Evaluating Logarithms WS Name Key

Evaluate each logarithm *without using a calculator*.

1. $\log_5 125 = x$ $5^x = 125$ $x = 3$
2. $\log_{11} 11 = x$ $11^x = 11$ $x = 1$
3. $\log_8 1 = x$ $8^x = 1$ $x = 0$
4. $\log_3 81 = x$ $3^x = 81$ $x = 4$
5. $\log_9 81 = x$ $9^x = 81$ $x = 2$
6. $\log_8 1 = x$ $8^x = 1$ $x = 0$
7. $\log_3 27 = x$ $3^x = 27$ $x = 3$
8. $\log_4 64 = x$ $4^x = 64$ $x = 3$
9. $\log_{10} 100 = x$ $10^x = 100$ $x = 2$
10. $\log_8 64 = x$ $8^x = 64$ $x = 2$
11. $\log_3 9 = x$ $3^x = 9$ $x = 2$
12. $\log_7 7 = x$ $7^x = 7$ $x = 1$
13. $\log_2 8 = x$ $2^x = 8$ $x = 3$
14. $\log_4 256 = x$ $4^x = 256$ $x = 4$
15. $\log_7 49 = x$ $7^x = 49$ $x = 2$
16. $\log_2 16 = x$ $2^x = 16$ $x = 4$
17. $\log_5 625 = x$ $5^x = 625$ $x = 4$
18. $\log_{10} 10{,}000 = x$ $10^x = 10{,}000$ $x = 4$

Now … stretch your mind … the log values don't have to be whole numbers!

19. $\log_{16} 4 = x$ $16^x = 4$ $4^{2x} = 4^1$ $x = \frac{1}{2}$
20. $\log_8 2 = x$ $8^x = 2$ $2^{3x} = 2^1$ $x = \frac{1}{3}$
21. $\log_{1/4} 64 = x$ $\left(\frac{1}{4}\right)^x = 64$ $4^{-x} = 4^3$ $x = -3$
22. $\log_{1/3} 81 = x$ $\left(\frac{1}{3}\right)^x = 81$ $3^{-x} = 3^4$ $x = -4$

Inverse Log Properties

$g(x) = \log_b x$ and $f(x) = b^x$ are inverses.

so ...

$\log_b b^x = x$ $\quad b^{\log_b x} = x$

Simplify the expression.

a. $5^{\log_5 x}$ $\quad x$

b. $\log_4 4^x$ $\quad x$

c. $3^{\log_3 25}$ $\quad 25$

d. $\log_5 125$ $\quad \log_5 5^3 \quad 3$

e. $10^{\log x}$ $\quad x$

f. $\log 1000^x \quad \log_{10} 10^{3x} \quad 3x$

Find inverse

1) $f(x) = \left(\frac{1}{3}\right)^x - 8$

$x = \frac{1}{3}^y - 8$

$+8 \quad +8$

$\log_{1/3}(x+8) = \log_{1/3}\frac{1}{3}^y$

$y = \log_{1/3}(x+8)$

2) $f(x) = e^{x+2}$

$\ln x = \ln e^{y+2}$

$\ln x = y + 2$

$y = \ln x - 2$

Finding Inverses of Exponential Functions WS
GPS Advanced Algebra

Name Key

For each of the following, find the inverse.

1. $f(x) = 3^x$

$x = 3^y$

$\log_3 x = y$

or $\log_3 x = \log_3 3^y$

$\log_3 x = y$

2. $f(x) = 3^x + 4$

$x = 3^y + 4$

$-4 \quad -4$

$(x-4) = 3^y$

$\log_3 (x-4) = \log_3 3^y$

$\log_3 (x-4) = y$

3. $f(x) = 3^{x-2}$

$x = 3^{(y-2)}$

$\log_3 x = \log_3 3^{(y-2)}$

$\log_3 x = y - 2$

$+2 \quad +2$

$y = \log_3 x + 2$

4. $f(x) = 3^{x+3} - 5$

$x = 3^{y+3} - 5$

$+5 \quad +5$

$(x+5) = 3^{y+3}$

$\log_3 (x+5) = \log_3 3^{y+3}$

$y + 3 = \log_3 (x+5)$

$-3 \quad -3$

$y = \log_3 (x+5) - 3$

5. $g(x) = \left(\frac{1}{2}\right)^x + 8$

$x = \left(\frac{1}{2}\right)^y + 8$

$-8 \quad -8$

$\log_{\frac{1}{2}} (x-8) = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^y$

$y = \log_{\frac{1}{2}} (x-8)$

6. $f(x) = e^{x+3}$

$\ln x = \ln e^{y+3}$

$\ln x = y + 3$

$y = \ln x - 3$

7. $f(x) = e^x - 4$

$x = e^y - 4$

$+4 \quad +4$

$\ln (x+4) = \ln e^y$

$\ln (x+4) = y$

8. $h(x) = e^{x+2} - 3$

$x = e^{y+2} - 3$

$+3 \quad +3$

$\ln (x+3) = \ln e^{y+2}$

$\ln (x+3) = y + 2$

$-2 \quad -2$

$y = \ln (x+3) - 2$

WHEN LUMBERJACKS PLAY MUSIC, WHY DO THEY USE A LARGE SOUP CAN INSTEAD OF A BASS DRUM?

Logarithms are exponents.

If $\log_b(a) = x$, then $b^x = a$. $\log_2(8) = 3$ because $2^3 = 8$.

Match each logarithmic expression with the value of x.

1) $\log_2 4 = x$ $2^x=4$ $x=2$	2) $\log_2 x = 3$ $2^3=x$ $x=8$	3) $\log_4 64 = x$ $4^x=64$ $x=3$	4) $\log_2 64 = x$ $2^x=64$ $x=5$
5) $\log_x 81 = 2$ $x^2=81$ $x=9$	6) $\log_5 x = 2$ $5^2=x$ $x=25$	7) $\log_x 7 = 1$ $x^1=7$ $x=7$	8) $\log_4 x = -\frac{1}{2}$ $4^{-1/2}=x$ $\frac{1}{4}^{1/2}=x$ $x=1/2$
9) $\log_8 x = \frac{1}{3}$ $8^{1/3}=x$ $x=2$	10) $\log_7 x = -1$ $7^{-1}=x$ $x=1/7$	11) $\log_{64} x = \frac{1}{2}$ $64^{1/2}=x$ $x=8$	12) $\log_5 x = 4$ $5^4=x$ $x=625$
13) $\log_3\left(\frac{1}{3}\right) = x$ $3^x=1/3$ $x=-1$	14) $\log_x 1000 = 3$ $x^3=1000$ $x=10$	15) $\log_{5/2}\left(\frac{2}{5}\right) = x$ $\frac{5}{2}^x=\frac{2}{5}$ $x=-1$	16) $\log_5 1 = x$ $5^x=1$ $x=0$
17) $\log_x 4 = \frac{2}{3}$ $x^{2/3}=4$ $x=8$	18) $\log_2\left(\frac{1}{4}\right) = x$ $2^x=1/4$ $x=-2$	19) $\log_2 x = 0$ $2^0=x$ $x=1$	20) $\log_8 x = -\frac{2}{3}$ $8^{-2/3}=x$ $\frac{1}{8}^{2/3}=x$ $x=.25$
21) $\log_{1/2} x = -1$ $\frac{1}{2}^{-1}=x$ $x=2$	22) $\log_{10} 10^8 = x$ $10^x=10^8$ $x=8$	23) $\log_4 x = 1$ $4^1=x$ $x=4$	24) $\log_{16} x = \frac{1}{2}$ $16^{1/2}=x$ $x=4$
25) $\log_{13} 169 = x$ $13^x=169$ $x=2$	26) $\log_x 125 = \frac{3}{4}$ $x^{3/4}=125$ $x=625$	27) $\log_{\sqrt{3}} x = 2$ $\sqrt{3}^2=x$ $x=3$	28) $\log_{1/3} x = -2$ $\frac{1}{3}^{-2}=x$ $x=9$
29) $\log_{2/3}\left(\frac{9}{4}\right) = x$ $\frac{2}{3}^x=\frac{9}{4}$ $x=-2$	30) $\log_x 0.1 = -1$ $x^{-1}=.1$ $x=10$	31) $\log_{81} x = \frac{1}{4}$ $81^{1/4}=x$ $x=3$	32) $\log_x 5 = \frac{1}{2}$ $x^{1/2}=5$ $25^{1/2}=x$ $x=25$
33) $\log_x\left(\frac{1}{4}\right) = -1$ $x^{-1}=1/4$ $x=4$	34) $\log_a 1 = x$ $a^x=1$ $x=0$	35) $\log_a a = x$ $a^x=a^1$ $x=1$	36) $\log_x .001 = -3$ $x^{-3}=.001$ $x=10$

Answers

A. 2	B. 6	C. $\frac{1}{7}$	D. 11	E. 4	F. –4	G. 625	H. 0	I. $\frac{1}{2}$	J. 5
L. 1	M. 3	N. 25	O. 10	R. –1	S. 8	T. –2	U. 7	W. $\frac{1}{4}$	Y. 9

		M			N
10	36	3	27	14	6

19	30	12	26	33	13

			R			S
15	16	5	18	34	31	2

9	35	20	21	28	17	

7	11	23

A
1

J			
4	25	22	24

29	8	32

Finding Inverses of Logarithmic Functions Using Properties of Logs

(We will be examining how to apply properties of logarithms to find their exponential inverse)

Remember: $y = b^x \iff \log_b y = x$

Example 1: Find the inverse of $y = \log_2 x + 1$

Hint: If $y = b^x \iff \log_b y = x$ and when we find the inverse we swap x and y...

$x = \log_2 y + 1$

$-1 \qquad -1$

$(x-1) = \log_2 y$

$\boxed{2^{(x-1)} = y}$

or $2^{(x-1)} = 2^{\log_2 y}$

$\boxed{2^{(x-1)} = y}$

Example 2: Find the inverse of $y = \log_4(x+2) - 5$

$$x = \log_y(y+2) - 5$$
$$+5 \qquad\qquad +5$$

$$4^{(x+5)} = \log_4(y+2)$$

$$4^{(x+5)} = y + 2$$
$$-2 \qquad -2$$

$$\boxed{y = 4^{(x+5)} - 2}$$

Example 3: Find the inverse of $y = \ln(x+7)$

$$e^{x} = e^{\ln(y+7)}$$

$$e^{x} = y + 7$$
$$-7 \qquad -7$$

$$\boxed{y = e^{x} - 7}$$

Adv. Alg. - INVERSES OF EXPONENTIAL/LOG FUNCTIONS Name Key

Find the inverse of each of the following functions.

1. $y = 3^x$

$x = 3^y$

$\log_3 x = \log_3 3^y$

$y = \log_3 x$

2. $y = \log_7 x$

$7^x = 7^{\log_7 y}$

$y = 7^x$

3. $y = \log_2 x + 1$

$x = \log_2 y + 1$

$-1 \quad -1$

$2^{(x-1)} = 2^{\log_2 y}$

$y = 2^{(x-1)}$

4. $y = 5^x - 1$

$x = 5^y - 1$

$+1 \quad +1$

$\log_5(x+1) = \log_5 5^y$

$y = \log_5(x+1)$

5. $y = 6^{x+3}$

$x = 6^{y+3}$

$\log_6 x = \log_6 6^{y+3}$

$y + 3 = \log_6 x$

$-3 \quad -3$

$y = \log_6 x - 3$

6. $y = \log_{\frac{1}{4}}(x+3)$

$x = \log_{1/4}(y+3)$

$(1/4)^x = y + 3$

$-3 \quad -3$

$y = \left(\frac{1}{4}\right)^x - 3$

7. $y = \log(x-9)$

$x = \log_{10}(y-9)$

$10^x = 10^{\log_{10}(y-9)}$

$10^x = y - 9$

$y = 10^x + 9$

8. $y = e^{x-2}$

$\ln x = \ln e^{y-2}$

$\ln x = y - 2$

$y = \ln x + 2$

9. $y = 2^{x-4} - 3$

$x = 2^{(y-4)} - 3$

$+3 \quad +3$

$\log_2(x+3) = \log_2 2^{(y-4)}$

$y - 4 = \log_2(x+3)$

$y = \log_2(x+3) + 4$

10. $y = \ln x + 5$

$x = \ln y + 5$

$-5 \quad -5$

$e^{(x-5)} = e^{\ln y}$

$y = e^{(x-5)}$

11. $y = \log_3(x-2) - 4$

$x = \log_3(y-2) - 4$

$+4 \quad +4$

$3^{(x+4)} = 3^{\log_3(y-2)}$

$y - 2 = 3^{x+4}$

$y = 3^{x+4} + 2$

12. $y = 4^{x+1} + 8$

$x = 4^{y+1} + 8$

$-8 \quad -8$

$\log_4(x-8) = \log_4 4^{y+1}$

$y = \log_4(x-8) - 1$

WHAT MATHEMATICAL TOPIC IS DISCUSSED BY THE MUSICIANS IN A GERMAN BEER HOUSE?

Match the letter of each logarithmic equation with the number of the exponential equation.

1) $2^3 = 8$	2) $3^2 = 9$	3) $9^{1/2} = 3$	4) $5^{-1} = \frac{1}{5}$
5) $2^0 = 1$	6) $4^3 = 64$	7) $8^{1/3} = 2$	8) $8^{2/3} = 4$
9) $a^b = x$	10) $b^x = a$	11) $x^a = b$	12) $a^x = b$

Logarithmic Form

A. $\log_8 4 = \frac{2}{3}$ 8	B. $\log_b x = a$	C. $\log_8\left(\frac{1}{3}\right) = 2$	E. $\log_8 2 = \frac{1}{3}$ 7
G. $\log_9 3 = \frac{1}{2}$ 3	H. $\log_4 64 = 3$ 6	H. $\log_2 1 = 0$ 5	K. $\log_{64} 4 = 3$
L. $\log_2 8 = 3$ 1	M. $\log_x b = a$ 11	R. $\log_a b = x$ 12	R. $\log_5\left(\frac{1}{5}\right) = -1$ 4
S. $\log_a x = b$ 9	T. $\log_3 9 = 2$ 2	X. $\log_3 2 = 9$	Y. $\log_b a = x$ 10

L	A	G	E	R
1	8	3	7	12

R	H	Y	T	H	M	S
4	6	10	2	5	11	9

WHAT TYPE OF DWELLING DOES "X" REPRESENT IN THE EQUATION 10^X = CABN?

Match the letter of each exponential equation with the number of the logarithmic equation.

1) $\log_2 32 = 5$	2) $\log_5 1 = 0$	3) $\log_5 125 = 3$	4) $\log_{\sqrt{2}} 2 = 2$
5) $\log_{64} 8 = \frac{1}{2}$	6) $\log_2\left(\frac{1}{8}\right) = -3$	7) $\log_{27} 9 = \frac{2}{3}$	8) $\log_3\left(\frac{1}{3}\right) = -1$

Exponential Form

A. $2^{-3} = \frac{1}{8}$ 6	B. $2^5 = 32$ 1	C. $64^{1/2} = 8$ 5	D. $5^2 = 32$
D. $125^3 = 5$	G. $5^3 = 125$ 3	H. $5^1 = 0$	I. $5^0 = 1$ 2
L. $3^{-1} = \frac{1}{3}$ 8	M. $3^{-1} = -3$	N. $27^{2/3} = 9$ 7	O. $(\sqrt{2})^2 = 2$ 4

L	O	G
8	4	3

C	A	B	I	N
5	6	1	2	7

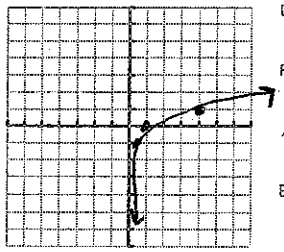
Graphing Logarithmic Functions Notes

x	y
4	1
16	2
1	0
1/4	-1

Think of it this way ...

$f(x) = \log_4 x$

$4^y = x$



Domain: $(0, \infty)$

Range: $\mathbb{R}$

Asymptote: $x = 0$

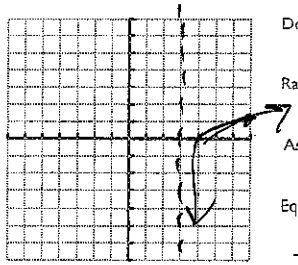
Equation of the inverse: $y = 4^x$

${}_4{}^{x} = {}_4{}^{\log_4 y}$

$4^x = y$

Think:

$f(x) = \log_4(x - 3)$ $\quad 4^y = (x - 3)$



Domain: $(3, \infty)$

Range: $\mathbb{R}$

Asymptote: $x = 3$

Equation of the inverse: $y = 4^x + 3$

x	y
19	2
7	1
4	0

${}_4{}^{x} = {}_4{}^{\log_4 (y-3)}$

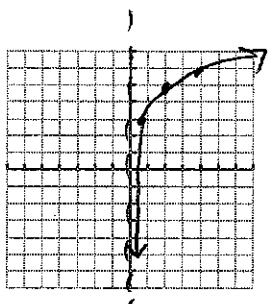
$y - 3 = 4^x$

$+3 \quad +3$

x	y + 4
2	1 + 4 = 5
4	2 + 4 = 6
1/2	-1 + 4 = 3

Think:

$f(x) = \log_2 x + 4$ $\quad 2^y = x$



Domain: $(0, \infty)$

Range: $\mathbb{R}$

Asymptote: $x = 0$

Equation of the inverse: $y = 2^{(x-4)}$

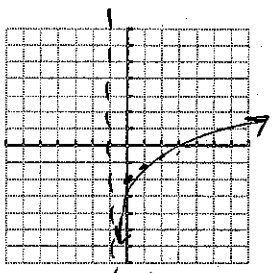
$x = \log_2 y + 4$

$-4 \quad -4$

${}_2{}^{(x-4)} = {}_2{}^{\log_2 y}$

Make a table of values like normal

$f(x) = \ln(x + 1) - 2$



Domain: $(-1, \infty)$

Range: $\mathbb{R}$

Asymptote: $x = -1$

Equation of the inverse: $y = e^{x+2} - 1$

x	y
0	-2
1	-1.3
2	-.9
3	-.61
4	-.39

$x = \ln(y + 1) - 2$

$+2 \quad +2$

$e^{(x+2)} = e^{\ln(y+1)}$

$e^{x+2} = y + 1$

$-1 \quad -1$

$y = e^{x+2} - 1$