Graphing Exponential and Logarithmic Functions

Day 1 – Exponential Functions ONLY – slides 1 and 2

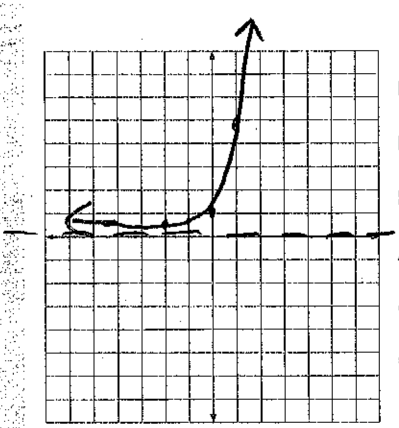
Day 2 – Exponential Functions ONLY – slides 3 and 4

Day 3 – Logarithmic Functions – slides 1, 2, 3, 4

Slide 1

$f(x) = 5^x + 0$ ← Horizontal Asymptote

X	y
-2	1/25
-1	1/5
0	1
1	5
2	25



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$(-\infty, \infty)$	
Range	$(0, \infty)$	
Intercept y →	$(0,1)$	
Asymptote	$y=0$	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

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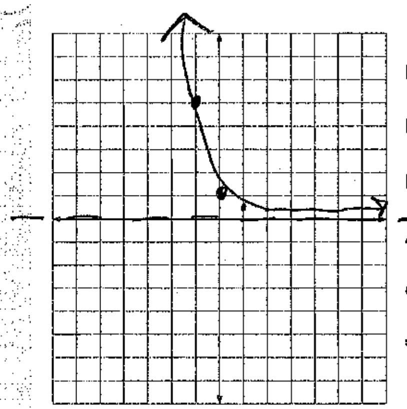
x	y
-2	25
-1	5
0	1
1	.2
2	.04

Slide 2

$f(x) = 5^{-x} = (1/5)^x + 0$ ← horizontal asymptote



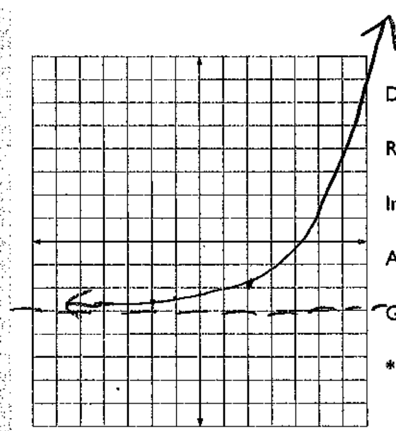
	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(0, \infty)$	
Intercept	$(0, 1)$	
Asymptote	$y = 0$	
Growth/Decay?	Decay	N/A

**Equation of the inverse of the exponential function

x	y
-2	-2.998
-1	-2.992
0	-2.96
1	-2.8
2	-2

Slide 3

$f(x) = 5^{(x-2)} - 3$ ← horizontal Asymptote



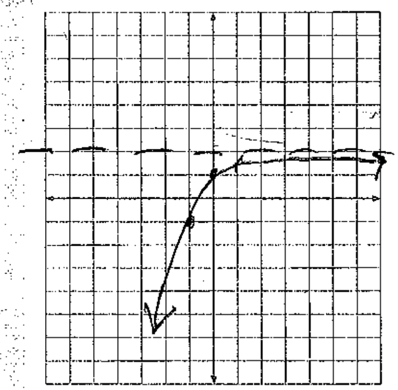
	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$(-\infty, \infty)$	
Range	$(-3, \infty)$	
Intercept	$(0, -2.96)$	
Asymptote	$y = -3$	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

Slide 4

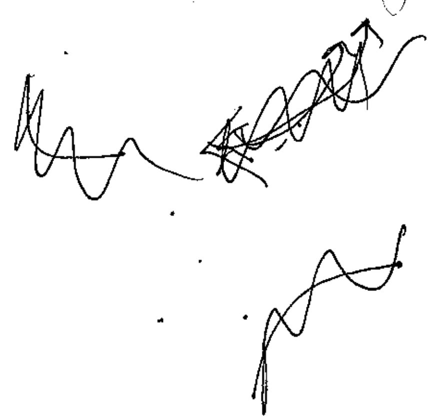
$f(x) = -(1/5)^x + 2$ horizontal Asymptote

x	y
-2	-23
-1	-3
0	1
1	1.8
2	1.96



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	ℝ	
Range	$(-\infty, 3)$	
Intercept	$(0,1)$	
Asymptote	$y=2$	
Growth/Decay?	reflected Decay	N/A

**Equation of the inverse of the exponential function

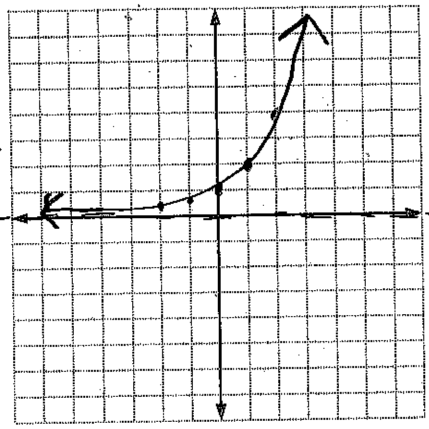


Worksheet—Graphing Exponential Functions I

Name Key

1. $f(x) = 2^x$ +0 ← Horizontal Asymptote

X	y
-2	.25
-1	.5
0	1
1	2
2	4

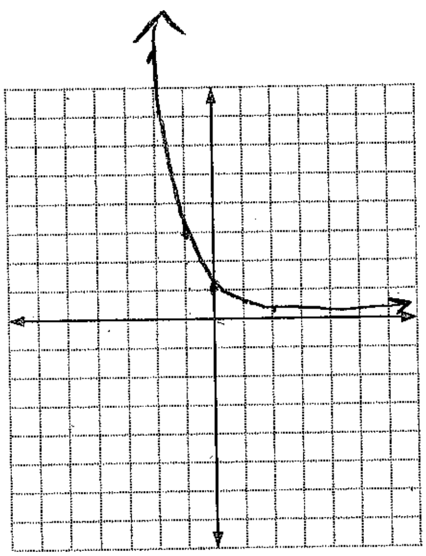


	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	ℝ	
Range	(0,∞)	
intercept	y-inter: (0,1)	
Asymptote	y=0	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

2. $f(x) = \frac{1^x}{3}$ +0 ← H.A.

X	y
-2	9
-1	3
0	1
1	.33
2	.11

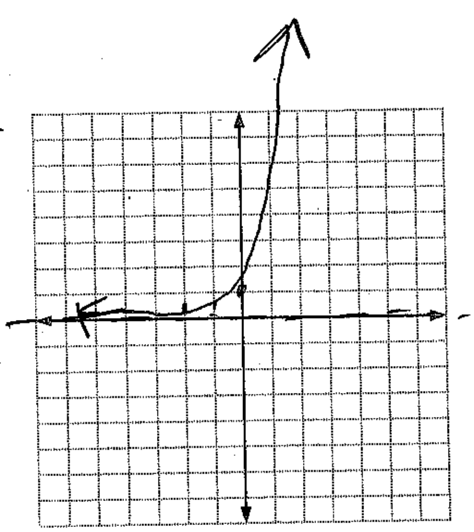


	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	ℝ	
Range	(0,∞)	
intercept	y-inter: (0,1)	
Asymptote	y=0	
Growth/Decay?	Decay	N/A

**Equation of the inverse of the exponential function

3. $f(x) = 10^x$ +0 ← H.A.

X	y
-2	.01
-1	.1
0	1
1	10
2	100

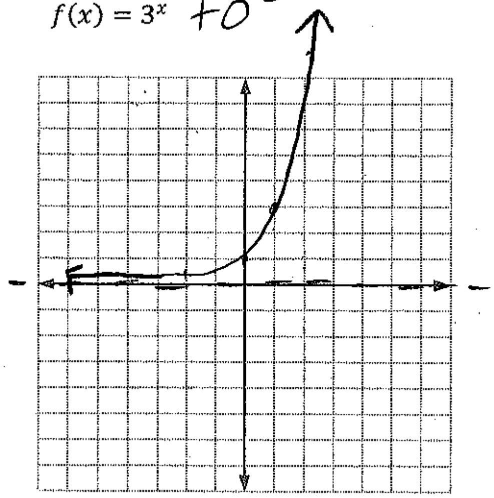


	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	ℝ	
Range	(0,∞)	
intercept	y-inter: (0,1)	
Asymptote	y=0	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

4. $f(x) = 3^x$ $+0$ ← H.A.

x	y
-2	1/9
-1	.33
0	1
1	3
2	9



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(0,\infty)$	
intercept	y-inter $(0,1)$	
Asymptote	$y=0$	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

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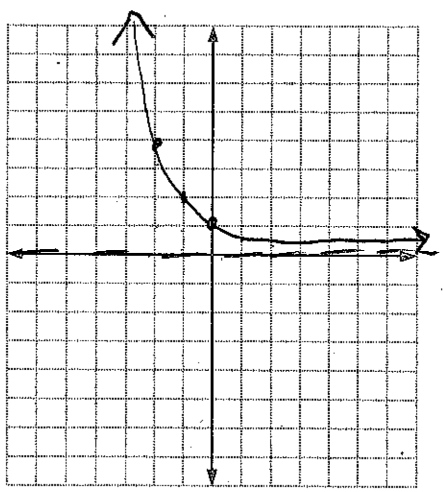
5. $f(x) = 2^{-x}$ $\rightarrow \left(\frac{1}{2}\right)^x + 0$ ↑ H.A.

x	y
-2	4
-1	2
0	1
1	1/2
2	.25



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(0,\infty)$	
intercept	y-inter: $(0,1)$	
Asymptote	$y=0$	
Growth/Decay?	Decay	N/A

**Equation of the inverse of the exponential function

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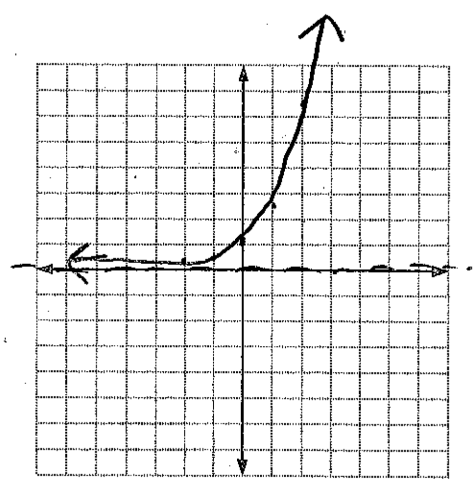
6. $f(x) = e^x$ $+0$ ↖ H.A.

x	y
-2	.13
-1	.36
0	1
1	2.7
2	7.3



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(0,\infty)$	
intercept	y-inter. $(0,1)$	
Asymptote	$y=0$	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

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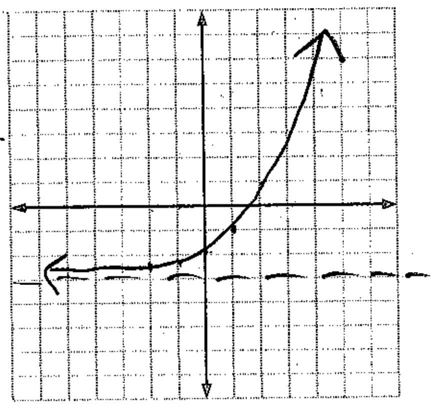
Worksheet—Graphing Exponential Functions II

Name Key

1. $f(x) = 2^x - 3$ ✓ H.A.

X	y
-2	-2.75
-1	-2.5
0	-2
1	-1
2	1

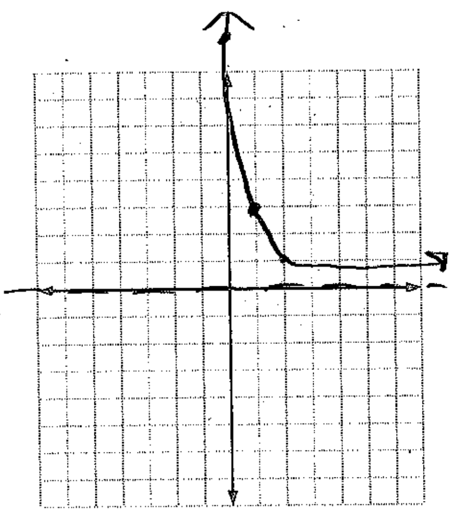


	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(-3, \infty)$	
Intercept	y-inter. $(0,-2)$	
Asymptote	$y=-3$	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

2. $f(x) = \left(\frac{1}{3}\right)^{x-2}$ $+0$ ✓ H.A.

X	y
-2	81
-1	27
0	9
1	3
2	1

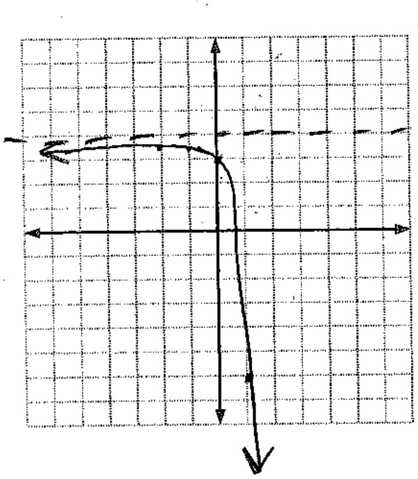


	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(0, \infty)$	
Intercept	$(0,9)$	
Asymptote	$y=0$	
Growth/Decay?	Decay	N/A

**Equation of the inverse of the exponential function

3. $f(x) = -10^x + 4$

X	y
-2	3.99
-1	3.9
0	3
1	-6
2	-96

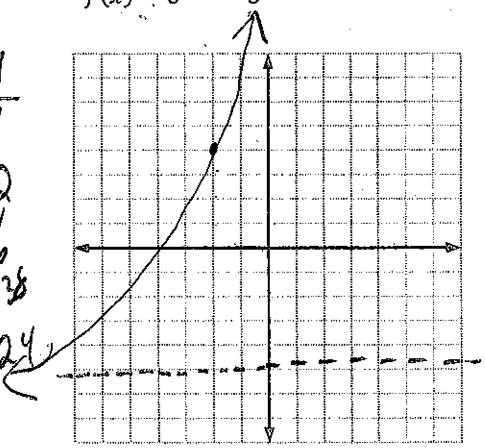


	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(-\infty, 4)$	
Intercept	$(0,3)$	
Asymptote	$y=4$	
Growth/Decay?	reflected growth	N/A

**Equation of the inverse of the exponential function

4. $f(x) = 3^{x+4} - 5$

X	y
-2	4
-1	22
0	76
1	238
2	724



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(-5, \infty)$	
Intercept	$(0, 76)$	
Asymptote	$y = -5$	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

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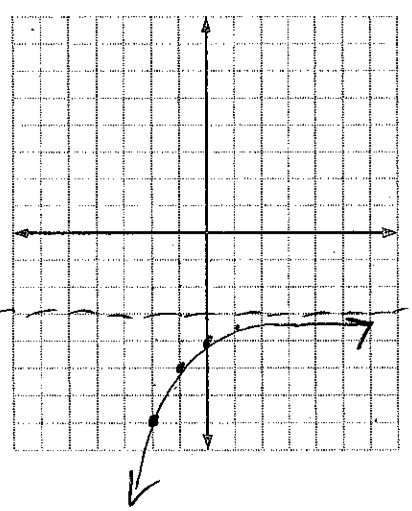
5. $f(x) = -2^{-x} - 3$

$f(x) = -(\frac{1}{2})^x - 3$

X	y
-2	-7
-1	-5
0	-4
1	-3.5
2	-3.25



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(-\infty, -3)$	
Intercept	$(0, -4)$	
Asymptote	$y = -3$	
Growth/Decay?	reflected decay	N/A

**Equation of the inverse of the exponential function

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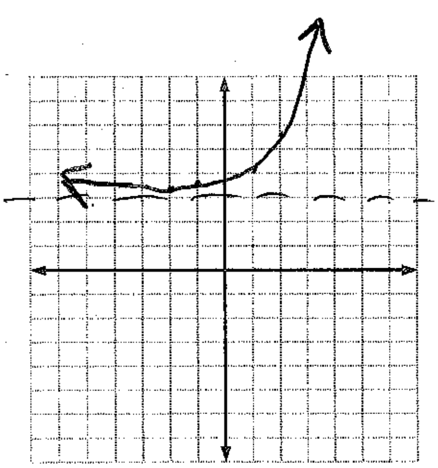
6. $f(x) = e^{x-1} + 3$

X	y
-2	3.04
-1	3.13
0	3.36
1	4
2	5.71



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	$\mathbb{R}$	
Range	$(3, \infty)$	
Intercept	$(0, 3.36)$	
Asymptote	$y = 3$	
Growth/Decay?	Growth	N/A

**Equation of the inverse of the exponential function

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NOTES ON INVERSES

INVERSES – Two functions are inverses, if and only if, when one function contains a point (a, b), the other function contains the point (b, a).

Example: f(x): { (3, 1) , (–2, 4) , (5, –1) } Domain of f(x) {3, -2, 5} Range of f(x) { 1, 4, -1 }

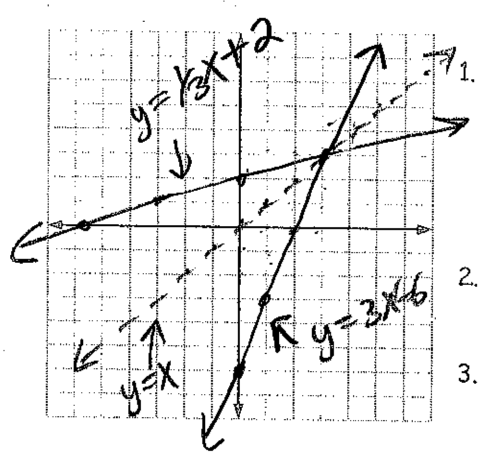
The inverse of f(x) will be { (1,3) (4,-2) (-1,5) } Domain of inverse { 1, 4, -1 }

Range of inverse { 3, -2, 5 }

****The **domain** of function f(x) has become the range of the inverse;

The **range** of function f(x) has become the domain of the inverse.

Graphing Inverse Functions



1. Graph $y = 3x - 6$ and list four points that appear on your graph.
   (3,3) (2,0) (1,-3) (0,-6)

2. Now graph $y = \frac{1}{3}x + 2$ on the same axes.

3. Switch the x and y coordinates in your original ordered pairs and list the new ordered pairs below. Are your new points on the graph of the second equation?
   (3,3) (0,2) (-3,1) (-6,0)

These two equations are **inverses** of each other. We can call one of them f(x) and the other f(x)$^{-1}$.

Look again at the graphs you drew. Now sketch in the graph of the line y = x on the same graph grid.

What do you notice?

The graphs are reflected over the line y = x !!

Finding Inverse Functions Algebraically

To find the equation of the inverse of a function algebraically, follow these steps:

1. Swith the x & y
2. Solve for y

Examples: Find the equation of the inverse of each of the following functions.

a. $f(x)=\frac{2}{3}x-1$

$x=\frac{2}{3}y-1$

$+1 \quad\quad +1$

$\frac{3}{2}(x+1)=\frac{2}{3}y\left(\frac{3}{2}\right)$

$\boxed{y=\frac{3}{2}x+\frac{3}{2}}$

b. $g(x)=2x^3+1$

$x=2y^3+1$

$-1 \quad\quad -1$

$\frac{x-1}{2}=\frac{2y^3}{2}$

$\sqrt[3]{\frac{x-1}{2}}=\sqrt[3]{y^3}$

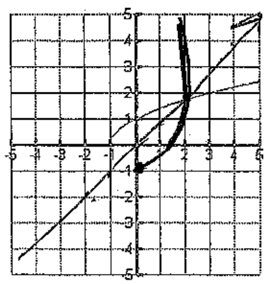
$\frac{\sqrt[3]{x-1}}{\sqrt[3]{2}}=y$

$y=\frac{\sqrt[3]{x-1}}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\boxed{\frac{\sqrt[3]{4(x-1)}}{2}}$

Inverses WS 1 Name Key

Graph the inverse of the following functions on the same set of axes. And identify the domain and range of the function and of its inverse.

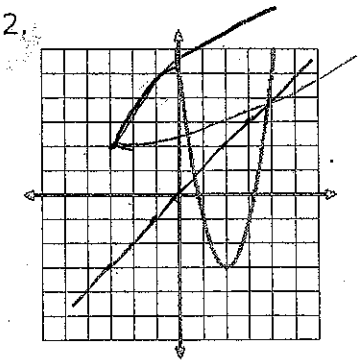
1.



Function
D: $[-1, \infty)$
R: $[0, \infty)$

Inverse
D: $[0, \infty)$
R: $[-1, \infty)$

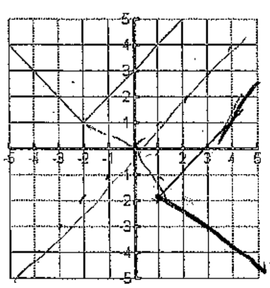
2.



Function
D: $\mathbb{R}$
R: $[-3, \infty)$

Inverse
D: $[-3, \infty)$
R: $\mathbb{R}$

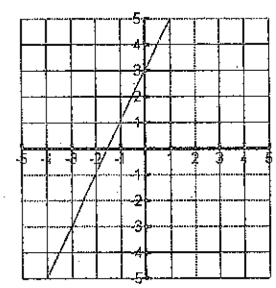
3.



Function
D: $\mathbb{R}$
R: $[1, \infty)$

Inverse
D: $[1, \infty)$
R: $\mathbb{R}$

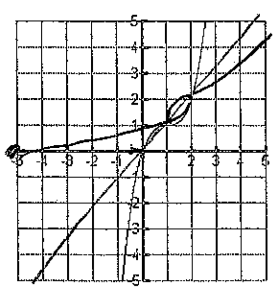
4.



Function
D: $\mathbb{R}$
R: $\mathbb{R}$

Inverse
D: $\mathbb{R}$
R: $\mathbb{R}$

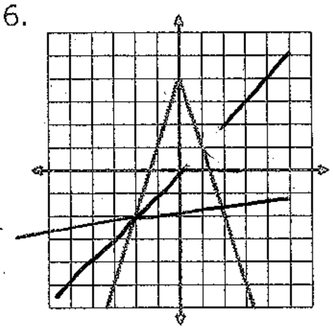
5.



Function
D: $\mathbb{R}$
R: $\mathbb{R}$

Inverse
D: $\mathbb{R}$
R: $\mathbb{R}$

6.



Function
D: $\mathbb{R}$
R: $(-\infty, 4]$

Inverse
D: $(-\infty, 4]$
R: $\mathbb{R}$

Find the inverse of the following equations.

7. $y = 3x - 1$

$x = 3y - 1$

$\frac{x+1}{3} = \frac{3y}{3}$

$\boxed{y = \frac{x+1}{3}}$

8. $y = \frac{1}{2}x - 5$

$x = \frac{1}{2}y - 5$

$2(x+5) = \frac{1}{2}y(2)$

$\boxed{y = 2x + 10}$

9. $y = \sqrt{x+3}$

$(x)^2 = (\sqrt{y+3})^2$

$x = y + 3$
$-3 \quad -3$

$\boxed{y = x - 3}$

10. $y = (x-2)^3 + 1$

$x = (y-2)^3 + 1$
$-1 \quad -1$

$\sqrt[3]{x-1} = \sqrt[3]{(y-2)^{\cancel{3}}}$

$\sqrt[3]{x-1} = y - 2$
$+2 \quad +2$

$\boxed{y = \sqrt[3]{x-1} + 2}$

11. $y = x^2 + 4$

$x = y^2 + 4$
$-4 \quad -4$

$\sqrt{x-4} = \sqrt{y^2}$

$\boxed{\sqrt{x-4} = y}$

12. $y = \sqrt[5]{5x+4}$

$(x)^5 = (\sqrt[5]{5y+4})^5$

$x^5 = 5y + 4$
$-4 \quad -4$

$\frac{5y}{5} = \frac{x^5 - 4}{5}$

$\boxed{y = \frac{x^5 - 4}{5}}$

13. $y = (x+2)^2$

$\sqrt{x} = \sqrt{(y+2)^{\cancel{2}}}$

$\sqrt{x} = y + 2$
$-2 \quad -2$

$\boxed{y = \sqrt{x} - 2}$

14. $y = \frac{1}{x+2}$

$(y+2)x = \frac{1}{\cancel{y+2}} \cdot \cancel{y+2}$

$\frac{x(y+2)}{x} = \frac{1}{x}$

$y + 2 = \frac{1}{x}$
$-2 \quad -2$

$\boxed{y = \frac{1}{x} - 2}$

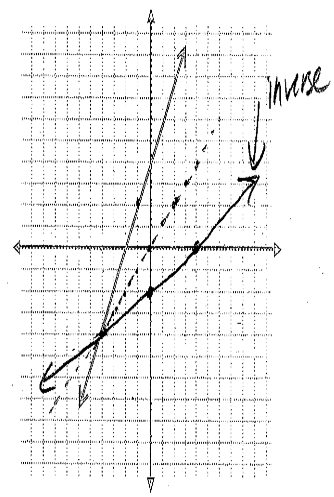
Inverses WS 2 Name ________________

The given coordinates are on $f(x)$, find the coordinates for $f^{-1}(x)$

1. $(-2,4)$ (4,-2)
2. $(4,7)$ (7,4)
3. $(0,11)$ (11,0)
4. $(-3,-8)$ (-8,-3)
5. $(10,10)$ (10,10)

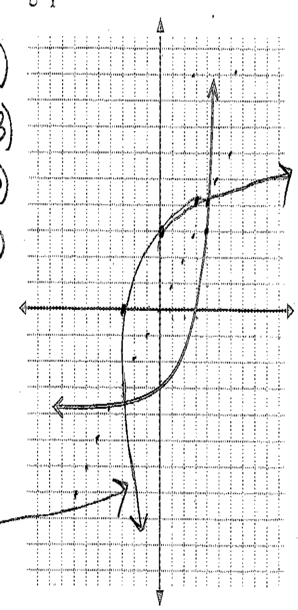
Answer each question regarding the function shown in the graph.

6. What is the x-intercept of the function? (-2,0)
7. What is the y-intercept of the function? (0,4)
8. What is the x-intercept of the inverse? (4,0)
9. What is the y-intercept of the inverse? (0,-2)
10. Name one other point on the function. (-1,2)
11. Name one other point on the inverse. (2,-1)
12. Graph the y = x line with a dotted line.
13. Graph the inverse.



Answer each question regarding the function shown in the graph.

14. What is the x-intercept of the function? (3,0)
15. What is the y-intercept of the function? (0,-3)
16. What is the x-intercept of the inverse? (-3,0)
17. What is the y-intercept of the inverse? (0,3)
18. Name one other point on the function. (4,3)
19. Name one other point on the inverse. (3,4)
20. Graph the y = x line with a dotted line.
21. Graph the inverse.



Composition of Functions

The COMPOSITION of f with g is

$$(f \circ g)(x) = f(g(x))$$

Ex. Given: $f(x) = 2x$, $g(x) = 8x - 6$

means take g(x) function + plug into x in f(x) function

$f(g(x)) =$

$f(8x-6) = 2(8x-6)$

$\boxed{f(g(x)) = 16x-12}$

Now, evaluate $f(g(5))$ two ways ...

$g(5) = 8(5) - 6$

$= 40 - 6$

$= 34$

$f(34) = 2(34)$

$\boxed{f(g(5)) = 68}$

Ex. Given: $f(x) = 2x$, $g(x) = 8x - 6$

$g(f(x)) =$

$g(2x) = 8(2x) - 6$

$= \boxed{16x - 6}$

$f(f(x)) =$

$f(2x) = 2(2x)$

$= \boxed{4x}$

Ex. Given: $f(x) = x^2 - 2x - 15$, $g(x) = x + 3$

$f(g(x))$

$f(x+3) = (x+3)^2 - 2(x+3) - 15$

$= x^2 + 6x + 9 - 2x - 6 - 15 = \boxed{x^2 + 4x - 12}$

$g(f(x))$

$g(x^2 - 2x - 15) = (x^2 - 2x - 15) + 3 = \boxed{x^2 - 2x - 12}$

$g(g(x))$

$g(x+3) = (x+3) + 3$

$= \boxed{x + 6}$

Composition of Functions WS Name ____________________

Use the following functions:

$f(x)=3x+5$ $g(x)=x^2-1$ $h(x)=-\frac{1}{2}x-2$ $k(x)=3x$

Find the following:

1. $f(h(2))$

$h(2)=-\frac{1}{2}(2)-2=-3$

$f(-3)=3(-3)+5$

$=-9+5$

$=\boxed{-4}$

2. $f\left(k\left(\frac{1}{2}\right)\right)$

$k(\frac{1}{2})=\frac{3}{2}$

$f(\frac{3}{2})=3(\frac{3}{2})+5$

$=\boxed{9.5}$

3. $g(f(4))$

$f(4)=3(4)+5=17$

$g(17)=(17)^2-1$

$=\boxed{288}$

4. $h(f(1))$

$f(1)=3(1)+5=8$

$h(8)=-\frac{1}{2}(8)-2$

$=\boxed{-6}$

5. $(h\circ g)(-4)$

$g(-4)=(-4)^2-1=15$

$h(15)=-\frac{1}{2}(15)-2$

$=\boxed{-9.5}$

6. $(k\circ f)(-5)$

$f(-5)=3(-5)+5=-10$

$k(-10)=3(-10)$

$=\boxed{-30}$

7. $(f\circ f)(3)$

$f(3)=3(3)+5=14$

$f(14)=3(14)+5$

$=\boxed{47}$

8. $(g\circ g)(-2)$

$g(-2)=(-2)^2-1=3$

$g(3)=3^2-1$

$=\boxed{8}$

9. $f(h(k(2)))$

$k(2)=3(2)=6$

$h(6)=-\frac{1}{2}(6)-2$

$=-3-2=-5$

$f(-5)=3(-5)+5=\boxed{-10}$

10. $h(f(g(-5)))$

$g(-5)=(-5)^2-1=24$

$f(24)=3(24)+5=77$

$h(77)=-\frac{1}{2}(77)-2$

$=-38.5-2$

$=\boxed{-40.5}$

11. $f(g(x))$

$f(x^2-1)=3(x^2-1)+5$

$=3x^2-3+5$

$=\boxed{3x^2+2}$

12. $f(f(x))$

$f(3x+5)=3(3x+5)+5$

$=9x+15+5$

$=\boxed{9x+20}$

13. $f(h(x))$

$f(-\frac{1}{2}x-2)=3(-\frac{1}{2}x-2)+5$

$=\boxed{-\frac{3}{2}x-1}$

14. $g(k(x))$

$g(3x)=(3x)^2-1$

$=\boxed{9x^2-1}$

15. $(h\circ k)(x)$

$h(3x)=-\frac{1}{2}(3x)-2$

$=\boxed{-\frac{3x}{2}-2}$

16. $(k\circ g)(x)$

$k(x^2-1)=3(x^2-1)$

$=\boxed{3x^2-3}$

17. $(g\circ f)(x)$

$g(3x+5)=(3x+5)^2-1$

$=9x^2+30x+25-1$

$=\boxed{9x^2+30x+24}$

18. $(h\circ g)(x)$

$h(x^2-1)=-\frac{1}{2}(x^2-1)-2$

$=-\frac{1}{2}x+\frac{1}{2}-2$

$=\boxed{-\frac{1}{2}x-1.5}$