*Verifying Inverses

Proofs

*Verifying Inverses

*f and g are inverses iff ...

*$f(g(x)) = x$ AND $g(f(x)) = x$

*Example 1

*Verify that f and g are inverses.

$$f(x)=\frac{5}{x-2} \qquad g(x)=\frac{5}{x}+2$$

$$f(g(x)) = \frac{5}{\frac{5}{x}+2-2} = \frac{5}{1} \div \frac{5}{x} = \frac{\cancel{5}}{1}\cdot\frac{x}{\cancel{5}} = \boxed{x}$$ ✓

$$g(f(x)) = \frac{5}{\frac{5}{x-2}}+2$$

$$= \frac{\cancel{5}}{1}\cdot\frac{x-2}{\cancel{5}}+2$$

$$= x-2+2 = \boxed{x}$$ ✓

*Example 2

*Verify that f and g are inverses.

$$f(x)=2x^3-1 \qquad g(x)=\sqrt[3]{\frac{x+1}{2}}$$

$$f(g(x)) = 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3-1$$

$$= \frac{\cancel{2}(x+1)}{\cancel{2}}-1$$

$$= x+1-1$$

$$= \boxed{x}$$ ✓

$$g(f(x)) =$$

$$= \sqrt[3]{\frac{2x^3-1+1}{2}}$$

$$= \sqrt[3]{\frac{2x^3}{2}}$$

$$= \sqrt[3]{x^3}$$

$$= \boxed{x}$$ ✓

Inverses WS 3 Name Key

Fill in the blank.

1. If $(-3, 1)$ is on f, then (1, -3) is on f^{-1}.

2. If $(-3, 0)$ is the x-intercept of f, then (0, -3) is the y-intercept of f^{-1}.

3. If $[-2,\infty)$ is the range of f, then $[-2, \infty)$ is the domain of f^{-1}.

4. If $[3,\infty)$ is the domain of f^{-1}, then $[3, \infty)$ is the range of f.

Verify that f and g are inverse functions (or not).
In order to do this you must prove that $f(g(x))=x$ and $g(f(x))=x$.

5. $f(x)=x+4;\ g(x)=x-4$

$f(g(x)) = (x-4)+4$
$= \boxed{x}$ ✓

$g(f(x)) = (x+4)-4$
$= \boxed{x}$ ✓

6. $f(x)=2x-4;\ g(x)=\frac{1}{2}x+2$

$f(g(x)) = 2(\frac{1}{2}x+2)-4$
$= x+4-4$
$= \boxed{x}$

$g(f(x)) = \frac{1}{2}(2x-4)+2$
$= x-2+2$
$= \boxed{x}$ ✓

7. $f(x)=x^2+2, x\ge 0;\ g(x)=\sqrt{x-2}$

$f(g(x)) = (\sqrt{x-2})^2+2$
$= x-2+2$
$= \boxed{x}$

$g(f(x)) = \sqrt{x^2+2-2}$
$= \sqrt{x^2}$ ✓
$= \boxed{x}$

8. $f(x)=\frac{1}{3}x^3-2;\ g(x)=\sqrt[3]{3x+6}$

$f(g(x)) = \frac{1}{3}(\sqrt[3]{3x+6})^3-2$
$= \frac{1}{3}(3x+6)-2 = x+2-2$
$= \boxed{x}$ ✓

$g(f(x)) = \sqrt[3]{3(\frac{1}{3}x^3-2)+6}$
$= \sqrt[3]{(x^3-6)+6}$
$= \sqrt[3]{x^3} = \boxed{x}$ ✓

9. $f(x)=3-x;\ g(x)=3-x$

$f(g(x)) = 3-(3-x)$
$= 3-3+x$
$= \boxed{x}$ ✓

$g(f(x)) = 3-(3-x)$
$= 3-3+x$
$= \boxed{x}$ ✓

Solving Exponential Equations

(with like bases)

Ex 1:

$$7^{2x} = 7^{3x-5}$$

* same bases cancel out

$2x = 3x - 5$

$-3x \quad -3x$

$-x = -5$

$\boxed{x = 5}$

Ex 2:

$* 5^3 = 125$

- $5^{4m} = 125^{m+2}$

$\cancel{5}^{4m} = \cancel{5}^{3(m+2)}$

$4m = 3m + 6$

$-3m \quad -3m$

$\boxed{m = 6}$

Ex 3:

- $9^x = 27$

$\cancel{3}^{2x} = \cancel{3}^{3}$

$\frac{2x}{2} = \frac{3}{2}$

$\boxed{x = 3/2}$

Ex 4:

o $(\frac{1}{2})^x = 16^{3x-1}$

$$\cancel{2}^{-x} = \cancel{2}^{4(3x-1)}$$

$$-x = 4(3x-1)$$

$$-x = 12x - 4$$

$$-12x \quad -12x$$

$$\frac{-13x}{-13} = \frac{-4}{-13}$$

$$\boxed{x = 4/13}$$

Exponential Equations (Like Bases) WS 1

Name ______________________

Solve each equation.

1. $\cancel{5}^x = \cancel{5}^{-3}$

$\boxed{x=-3}$

2. $6^x = 216$

$\cancel{6}^x = \cancel{6}^3$

$\boxed{x=3}$

3. $7^y = \frac{1}{49}$

$\cancel{7}^y = \cancel{7}^{-2}$ $\boxed{y=-2}$

4. $10^x = .001$

$\cancel{10}^x = \cancel{10}^{-3}$

$\boxed{x=-3}$

5. $2^{2x} = \frac{1}{8}$

$\cancel{2}^{2x} = 2^{-3}$

$\frac{2x}{2} = \frac{-3}{2}$

$\boxed{x=-3/2}$

6. $\left(\frac{1}{5}\right)^{x-3} = 125$

$\cancel{5}^{-(x-3)} = \cancel{5}^3$

$-x+3=3$

$\boxed{x=0}$

7. $\cancel{3}^y = \cancel{3}^{3y+1}$

$y = 3y+1$

$-2y = 1$

$\boxed{y=-1/2}$

8. $\cancel{5}^{3y+4} = \cancel{5}^y$

$3y+4 = y$

$4 = -2y$

$\boxed{y=-2}$

9. $3^x = 9^{x+1}$

$\cancel{3}^x = \cancel{3}^{2(x+1)}$

$x = 2(x+1)$

$x = 2x+2$

$-2x \quad -2x$

$-x = 2$

$\boxed{x=-2}$

10. $\cancel{2}^5 = \cancel{2}^{2x-1}$

$5 = 2x-1$

$6 = 2x$

$\boxed{x=3}$

11. $8^{x-1} = 16^{3x}$

$\cancel{2}^{3(x-1)} = \cancel{2}^{4(3x)}$

$3x-3 = 12x$

$\frac{-3}{9} = \frac{9x}{9}$ $\boxed{x=-1/3}$

12. $2^{x+3} = \frac{1}{16}$

$\cancel{2}^{x+3} = \cancel{2}^{-4}$

$x+3 = -4$

$\boxed{x=-7}$

Answers: 1) –3 2) 3 3) –2 4) –3 5) –3/2 6) 0 7) –1/2 8) –2 9) –2 10) 3 11) –1/3 12) –7

Interest compounded continuously $A = Pe^{rt}$ Growth/Decay $y = ae^{kt}$

Interest compounded frequently but not continuously $A = P\left(1+\frac{r}{n}\right)^{nt}$

t = # years
r = rate (make decimal)
n = # times compounded

Write an equation for each of the following. Then use your calculator to find the answer.

1. Thrifty Thelma invests \$7500 in an account paying 4% interest compounded quarterly. How much will be in Thelma's account at the end of 6 years?

$A = 7500\left(1 + \frac{.04}{4}\right)^{4(6)}$

$A = \$9523.01$

2. Sam the Saver invests \$500 in an account that pays 3.5% interest compounded continuously. How much money will Sam have after 3 years?

$A = 500e^{.035(3)}$

$= \$555.36$

3. If you invest \$2100 in a savings account that pays 2.25% interest compounded monthly, how much money will you have at the end of one year?

$A = 2100\left(1 + \frac{.0225}{12}\right)^{12(1)}$

$= \$2147.74$

4. Ted invested \$675 in an account that pays 3.4% interest compounded continuously. How much will be in his account after 6 months?

$A = 675e^{.034(.5)}$

$= \$686.57$

5. Your parents just won the Mega Millions Lottery. Because they love you so much, they decide to give you some of their winnings; however, they don't want you to have the money until your 22nd birthday. So they invest \$15,000 in a trust fund that pays $3\frac{5}{8}\%$ interest compounded continuously.

Assuming that you are 17 years old right now, how much money will you get when you are 22?

$A = 15{,}000e^{(.03625 \cdot 5)}$

$= \$17980.72$

6. After t years, the value of a car that costs \$20,000 when it was new is modeled by $V(t) = 20{,}000\left(\frac{3}{4}\right)^t$.

Determine the value of the car 3 years after it was purchased.

$$V(3) = 20{,}000\left(\frac{3}{4}\right)^3$$

$$= 8437.5$$

7. A certain bacteria grows at an exponential rate with constant value of k = .0324. If there are 50 bacterium in a dish at the beginning of the day, how many will there be at the end of the day, 8 hours later?

$$A = 50e^{(.0324(8))}$$

$$A = 64.79$$

$\rightarrow 64$ * can't have .79 of a bacteria

8. The population of a town increases according to the model $P(t) = 2500e^{0.0293t}$ where t is time in years and $t = 0$ corresponds to 1990. What will the population of the town be in 2010?
What was the population of the town in 1985?

* 2010 - 1990 = 20

$$P(20) = 2500e^{.0293(20)}$$

$$= 4491.97$$

4491 - can't have .97 of a person

$$P(-5) = 2500e^{.0293(-5)}$$

$$= 2159.3$$

= 2159 - can't have .3 of a person

Answers: 1. \$9523.01 2. \$555.36 3. \$2147.74 4. \$686.57

Adv Alg - Exponential Functions
More application problems

Name ____________________

1. The amount in trillions of cubic feet of natural gas consumed in the United States from 1940 to 1970 can be modeled by the function $y = a(1.07)^t$, where t is the number of years since 1940.

(a) Assuming 2.91 trillion cubic feet of natural gas was consumed in the US in 1940, estimate how many cubic feet were consumed in 1937.

$y = 2.91(1.07)^{-3}$ $\boxed{y \approx 2.38 \text{ trillion cubic feet}}$

(b) Predict the natural gas usage for 2012.

$y = 2.91(1.07)^{72}$

$\boxed{y \approx 379.77 \text{ tril ft}^3}$

$$\begin{array}{r} 2012 \\ -1940 \\ \hline 72 \end{array}$$

2. From 1971 to 1995, the average number of transistors on a computer chip can be modeled by the function $y = a(1.59)^t$, where t represents the number of years since 1971.

(a) Assuming there was an average of 2300 transistors per chip in 1971, estimate the number of transistors on each chip in 1998.

$y = 2300(1.59)^{27}$

$\boxed{y \approx 1593105149}$

(b) How many transistors were used on each chip in 2011?

$y = 2300(1.59)^{40}$

$\boxed{y = 2.615849832 \times 10^{11}}$

3. Connor the contractor likes to buy houses, remodel them, live in them for a while and then sell the house and buy another one. Seven and a half years ago, he paid \$137,000 for the house he now lives in. During that time, house values have appreciated according to the model $y = a(1.024)^t$ where a is the initial value and t is time in years.

(a) How much is his house worth now?

$y = 137{,}000(1.024)^{7.5}$

$\boxed{y = \$163{,}670.44}$

(b) Connor decides to use a real estate agent to sell the house. He must pay the agent a commission of 3.5% of the selling price of the house. Connor has found a buyer willing to pay \$173,900 for the house. After the real estate agent is paid, how much profit will Connor make on the sale of the house?

$173{,}900(.035) = \$6086.50$

$173{,}900 - 6086.50 = \$167813.50$ →

$$\begin{array}{r} 167813.50 \\ -137{,}000.00 \\ \hline \boxed{\$30{,}813.50} \end{array}$$

4. Since 1972 the US Fish and Wildlife Service has kept a list of endangered species in the United States. In 1972 there were 119 species on the endangered list. For the years from 1972 to 1998, the number of species on the list can be modeled by the equation $y = ae^{0.0917t}$ where t is the number of years since 1972. Estimate the number of species on the endangered list at the present time.

$y = 119e^{.0917(43)}$

$= 6137.81$

$\rightarrow \boxed{6137 \text{ endangered species}}$

$$\begin{array}{r} 2015 \\ -1972 \\ \hline 43 \end{array}$$

5. One hundred grams of radium is stored in a container. The amount (in grams) of radium present after t years can be modeled by the function $y = ae^{-0.00043t}$. How much of the radium will be present after 10,000 years?

$y = 100e^{-.00043(10,000)}$

$\boxed{y = 1.36 \text{ radium present}}$

6. The area of a wound decreases exponentially with time. The area of a wound after t days can be modeled by the function $y = ae^{-0.05t}$. If the initial wound area is 4 square centimeters, how much of the wound area is present after 14 days?

$y = 4e^{-.05(14)}$

$y \approx 1.99 \text{ cm}^2$

7. Little Susie is 11 years old today.
Her parents want to begin a savings account to help pay Susie's college expenses. So, as a birthday gift, Susie's parents open a savings account for her with \$5725 at a bank that compounds interest monthly at an interest rate of $1\frac{3}{8}$%.
Susie's grandparents opened a college savings account for her on the day she was born. They invested \$250 in an account that compounds interest continuously at a rate of $2\frac{1}{4}$%.
Susie will begin college on her 18th birthday.

(a) How much money will be in the account her parents established for her?

$y = 5725\left(1 + \frac{.01375}{12}\right)^{12(7)}$

$y = \$6303.07$

(b) How much money will be in the account her grandparents established for her?

$y = 250e^{.0225(18)}$

$y = \$374.83$

8. Two brothers won \$16,000 in the lottery and decided to split the money evenly.
John invested \$4000 at an rate of 2% compounded continuously at one bank and \$4000 at another bank that paid 3% interest compounded weekly.
James invested 75% of his share at a bank that pays $2\frac{1}{2}$% compounded continuously and the rest at a bank paying $3\frac{1}{4}$% compounded quarterly.

(a) At the end of 2 years, how much money will John have?

$y = 4000e^{.02(2)}$

$\boxed{y = 4163.24}$

$y = 4000\left(1 + \frac{.03}{52}\right)^{52(2)}$

$\boxed{y = 4247.27}$

$\boxed{\$8410.50}$

(b) At the end of the 2 years, how much money will James have?

$y = 6,000e^{.025(2)}$

$y = \$6307.63$

$y = 2,000\left(1 + \frac{.0325}{4}\right)^{4(2)}$

$y = 2133.76$

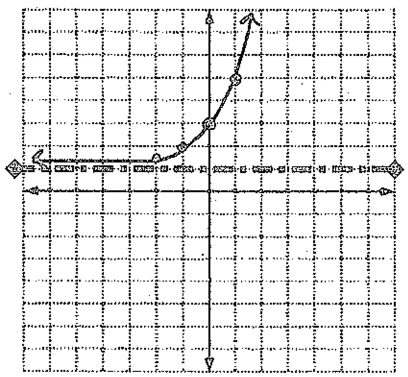
$\boxed{\$8441.39}$

Graphing Exponential Equations and Their Inverse Functions - NOTES

- What do we need to find to graph an exponential function?

- To graph:
  1) Make a T-Chart centered where the exponent will = 0.
  2) Find the y-intercept (will always have one)
  3) Sketch the asymptote
  4) Sketch the graph
  5) Determine the domain, range and equation of the asymptote

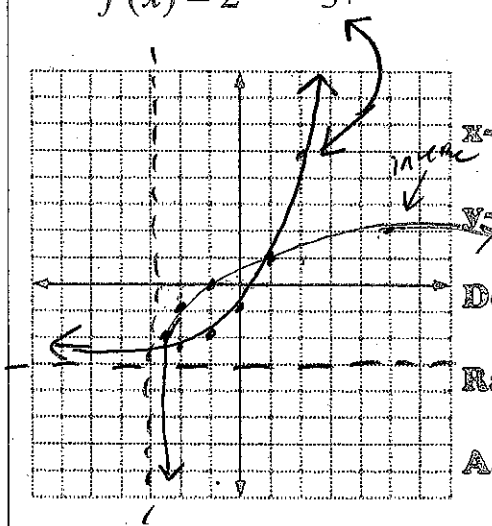
ALL EXPONENTIAL FUNCTIONS WILL HAVE AN ASYMPTOTE!



Example 1: Graph the following function and its inverse.

$$f(x) = 2^{x+1} - 3$$

X	y
-2	-2.5
-1	-2
0	-1
1	1
2	5



	Function	Inverse
x-int:	≈(.63,0)	(-1,0)
y-int:	(0,-1)	(0,.63)
Dom:	ℝ	(-3,∞)
Range:	(-3,∞)	ℝ
Asymp:	$y=-3$	$X=-3$

inverse

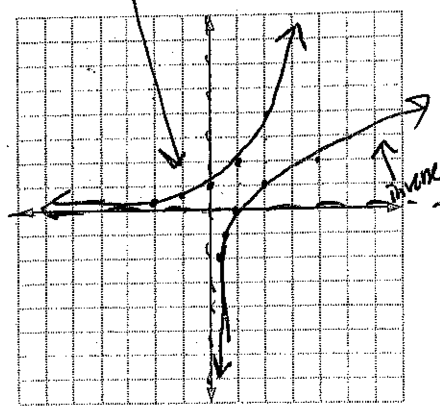
X	y
-2.5	-2
-2	-1
-1	0
1	1
5	2

Worksheet—Graphing Exponential Functions and Their Inverses

Name ______________________

1. $f(x) = 2^x$

x	y
-2	.25
-1	.5
0	1
1	2
2	4



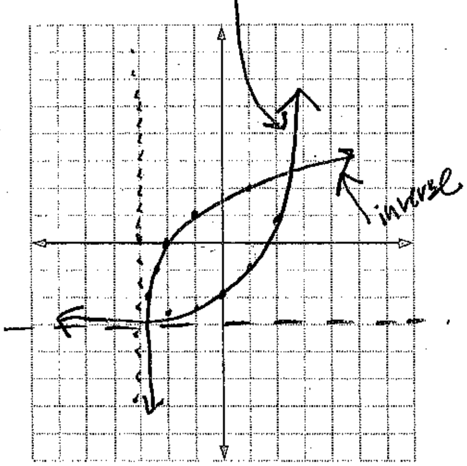
	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(0,\infty)$
Range	$(0,\infty)$	$\mathbb{R}$
Y intercept	$(0,1)$	none
X intercept	none	$(1,0)$
Asymptote	$y=0$	$x=0$

inverse

x	y
.25	-2
.5	-1
1	0
2	1
4	2

2. $f(x) = 2^x - 3$

x	y
-2	-2.75
-1	-2.5
0	-2
1	-1
2	1



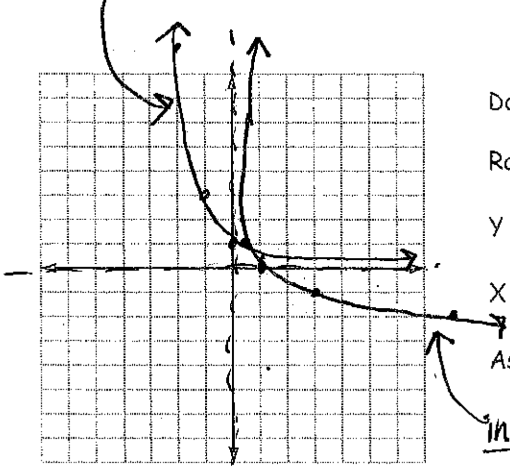
	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(-3,\infty)$
Range	$(-3,\infty)$	$\mathbb{R}$
Y intercept	$(0,-2)$	$(0,1.58)$
X intercept	$(1.58,0)$	$(-2,0)$
Asymptote	$y=-3$	$x=-3$

inverse

x	y
-2.75	-2
-2.5	-1
-2	0
-1	1
1	2

3. $f(x) = \frac{1^x}{3}$

x	y
-2	9
-1	3
0	1
1	.33
2	.11



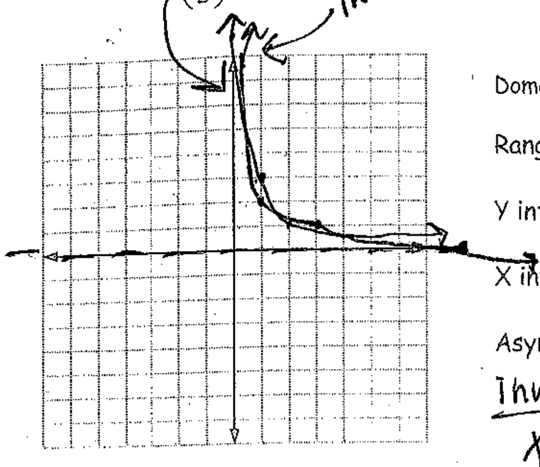
	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(0,\infty)$
Range	$(0,\infty)$	$\mathbb{R}$
Y intercept	$(0,1)$	none
X intercept	none	$(1,0)$
Asymptote	$y=0$	$x=0$

inverse

x	y
9	-2
3	-1
1	0
.33	1
.11	2

4. $f(x)=\left(\frac{1}{3}\right)^{x-2}$

X	y
-2	81
-1	27
0	9
1	3
2	1



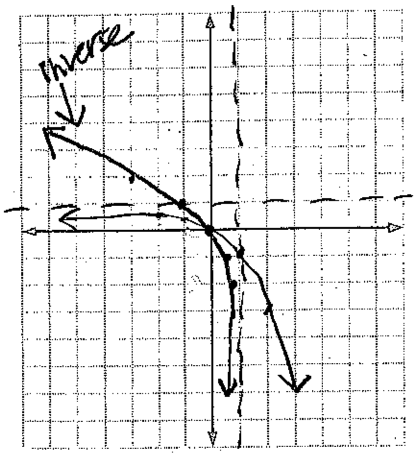
	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(0,\infty)$
Range	$(0,\infty)$	$\mathbb{R}$
Y intercept	$(0,9)$	none
X intercept	none	$(9,0)$
Asymptote	$y=0$	$x=0$

Inverse

X	y
81	-2
27	-1
9	0
3	1
1	2

5. $f(x)=-2^x+1$

X	y
-2	.75
-1	.5
0	0
1	-1
2	-3



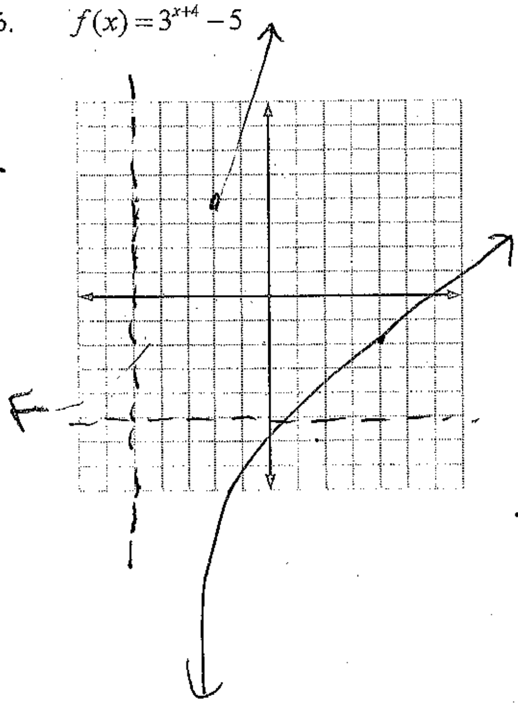
	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(-\infty,1)$
Range	$(-\infty,1)$	$\mathbb{R}$
Y intercept	$(0,0)$	$(0,0)$
X intercept	$(0,0)$	$(0,0)$
Asymptote	$y=1$	$x=1$

inverse

X	y
.75	-2
.5	-1
0	0
-1	1
-3	2

6. $f(x)=3^{x+4}-5$

X	y
-2	4
-1	22
0	76
1	238
2	



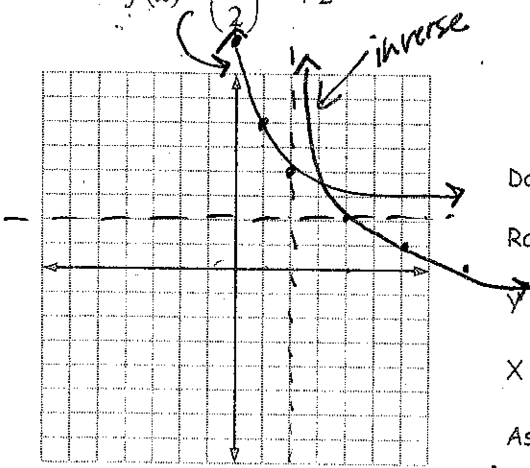
	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(-5,\infty)$
Range	$(-5,\infty)$	$\mathbb{R}$
Y intercept	$(0,76)$	$(0,-2.53$
X intercept	$(-2.53,0)$	$(76,0)$
Asymptote	$y=-5$	$x=-5$

Inverse

X	y
4	-2
22	-1
76	0
238	1

7. $f(x) = \left(\frac{1}{2}\right)^{x-3} + 2$

x	y
-2	34
-1	18
0	10
1	6
2	4



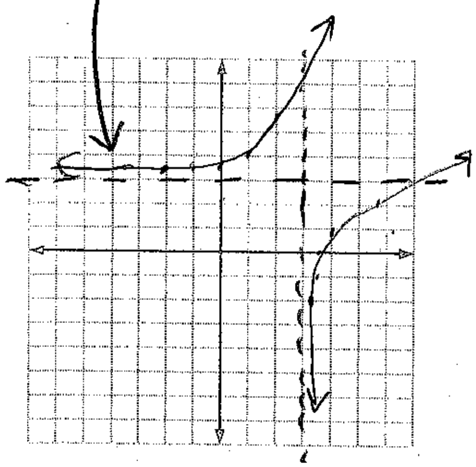
	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(2,\infty)$
Range	$(2,\infty)$	$\mathbb{R}$
Y intercept	$(0,10)$	none
X intercept	none	$(10,0)$
Asymptote	$y=2$	$x=2$

inverse

x	y
34	-2
18	-1
10	0
6	1
4	2

8. $f(x) = e^{x-1} + 3$

x	y
-2	3.05
-1	3.13
0	3.36
1	4
2	5.71



	EXPONENTIAL FUNCTION	INVERSE FUNCTION
Domain	$\mathbb{R}$	$(3,\infty)$
Range	$(3,\infty)$	$\mathbb{R}$
Y intercept	$(0,3.36)$	none
X intercept	none	$(3.36,0)$
Asymptote	$y=3$	$x=3$

inverse

x	y
3.05	-2
3.13	-1
3.36	0
4	1
5.71	2