Label the mean and three standard deviations from the mean.



Answer the following questions based on the data:

- a) What percentage of scores are between scores 61 and 82?
- b) What percentage of scores are between scores 75 and 82?
- c) What percentage of scores are between scores 61 and 89?
- d) What percentage of scores is less than a score of 61?
- e) What percentage of scores is greater than a score of 96?
- f) Approximately how many algebra students scored between 61 and 89?
- g) Approximately how many algebra students scored between 68 and 82?
- h) Approximately how many algebra students scored between 61 and 75?
- i) Approximately how many algebra students scored between 89 and 96?
- j) Approximately how many algebra students scored higher than 89?

## 9. Here are the scores for a recent test in M414 Statistics.

90	90	95	100	80	80	75	80	70	60	95	100	100
100	75	80	90	90	90	70	70	80	85	90	90	85

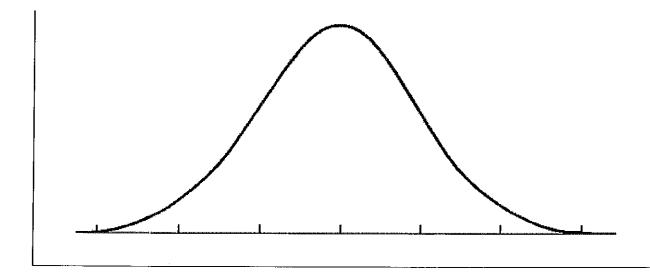
Answer the following questions regarding this set of data.

Median =	Mean =	Mode =

How many scores are within 1 standard deviation of the mean? \_\_\_\_\_

How many scores are within 2 standard deviations of the mean?

## Hint: Drawing the curve will help answer the last two questions!!!



# NORMAL DISTRIBUTIONS



## Z - SCORE

- When the data value in question does not fall on exactly one, two, or three standard deviations from the mean, we can no longer rely on The Empirical Rule.
- What do we do?
- Use z-Score!

$$z = \frac{x - \overline{x}}{\sigma}$$

## Normal Distributions and z-Scores

(using a z-table)

## Example 1:

Find the probability a data value falls below 1.15 standard deviations from the mean.

## Example 2:

Find the probability a data value falls below 0.82 standard deviations from the mean.

#### Example 3:

A normal distribution has a mean of 70 and a standard deviation of 10. Find the probability that a randomly selected data value from the distribution is in the given interval. Draw a sketch to represent each interval. Use your z-table for probabilities.

- a.  $P(x \le 65)$
- b.  $P(x \ge 47)$
- c.  $P(39 \le x \le 82)$

## Example 4:

The data for the SAT is normally distributed with a mean of 1000 and standard deviation 180.

- (a) What percent of students that score under 1200?
- (b) What percent of test takers score ABOVE 1200?
- (c) What is the probability that a student scores between 900 and 1300?

What is a standard normal distribution?

Advanced Algebra		
Normal Distributions	WS	3

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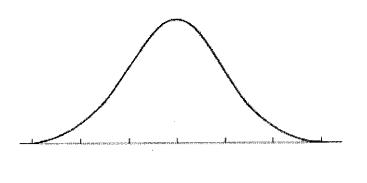
#### Non-Standard Normal Distribution

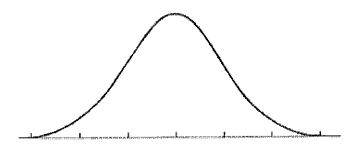
For each question, draw a sketch and find each probability.

- 1. According to a survey conducted by television advertisers, the average adult American watches an average of 6.98 hours of television per day. The data is normally distributed with a standard deviation of 3.80 hours. Find the probability that a randomly selected person watches more than 8.00 hours of television per day.
- 2. Insurance companies have determined that US males between the ages of 16 and 24 drive an average of 10,718 miles each year with a standard deviation of 3763 miles. Assume the data is normally distributed. For a randomly selected male in that age group, find the probability that he drives less than 12,000 miles per year.
- 3. A company that manufactures sleeping bags says that the average amount of down in an adult sleeping bag is 32 ounces with a standard deviation of 0.9 ounces. What is the probability that a bag chosen at random has more than 33.2 ounces of down?
- 4. The heights of six-year old girls are normally distributed with a mean of 117.80 cm and a standard deviation of 5.52 cm. Find the probability that a randomly selected six-year old girl has a height between 117.80 cm and 120.56 cm.
- 5. The average heating bill for a residential area is \$123 for the month of November with a standard deviation of \$8. If the amount of the heating bills are normally distributed, find the probability that the average bill for a randomly selected resident is more than \$125.
- 6. An IQ test has a mean of 100 with a standard deviation of 15. What is the probability that a randomly selected adult has an IQ between 85 and 115?
- 7. The average adult spends 5 hours per week on a home computer, with a standard deviation of 1 hour. What percent of adults spend more than 6 hours per week on their home computer?

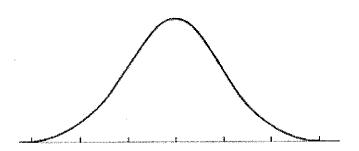
## **Non-Standard Normal Distribution**

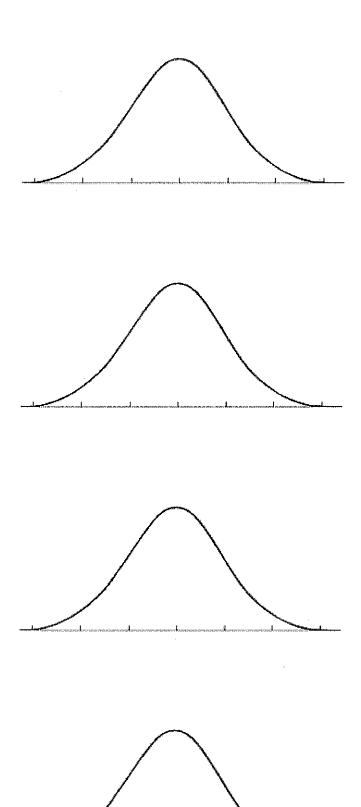
1.	Assume that body temperatures of normal healthy persons are normally distributed with a mean of 98.2°F and a standard deviation of 0.62°F. If we define a fever to be a body temperature above 100°F, what percentage of normal and healthy persons would be considered to have a fever?
2.	On one measure of attractiveness, scores are normally distributed with a mean of 5.9 and a standard deviation of 0.7. What percent of the population has a measure of attractiveness greater than 7.0?
3.	Scores on an anti-aircraft exam are normally distributed with a mean of 99.6 and a standard deviation of 25.8. For a randomly selected subject, find the probability that a score will fall between 110.00 and 150.00.
4.	For a certain population, scores on the Miller Analogies Test are normally distributed with a mean of 58.8 and a standard deviation of 15.9. If subjects who score below 27.00 are to be given special training, what is the percentage of subjects who will be given the special training?
5.	Scores on the biology portion of the Medical College Admissions Test are normally distributed with a mean of 8.0 and a standard deviation of 2.6. Among 600 individuals taking this test, how many are expected to score between 6.0 and 7.0?
6.	The Chemco Company, which manufactures car tires, finds that the tires last distances that are normally distributed with a mean of 35,600 mi. and a standard deviation of 4275 mi. The manufacturer wants to guarantee the tires so that only 3% will be replaced because of failure before the guaranteed number of miles. For how many miles should the tires be guaranteed?
7.	Two different one mile routes were set up for 1200 P.E. students. The times to complete the downhill course are normally distributed with a mean of 420 seconds and a standard deviation is 75 seconds. What percentage of students finished the downhill course between 350 and 550 seconds?











# Standard Error, Margin of Error and Confidence Intervals

Aug 29-10:14 AM

## Standard Error of the Mean:

Definition: An indication of how well the mean of a sample estimates the mean of a population.

$$SEM = \frac{\sigma}{\sqrt{n}}$$

When we take a sample from a population and find its mean, it is only the mean of that particular sample. No matter how good the sample is, it does not represent the population in its entirety - that is why we usually make inferences with statistics.

## 13. Notes - SE and Margin of Error.notebook

Standard Error of the Mean tells us what the spread of sample means should look like if we took many samples.

For example: roll a 6 sided dice 3 times and add up the sum. What would the mean of the distrubtion be?

 Trial 1: 2, 4, 6 = 12
 Trial 4: 2, 2, 2 = 6
 Trial 7: 5, 1, 1 = 7

 Trial 2: 5, 3, 5 = 13
 Trial 5: 6, 2, 3 = 11
 Trial 8: 3, 4, 5 = 12

 Trial 3: 2, 3, 5 = 10
 Trial 6: 2, 3, 4 = 9
 Trial 9: 1, 6, 4 = 11

12 + 13 + 10 = 35 6 + 11 + 9 = 26 7 + 12 + 11 = 30

Each group of 3 trials made up a different sum, which is different that what the population mean would be (10.5). The SEM tells us that this is expected, but that it is expected within a certain range. For this sample problem, the SEM for the first 3 trial groups would be:

 $\frac{\sigma}{\sqrt{n}} = \frac{1.53}{\sqrt{3}} = .88$  11.67 ± .88 = {10.79, 12.55}

If we included all 9 trials individually, the SEM would be:  $\frac{\sigma}{\sqrt{n}} = \frac{2.37}{\sqrt{9}} = .79$ 

10.11 ± .79 = {9.32, 10.9}

So as you see, when we increase the number of trials, our variation from the true mean should be lowered since we are looking at something closer and closer to the population when we increase the sample size.

Sep 2-3:41 PM

How can we use the SEM?

We use the SEM to calculate the margin of error for a statistic

Since the SEM helps to define what the population mean should be in relation to what our sample is, we can come up with a margin of error for a sample

Formula:

$$E = Z \cdot \frac{\sigma}{\sqrt{n}}$$

In the formula, E represents the margin of error. Z represents our critical value (in the form of a z-score), sigma represents our standard deviation and n represents the number of samples taken.

Confidence Level	Critical Value
99%	2.58
95%	1.96
90%	1.65
80%	1.28

#### Example 1:

A random sample of 35 students makes an average of \$2,500 during the summer with a standard deviation of \$345. Find the margin of error for a confidence level of 90%.

mean = 2500 standard deviation = 345 n = 35

SEM =

Margin of Error =

Sep 2-4:07 PM

We can also use confidence intervals to give us an appropriate range of values which should include the mean. This is called a confidence interval. The confidence interval using the margin of error to create a range of values.

Formula:

$$CI = \overline{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

For example:

1. You want to rent an unfurnished one-bedroom apartment in Boston next year. The mean monthly rent for a simple random sample of 32 apartments advertised in the local newspaper is \$1,400. Assume that the standard deviation is known to be \$220.

Find the margin of error for a 99% confidence interval:

Give the 99% confidence interval for mean rent of an apartment in Boston:

## 13. Notes - SE and Margin of Error.notebook

2. We have IQ test scores of 31 seventh-grade girls in a Georgia school district. We have calculated that sample mean is 105.84 and the standard deviation is 14.27.
Find the margin of error for a 95% confidence interval:
Give a 95% confidence interval for the average score in the population.
Con 2 2:40 DM

Sep 3-2:18 PM

Name		
Date	Period	

For the following data sets, you rolled a single dice and recorded the results. Calculate the Standard Error of the Mean for each of the data sets.

1. {1, 2, 5, 5, 6}

2. {1, 2, 3, 4, 4, 5, 5, 6, 6}

3. {1, 1, 2, 3, 3, 3, 4, 5, 5, 6}

4. You sample the weights of 500 bananas in a shipping crate and find the mean weight is 95g with a standard deviation of 6g. Find a 90% confidence interval for the actual mean weight of bananas.

5. Looking through the new car ads in the newspaper, you find the average cost for the car you are looking to buy from 20 ads is \$23,749 with a standard deviation of \$896. Find a 99% confidence interval for the true mean cost of your car.

6. A sample of 40 tires has a mean circumference of 116cm with a standard deviation of 2.1cm. Find an 80% confidence interval for the population mean circumference of the tires.

CCGPS Advanced Algebra
Confidence Intervals

Name		 

- 1) On a study on body temperatures, 106 temperatures were taken. The mean and standard deviation of these 106 scores were 98.44°F and 0.30°F, respectively. Construct a 95% confidence interval for the mean of all body temperatures.
- 2) In a time use study 20 randomly selected managers were found to spend a mean time of 2.4 hours per day on paperwork. The standard deviation of the 20 scores was 1.30 hours. Construct a 99% confidence interval for the mean time spent on paperwork by all managers.
- 3) A random sample of 19 women results in a mean height of 63.85 inches. Other studies have shown that women's heights are normally distributed with a standard deviation of 2.5 inches. Construct a 90% confidence interval for the mean height of all women.
- 4) The National Center for Education Statistics surveyed 4400 college graduates about the lengths of time required to earn their bachelor's degrees. The mean was 5.15 years and the standard deviation was 1.68 years. Based on the above information, construct a 90% confidence interval for the mean time required to earn a bachelor's degree by all college students.
- 5) A random sample of 60 female members of health clubs in Los Angles showed that they spend on average 4 hours per week doing physical exercise with a standard deviation of .75 hours. Find a 95% confidence interval for the population mean.
- 6) A random sample of 20 married women showed that the mean time spent on housework by them was 29.8 hours a week with a standard deviation of 6.7 hours. Find a 95% confidence interval for the mean time spent on housework per week by all married women.
- 7) A fleet of 100 airplanes has an air time (time spent flying) standard deviation of 14,9 hours. A sample of 32 of these planes gave a mean air time of 49 hours. Construct a 80% confidence interval on the mean air time for this fleet.
- Automotive engineers are continually improving their products/ Suppose a new type of brake light has been developed by General Motors. As part off a product safety evaluation program General Motors' engineers wish to estimate the mean driver response time to the new brake light. Fifty drivers are selected at random and the response time (in seconds) for each driver is recorded, yielding the following results:  $\bar{x} = .72$  and s = .022. Construct a 99% confidence interval for the mean response time.
- 9) A random sample of 45 life insurance policy holders showed that the average premiums paid on their life insurance policies was \$340 per year with a standard deviation of \$62. Construct a 90% confidence interval for the population mean.

- 10) A process has been developed that can transform ordinary iron into a kind of super iron called *metallic glass*. Metallic glass is three to four times stronger than the toughest steel alloys. To estimate the mean temperature,  $\mu$ , at which a particular type of metallic glass becomes brittle, 25 pieces of this metallic glass were randomly sampled from a recent production run. Each piece was subjected to higher and higher temperatures until it became brittle. The temperature at which brittleness first appeared was recorded for each piece in the sample. The following results were obtained:  $\bar{x} = 480^{\circ}\text{F}$  and  $s = 11^{\circ}\text{F}$ . Construct a 95% confidence interval to estimate  $\mu$ .
- 11) Health insurers and the federal government are both putting pressure on hospitals to shorten the average length of stay (LOS) of their patients. A random sample of 27 hospitals in one state had a mean LOS in 1998 of 3.8 days and a standard deviation of 1.2 days. Construct a 90% confidence interval to estimate the population mean of the LOS for the state's hospitals in 1998.
- 12) A random sample of 50, 8 ounce cups of black "Early Riser" coffee dispensed by a new machine gave a mean of 11.0 mg. of caffeine. It is known from previous studies that the standard deviation for 8 oz. cups of black "Early Riser" coffee dispensed by this machine was 7.1 mg. Construct a 90% confidence interval for the mean caffeine content for cups dispensed by this machine.
- 13) The U.S Bureau of the Census conducted a survey of 5000 people and found that the mean income for a person with a bachelor's degree was \$38,973. It is known from previous studies nationwide that the standard deviation in income for a person with a bachelor's degree is \$6,340. Construct a 95% confidence interval for the mean income nationwide for persons with a bachelor's degree.
- 14) A bank took a sample of 100 of its delinquent credit card accounts and found that the mean owed on these accounts was \$2,130. It is known that the standard deviation for all delinquent credit card accounts at this bank is \$578. Give a 99% confidence interval for the mean amount owed on all delinquent credit card accounts for this bank.
- 15) A random sample of 100 movie theaters showed that the mean price of a movie was \$7.00 with a standard deviation of \$.80. Construct a 90% confidence interval for the mean price of a movie.

Confidence Intervals for a Proportion

Sep 7-4:44 PM

Confidence Interval of a Proportion

Formula: 
$$\hat{p} \pm z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Similar to the Confidence Interval for the Mean, we are finding a range from which 99%, 95%, 90% or 80% of all SAMPLES of the same size would have the same proportion. This margin of error give us the ability to validate our findings statistically.

#### Example:

Alcoholism on college campuses is a very serious problem. But how common is it? A survey of 17,096 students in US four-year colleges collected information on drinking behavior and alcohol-related problems. (Henry Wechsler et al., "Health and Behavioral Consequences of Binge Drinking in College", Journal of the American Medical Association, 272 (1994). The researchers defined "frequent binge drinking" as having five or more drinks in a row three or more times in the past two weeks. According to this definition 3,314 students were classified as frequent binge drinkers. Find a 99% confidence interval for the true percentage of collegiate binge drinkers

First, we need to find a point-estimate for the statistic since one was not given to us.

Second we apply the point-estimate and 'n' to the formula

This solution gives us our 99% CI for the problem.

Sep 7-4:46 PM

## Example 2

An exit poll for a local election has Candidate A ahead with 53.2% of the vote. If they have polled 982 individuals, what is the 95% confidence interval for the percent of voters who have voted for Candidate A?

# CCGPS Advanced Algebra Confidence Intervals WS 3

Name	-
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- 1) In a Roper poll of 3000 working men, 56% said "they feel guilty that they don't spend more time with their families." Construct a 95% confidence interval for the proportion of all working men who hold this view.
- 2) In a *Time/CNN* telephone poll of 1012 adult Americans, 11% of the respondents said that Ronald Regan was a great president. Give a 99% confidence interval for the proportion of all adult Americans who think that Regan was a great president.
- 3) U.S. firms are increasingly looking to the international market for expansion and growth. A random sample of 15 of the *Fortune 1,000* firms is selected, and the percentage of their revenues from foreign sales is recorded for each. The mean is 23.8% and the standard deviation is 9.4%. Use a 99% confidence interval to estimate the mean percentage of foreign sales for all large U.S. firms.
- 4) In an exit poll, HLN declares that candidate A is ahead by a percentage of 52%. If the exit poll included 325 results, what is the margin of error at a 90% confidence level? Could you conclusively say that candidate A will win the election?
- 5) You are writing a story for the Greyhound Tracks about the new school lunch program. Your conduct a simple random survey (SRS) of 200 students and 79% of them said they did not like the new lunches. Construct a 80% confidence interval for the proportion of students who don't like the new lunch program. How would you say this is your article?
- 6) Dr. Downs wants to evaluate his teachers using student surveys. He conducts and random sample of 120 students and 42% of them respond that Mr. Mule should be fired for being the worst teacher at the school! If there are 2,126 students at Pope, give a 95% confidence interval for the mean number of students who think Mr. Mule should be fired.
- 7) To estimate the mean length,  $\mu$ , of western rattlesnakes, 10 such snakes are randomly selected and their lengths are recorded. The sample mean is 42.36 inches and the standard deviation is 2.158 inches. Find a 90% confidence interval for the mean length of all western rattlesnakes.
- 8) The A. C. Nielson Company publishes information on television viewing by Americans. A random sample of 20 U.S. households shows that 81% watch at least 6 hours per day. Find a 95% confidence interval for the mean percentage of all U.S. households that watch at least 6 hours of TV per day.
- 9) As an aid in the establishment of personnel requirements, the director of a hospital wishes to estimate the mean number of people who are admitted to the emergency room during a 24-hour period. The director randomly selects 64 different 24-hour periods and determines the number of admissions for each. For this sample, the mean is 19.8. The standard deviation is known to be 5. Find a 99% confidence interval for estimating the mean number of admissions per 24-hour period.
- 10) A sample of 15 private-duty nurses showed an average weekly wage of \$480.75. The standard deviation of the sample was \$56. Find the 99% confidence interval of the true mean.

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Unit	6B	Test	F	Review

Name	 
Date	 Period

#### Normal Distributions and the Empirical Rule

For the Empirical Rule...

- \_\_\_\_\_ of all observations fall within one standard deviation of the mean
- of all observations fall within two standard deviations of the mean
- of all observations fall within three standard deviations of the mean10
- 1) A normal distribution has a mean of 28 and a standard deviation of 6. Find the probability that a randomly selected x value from the distribution is in the interval from 16 to 40.
- 2) The distribution of heights of young women aged 18 to 24 is approximately normal with mean 62.5 inches and s 2.5 inches. Use the Empirical Rule
  - a. Draw a normal distribution for this data set, showing the Empirical Rule. Use the Empirical rule for the remaining questions.
  - b. What percent of women are taller than 67.5 in?
  - c. Between what heights do the middle 95% of women fall?
  - d. What percent of women are shorter than 5 feet?
  - e. A height of 65 inches corresponds to what percentile of adult female (18-24) heights?
- 3) Given an approximately normal distribution with a mean of 175 and a standard deviation of 32.... (Use the Empirical Rule)
  - a. Draw a normal curve and label 1, 2 and 3 standard deviations from the mean.
  - b. What percent of values are within the interval (143, 207)?
  - c. What percent of values are within the interval (79, 143)?
  - d. What percent of values are outside the interval (111, 239)?
  - e. What percent of values are less than 143 or greater than 271?

- 4) The weights of 1500 fish in a lake are normally distributed with a mean of 5kg and a standard deviation of 0.4kg. Use the Empirical Rule.
  - a. About how many fish weigh 4.6 kg or more?
  - b. About how many fish weigh less than 4.2 kg?
  - c. About how many fish weigh between 4.6 kg and 5.8 kg?
  - d. About how many fish weigh between 4.2 kg and 6.2 kg?
- 5) A park ranger samples 27 trees in a wooded area and found that the mean diameter of the trees is 15.2 inches with a standard deviation of 3.5 inches. Suppose that this sample of trees provides an accurate description of the entire forest and that the trees diameters are normally distributed.
  - a. What range of diameters would encompass the middle 95% of trees?
  - b. What PERCENT of the trees in the forest would you expect to be under 8.2 inches in diameter?
  - c. What is the PROBABILITY that a randomly selected tree will be between 11.7 and 15.2 inches in diameter?
  - d. If there are 1, 540 trees in the park, about HOW MANY trees are more than 18.7 inches in diameter?

# Normal Distributions and Z-Scores (Use your Z-Table)

- 6) A normal distribution has a mean of 90 and a standard deviation of 13. Find the z-score for each data point given below.
  - a. 76
  - b. 96
  - c. 120
  - d. 59
- 7) On one measure of attractiveness, scores are normally distributed with a mean of 5.9 and a standard deviation of 0.6
  - a. What percent of the population would be rated 7.0 or better?
  - b. Find  $P(x \le 5.0)$
  - c. Find  $P(6.1 \le x \le 7.5)$
- 8) Scores on an anti-aircraft exam are normally distributed with a mean of 99.6 and a standard deviation of 24.7. For a randomly selected subject, find the probability that a score will fall between a 105 and a 135.
- 9) For a certain population, scores on the Miller Analogies Test are normally distributed with a mean of 58.7 and a standard deviation of 15.9. If subjects who score under a 28.00 are to be given special training, what percentage of subjects will get special training?
- 10) Scores on the biology portion of the Medical College Admissions Test (MCAT) are normally distributed with a mean of 8.0 and a standard deviation of 1.6. If 545 students take the test, how many are expected to score

#### above a 11.5 or below a 5.0?

11) The Brake Stop wants to offer a guarantee to its customers that its brake pads will last for a certain number of miles. They find their brake pads last an average of 40,000 miles with a standard deviation of 3,580 miles. They want to guarantee their brake pads so that only 2% of customers need to have the pads replaced before the warranty expires. How many miles should they guarantee their brake pads for?

## Margin of Error and Confidence Intervals

- 12. You sample the weights of 300 apples and find the mean weight to be 3.7 oz. with a standard deviation of 0.94 oz. Find an 80% confidence interval for the true mean of apples.
- 13. In a poll of 2,548 teachers, 71% said "they feel elementary students need to spend more time at recess". Construct a 95% confidence interval for the true proportion of teachers who share this view.
- 14. A sample of 89 Bic pens has a mean ink volume of 12 mL with a standard deviation of 3.28 mL. Find a 99% confidence interval the actual ink volume in all Bic pens.
- 15. 1,925 high school students were asked if they had used illegal drugs during the school year. 233 of the students responded that they had. Construct a 95% confidence interval for the true proportion of high school students who use illegal drugs.
- 16. Construct a probability distribution for the following:

A team plays a series of 4 games in a tournament. The following displays the probability of winning a certain number of games.

Outcome	1	2	3	4
P(x)	0.55	0.29	0.12	0.04