

# Advanced Algebra – Unit 4A

## Logarithms

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**Monday, February 2<sup>nd</sup>**

***Graphing Exponential Functions***

Outline w/PowerPoint (Day 1):

Graphing an Exponential Parent

CW/HW: Graphing Exp & Logs WS 1  
(exponentials ONLY)

**Tuesday, February 3<sup>rd</sup>**

***Graphing Exponential Functions***

Outline w/PowerPoint (Day 2):

Transformations of Exponentials

CW/HW: Graphing Exp & Logs WS 2  
(exponentials ONLY)

**Wednesday, February 4<sup>th</sup>**

***Inverse Functions***

Inverses Notes Outline w/SMART

CW/HW: Inverses WS 1

**Thursday, February 5<sup>th</sup>**

***Inverse Functions 2***

Go over Inverses WS 1

CW/HW: Inverses WS 2

**Friday, February 6<sup>th</sup>**

***Composition of Functions***

Notes & Examples PowerPoint

CW/HW: Composition WS

**Monday, February 9<sup>th</sup>**

***Verifying Inverses***

Verifying Inverses Notes &

Examples PowerPoint

CW/HW: Inverses WS 3

**Tuesday, February 10<sup>th</sup>**

Quiz: Graphing, Inverses and  
Composition

**Wednesday, February 11<sup>th</sup>**

Solving Exponential Equations Notes

(with like bases)

CW/HW: Solving Exponential  
Equations WS (with like bases)

**Thursday, February 12<sup>th</sup>**

Applications with exponentials

Walk-Through Problems

CW/HW: More Exponential  
Applications WS

**Friday, February 13<sup>th</sup>**

Graphing Inverses of Exponential

Equations Notes [Smart]

CW/HW: Graphing Exponential  
Functions & Their Inverse WS

**Monday, February 16<sup>th</sup>**

Intro to logs Notes [Smart]

CW/HW: Converting Forms WS &  
Logs WS1

**Tuesday, February 17<sup>th</sup>**

Inverses of exponential functions  
notes [Smart]

CW/HW: Finding Inverses of  
Exponential Functions WS

**Wednesday, February 18<sup>th</sup>**

CFA

**Thursday, February 19<sup>th</sup>**

Review

**Friday, February 20<sup>th</sup>**

Test


# Graphing Exponential and Logarithmic Functions

Day 1 – Exponential Functions ONLY – slides 1 and 2

Day 2 – Exponential Functions ONLY – slides 3 and 4

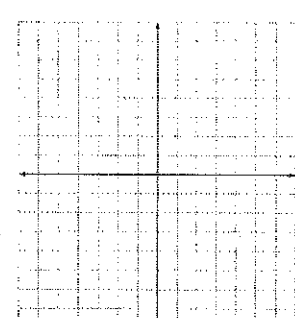
Day 3 – Logarithmic Functions – slides 1, 2, 3, 4

Slide 1



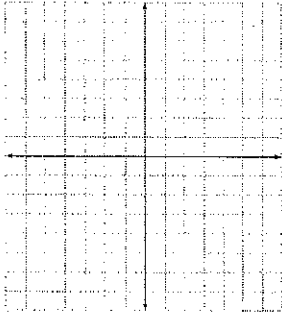
$$f(x) = 5^x$$

	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain		
Range		
Intercept		
Asymptote		
Growth/Decay?		N/A
**Equation of the inverse of the exponential function		



Slide 2

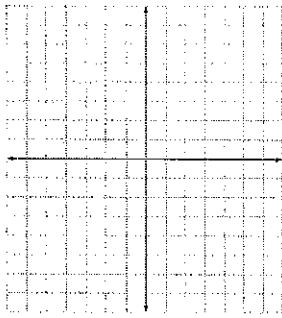
$f(x) = 5^{-x}$



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	_____	_____
Range	_____	_____
Intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function	_____	

Slide 3

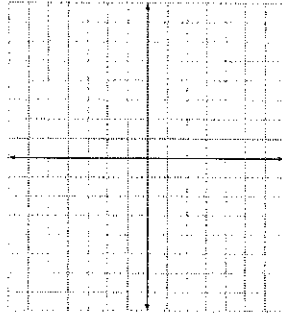
$f(x) = 5^{(x-2)} - 3$



	EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION
Domain	_____	_____
Range	_____	_____
Intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function	_____	

Slide 4

$$f(x) = -(1/5)^x + 2$$



EXPONENTIAL  
FUNCTION

\*\*LOGARITHMIC  
FUNCTION

Domain \_\_\_\_\_

Range \_\_\_\_\_

Intercept \_\_\_\_\_

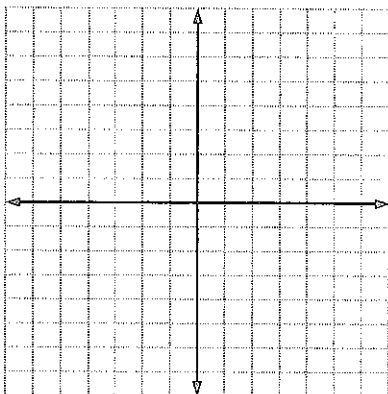
Asymptote \_\_\_\_\_

Growth/Decay? \_\_\_\_\_ N/A

\*\*Equation of the inverse of the exponential function

\_\_\_\_\_

1.  $f(x) = 2^x$



EXPONENTIAL  
FUNCTION

\*\*LOGARITHMIC  
FUNCTION

Domain \_\_\_\_\_

Range \_\_\_\_\_

intercept \_\_\_\_\_

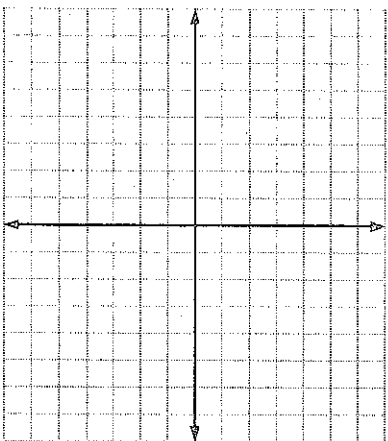
Asymptote \_\_\_\_\_

Growth/Decay? \_\_\_\_\_ N/A

\*\*Equation of the inverse of the exponential function

\_\_\_\_\_

2.  $f(x) = \frac{1}{3}^x$



EXPONENTIAL  
FUNCTION

\*\*LOGARITHMIC  
FUNCTION

Domain \_\_\_\_\_

Range \_\_\_\_\_

intercept \_\_\_\_\_

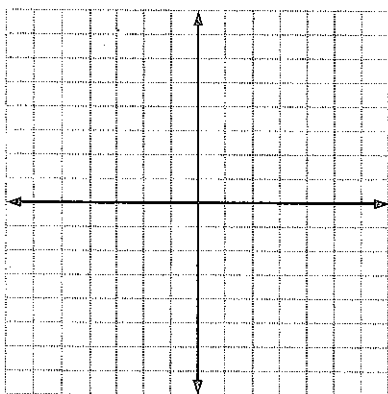
Asymptote \_\_\_\_\_

Growth/Decay? \_\_\_\_\_ N/A

\*\*Equation of the inverse of the exponential function

\_\_\_\_\_

3.  $f(x) = 10^x$



EXPONENTIAL  
FUNCTION

\*\*LOGARITHMIC  
FUNCTION

Domain \_\_\_\_\_

Range \_\_\_\_\_

intercept \_\_\_\_\_

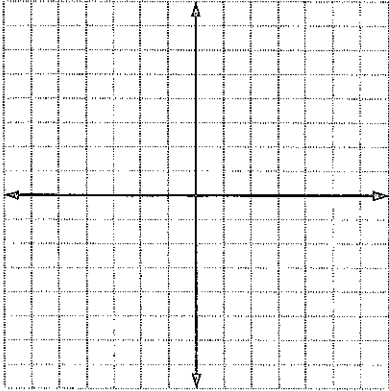
Asymptote \_\_\_\_\_

Growth/Decay? \_\_\_\_\_ N/A

\*\*Equation of the inverse of the exponential function

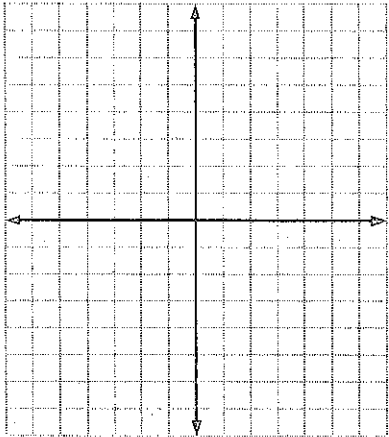
\_\_\_\_\_

4.  $f(x) = 3^x$



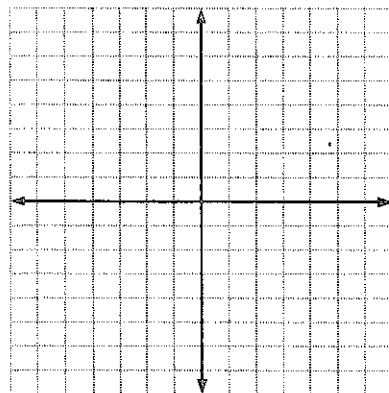
	<u>EXPONENTIAL FUNCTION</u>	<u>**LOGARITHMIC FUNCTION</u>
Domain	_____	_____
Range	_____	_____
intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function		
_____		

5.  $f(x) = 2^{-x}$



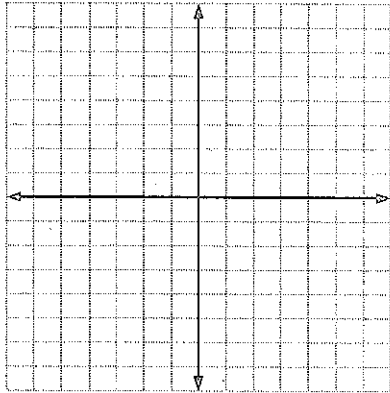
	<u>EXPONENTIAL FUNCTION</u>	<u>**LOGARITHMIC FUNCTION</u>
Domain	_____	_____
Range	_____	_____
intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function		
_____		

6.  $f(x) = e^x$



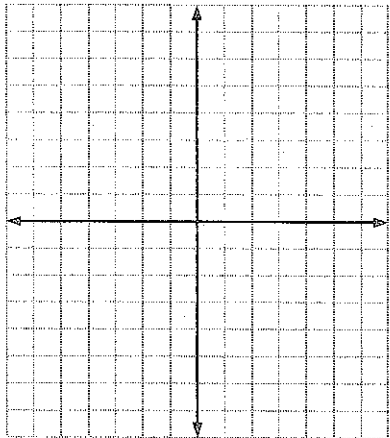
	<u>EXPONENTIAL FUNCTION</u>	<u>**LOGARITHMIC FUNCTION</u>
Domain	_____	_____
Range	_____	_____
intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function		
_____		

1.  $f(x) = 2^x - 3$



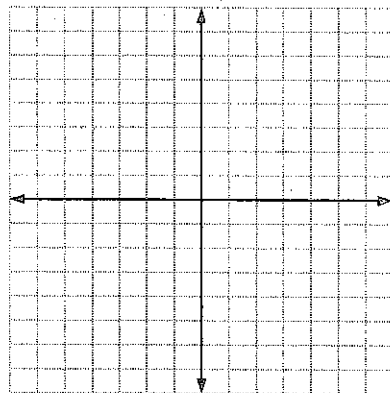
	<u>EXPONENTIAL FUNCTION</u>	<u>**LOGARITHMIC FUNCTION</u>
Domain	_____	_____
Range	_____	_____
Intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function	_____	

2.  $f(x) = \left(\frac{1}{3}\right)^{x-2}$



	<u>EXPONENTIAL FUNCTION</u>	<u>**LOGARITHMIC FUNCTION</u>
Domain	_____	_____
Range	_____	_____
Intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function	_____	

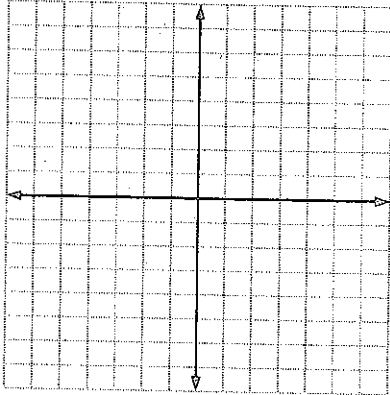
3.  $f(x) = -10^x + 4$



	<u>EXPONENTIAL FUNCTION</u>	<u>**LOGARITHMIC FUNCTION</u>
Domain	_____	_____
Range	_____	_____
Intercept	_____	_____
Asymptote	_____	_____
Growth/Decay?	_____	N/A
**Equation of the inverse of the exponential function	_____	

4.

$$f(x) = 3^{x+4} - 5$$



EXPONENTIAL  
FUNCTION

\*\*LOGARITHMIC  
FUNCTION

Domain

\_\_\_\_\_

\_\_\_\_\_

Range

\_\_\_\_\_

\_\_\_\_\_

Intercept

\_\_\_\_\_

\_\_\_\_\_

Asymptote

\_\_\_\_\_

\_\_\_\_\_

Growth/Decay?

\_\_\_\_\_

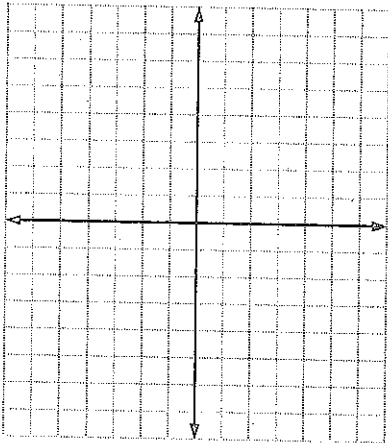
N/A

\*\*Equation of the inverse of the exponential function

\_\_\_\_\_

5.

$$f(x) = -2^{-x} - 3$$



EXPONENTIAL  
FUNCTION

\*\*LOGARITHMIC  
FUNCTION

Domain

\_\_\_\_\_

\_\_\_\_\_

Range

\_\_\_\_\_

\_\_\_\_\_

Intercept

\_\_\_\_\_

\_\_\_\_\_

Asymptote

\_\_\_\_\_

\_\_\_\_\_

Growth/Decay?

\_\_\_\_\_

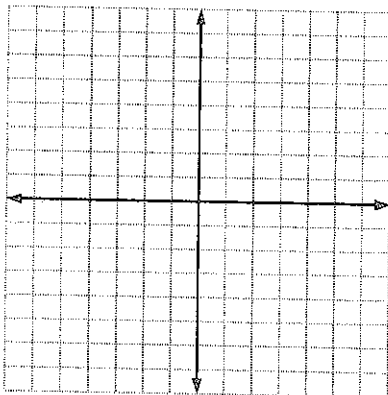
N/A

\*\*Equation of the inverse of the exponential function

\_\_\_\_\_

6.

$$f(x) = e^{x-1} + 3$$



EXPONENTIAL  
FUNCTION

\*\*LOGARITHMIC  
FUNCTION

Domain

\_\_\_\_\_

\_\_\_\_\_

Range

\_\_\_\_\_

\_\_\_\_\_

Intercept

\_\_\_\_\_

\_\_\_\_\_

Asymptote

\_\_\_\_\_

\_\_\_\_\_

Growth/Decay?

\_\_\_\_\_

N/A

\*\*Equation of the inverse of the exponential function

\_\_\_\_\_



## NOTES ON INVERSES

INVERSES – Two functions are inverses, if and only if, when one function contains a point  $(a, b)$ , the other function contains the point \_\_\_\_\_.

Example:  $f(x) = \{(3, 1), (-2, 4), (5, -1)\}$  Domain of  $f(x)$  \_\_\_\_\_ Range of  $f(x)$  \_\_\_\_\_

The inverse of  $f(x)$  will be { \_\_\_\_\_ } Domain of inverse \_\_\_\_\_

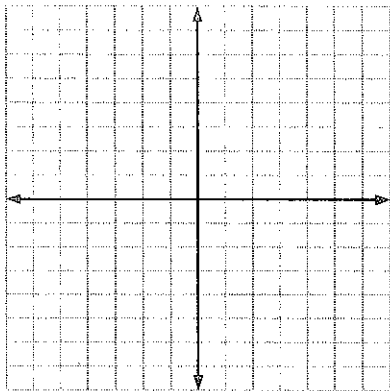
Range of inverse \_\_\_\_\_

\*\*\*\*The domain of function  $f(x)$  has become the \_\_\_\_\_ of the inverse;

The range of function  $f(x)$  has become the \_\_\_\_\_ of the inverse.

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## Graphing Inverse Functions



1. Graph  $y = 3x - 6$  and list four points that appear on your graph.

2. Now graph  $y = \frac{1}{3}x + 2$  on the same axes.

3. Switch the  $x$  and  $y$  coordinates in your original ordered pairs and list the new ordered pairs below. Are your new points on the graph of the second equation?

These two equations are inverses of each other. We can call one of them \_\_\_\_\_ and the other \_\_\_\_\_.

Look again at the graphs you drew. Now sketch in the graph of the line  $y = x$  on the same graph grid.

What do you notice?

## Finding Inverse Functions Algebraically

To find the equation of the inverse of a function algebraically, follow these steps:

1.

2.

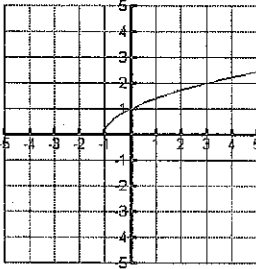
Examples: Find the equation of the inverse of each of the following functions.

a.  $f(x) = \frac{2}{3}x - 1$

b.  $g(x) = 2x^3 + 1$

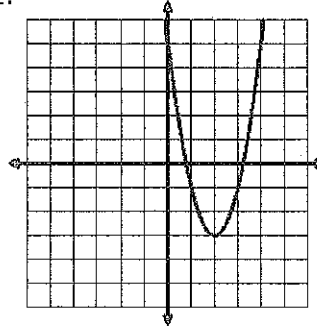
Graph the inverse of the following functions on the same set of axes. And identify the domain and range of the function and of its inverse.

1.



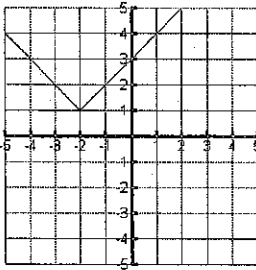
Function  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_  
Inverse  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_

2.



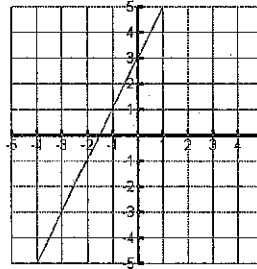
Function  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_  
Inverse  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_

3.



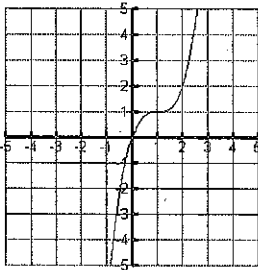
Function  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_  
Inverse  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_

4.



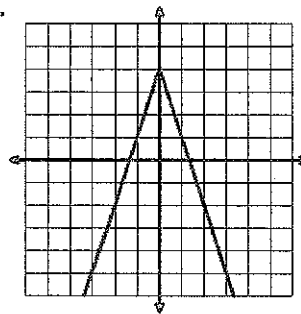
Function  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_  
Inverse  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_

5.



Function  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_  
Inverse  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_

6.



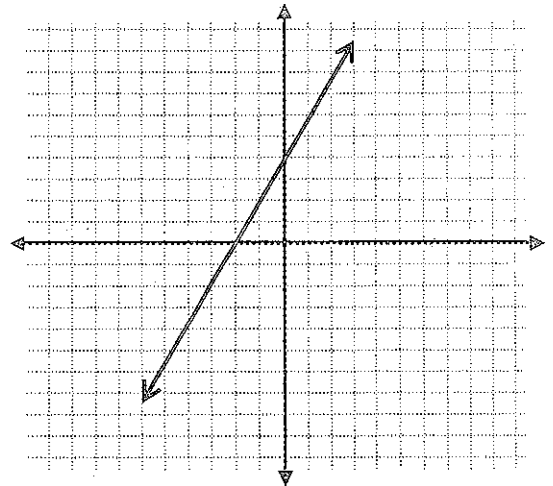
Function  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_  
Inverse  
 D: \_\_\_\_\_  
 R: \_\_\_\_\_

The given coordinates are on  $f(x)$ , find the coordinates for  $f^{-1}(x)$

1.  $(-2, 4)$
2.  $(4, 7)$
3.  $(0, 11)$
4.  $(-3, -8)$
5.  $(10, 10)$

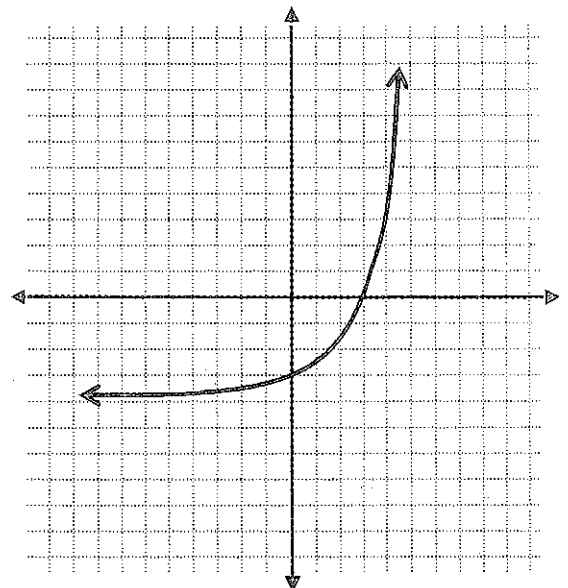
Answer each question regarding the function shown in the graph.

6. What is the x-intercept of the function?
7. What is the y-intercept of the function?
8. What is the x-intercept of the inverse?
9. What is the y-intercept of the inverse?
10. Name one other point on the function.
11. Name one other point on the inverse.
12. Graph the  $y = x$  line with a dotted line.
13. Graph the inverse.



Answer each question regarding the function shown in the graph.

14. What is the x-intercept of the function?
15. What is the y-intercept of the function?
16. What is the x-intercept of the inverse?
17. What is the y-intercept of the inverse?
18. Name one other point on the function.
19. Name one other point on the inverse.
20. Graph the  $y = x$  line with a dotted line.
21. Graph the inverse.



## Composition of Functions

The COMPOSITION of  $f$  with  $g$  is

$$(f \circ g)(x) = f(g(x))$$

Ex. Given:  $f(x) = 2x$ ,  $g(x) = 8x - 6$

$$f(g(x)) =$$

Now, evaluate  $f(g(5))$  two ways ...

Ex. Given:  $f(x) = 2x$ ,  $g(x) = 8x - 6$

$g(f(x)) =$

$f(g(x)) =$

Ex. Given:  $f(x) = x^2 - 2x - 15$ ,  $g(x) = x + 3$

$f(g(x))$

$g(f(x))$

$g(g(x))$

---

Use the following functions:

$$f(x) = 3x + 5$$

$$g(x) = x^2 - 1$$

$$h(x) = -\frac{1}{2}x - 2$$

$$k(x) = 3x$$

---

Find the following:

1.  $f(h(2))$

2.  $f\left(k\left(\frac{1}{2}\right)\right)$

3.  $g(f(4))$

4.  $h(f(1))$

5.  $(h \circ g)(-4)$

6.  $(k \circ f)(-5)$

7.  $(f \circ f)(3)$

8.  $(g \circ g)(-2)$

9.  $f(h(k(2)))$

10.  $h(f(g(-5)))$

11.  $f(g(x))$

12.  $f(f(x))$

13.  $f(h(x))$

14.  $g(k(x))$

15.  $(h \circ k)(x)$

16.  $(k \circ g)(x)$

17.  $(g \circ f)(x)$

18.  $(h \circ g)(x)$

# \*Verifying Inverses

Proofs

# \*Verifying Inverses

\* $f$  and  $g$  are inverses iff ...

$$*f(g(x)) = x \text{ AND } g(f(x)) = x$$



**\*Example 1**

\*Verify that  $f$  and  $g$  are inverses.

$$f(x) = \frac{5}{x-2} \quad g(x) = \frac{5}{x} + 2$$

**\*Example 2**

\*Verify that  $f$  and  $g$  are inverses.

$$f(x) = 2x^3 - 1 \quad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

Fill in the blank.

1. If  $(-3, 1)$  is on  $f$ , then \_\_\_\_\_ is on  $f^{-1}$ .
  2. If  $(-3, 0)$  is the x-intercept of  $f$ , then \_\_\_\_\_ is the y-intercept of  $f^{-1}$ .
  3. If  $[-2, \infty)$  is the range of  $f$ , then \_\_\_\_\_ is the domain of  $f^{-1}$ .
  4. If  $[3, \infty)$  is the domain of  $f^{-1}$ , then \_\_\_\_\_ is the range of  $f$ .
- 

Verify that  $f$  and  $g$  are inverse functions (or not).

In order to do this you must prove that  $f(g(x)) = x$  and  $g(f(x)) = x$ .

5.  $f(x) = x + 4$ ;  $g(x) = x - 4$

6.  $f(x) = 2x - 4$ ;  $g(x) = \frac{1}{2}x + 2$

7.  $f(x) = x^2 + 2, x \geq 0$ ;  $g(x) = \sqrt{x - 2}$

8.  $f(x) = \frac{1}{3}x^3 - 2$ ;  $g(x) = \sqrt[3]{3x + 6}$

9.  $f(x) = 3 - x$ ;  $g(x) = 3 - x$

# Solving Exponential Equations

(with like bases)

Ex 1:

$$0 \quad 7^{2x} = 7^{3x-5}$$

Ex 2:

$$0 \quad 5^{4m} = 125^{m+2}$$

Ex 3:

$$0 \quad 9^x = 27$$

Ex 4:

$$0 \quad \left(\frac{1}{2}\right)^x = 16^{3x-1}$$

Solve each equation.

1. $5^x = 5^{-3}$	2. $6^x = 216$
3. $7^y = \frac{1}{49}$	4. $10^x = .001$
5. $2^{2x} = \frac{1}{8}$	6. $\left(\frac{1}{5}\right)^{x-3} = 125$
7. $3^y = 3^{3y+1}$	8. $5^{3y+4} = 5^y$
9. $3^x = 9^{x+1}$	10. $2^5 = 2^{2x-1}$
11. $8^{x-1} = 16^{3x}$	12. $2^{x+3} = \frac{1}{16}$

Answers:      1) -3   2) 3   3) -2   4) -3   5) -3/2   6) 0   7) -1/2   8) -2   9) -2   10) 3   11) -1/3   12) -7

Interest compounded continuously  $A = Pe^{rt}$ Growth/Decay  $y = ae^{kt}$ Interest compounded frequently but not continuously  $A = P\left(1 + \frac{r}{n}\right)^{nt}$ 

Write an equation for each of the following. Then use your calculator to find the answer.

1. Thrifty Thelma invests \$7500 in an account paying 4% interest compounded quarterly. How much will be in Thelma's account at the end of 6 years?

2. Sam the Saver invests \$500 in an account that pays 3.5% interest compounded continuously. How much money will Sam have after 3 years?

3. If you invest \$2100 in a savings account that pays 2.25% interest compounded monthly, how much money will you have at the end of one year?

4. Ted invested \$675 in an account that pays 3.4% interest compounded continuously. How much will be in his account after 6 months?

5. Your parents just won the Mega Millions Lottery. Because they love you so much, they decide to give you some of their winnings; however, they don't want you to have the money until your 22<sup>nd</sup> birthday. So they invest \$15,000 in a trust fund that pays  $3\frac{5}{8}\%$  interest compounded continuously.

Assuming that you are 17 years old right now, how much money will you get when you are 22?

1. The amount in trillions of cubic feet of natural gas consumed in the United States from 1940 to 1970 can be modeled by the function  $y = a(1.07)^t$ , where  $t$  is the number of years since 1940.

(a) Assuming 2.91 trillion cubic feet of natural gas was consumed in the US in 1940, estimate how many cubic feet were consumed in 1937.

(b) Predict the natural gas usage for 2012.

2. From 1971 to 1995, the average number of transistors on a computer chip can be modeled by the function  $Y = a(1.59)^t$ , where  $t$  represents the number of years since 1971.

(a) Assuming there was an average of 2300 transistors per chip in 1971, estimate the number of transistors on each chip in 1998.

(b) How many transistors were used on each chip in 2011?

3. Connor the contractor likes to buy houses, remodel them, live in them for a while and then sell the house and buy another one. Seven and a half years ago, he paid \$137,000 for the house he now lives in. During that time, house values have appreciated according to the model  $y = a(1.024)^t$  where  $a$  is the initial value and  $t$  is time in years.

(a) How much is his house worth now?

(b) Connor decides to use a real estate agent to sell the house. He must pay the agent a commission of 3.5% of the selling price of the house. Connor has found a buyer willing to pay \$173,900 for the house. After the real estate agent is paid, how much profit will Connor make on the sale of the house?

4. Since 1972 the US Fish and Wildlife Service has kept a list of endangered species in the United States. In 1972 there were 119 species on the endangered list. For the years from 1972 to 1998, the number of species on the list can be modeled by the equation  $y = ae^{0.0917t}$  where  $t$  is the number of years since 1972. Estimate the number of species on the endangered list at the present time.



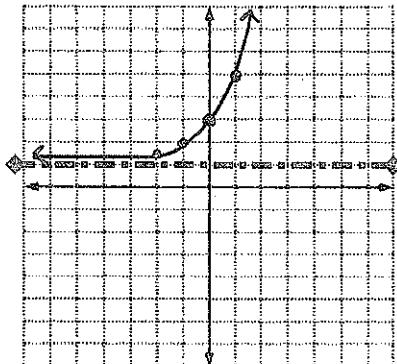
**Graphing Exponential Equations  
and Their Inverse Functions - NOTES**

- What do we need to find to graph an exponential function?

- To graph:

- 1) Make a T-Chart centered where the exponent will = 0.
- 2) Find the y-intercept (will always have one)
- 3) Sketch the asymptote
- 4) Sketch the graph
- 5) Determine the domain, range and equation of the asymptote

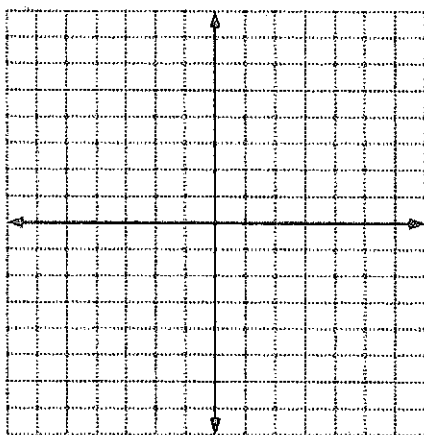
**ALL EXPONENTIAL FUNCTIONS WILL HAVE AN ASYMPTOTE!**



**Example 1: Graph the following function and its inverse.**

$$f(x) = 2^{x+1} - 3$$

	Function	Inverse
x-int:	_____	_____
y-int:	_____	_____
Dom:	_____	_____
Range:	_____	_____
Asymp:	_____	_____



x-int: \_\_\_\_\_

y-int: \_\_\_\_\_

Dom: \_\_\_\_\_

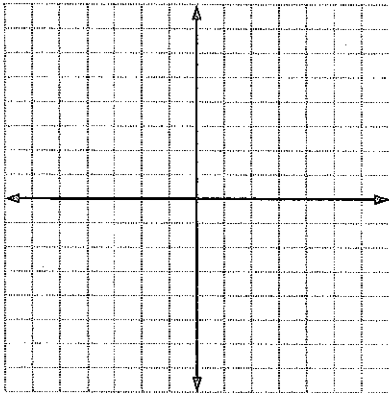
Range: \_\_\_\_\_

Asymp: \_\_\_\_\_

Worksheet—Graphing Exponential Functions and Their Inverses

Name \_\_\_\_\_

1.  $f(x) = 2^x$



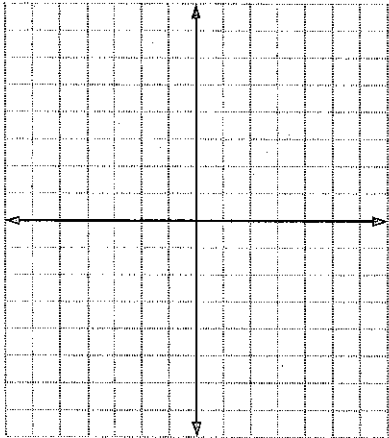
Domain \_\_\_\_\_  
 Range \_\_\_\_\_  
 Y intercept \_\_\_\_\_  
 X intercept \_\_\_\_\_  
 Asymptote \_\_\_\_\_

EXPONENTIAL  
FUNCTION

INVERSE  
FUNCTION

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

2.  $f(x) = 2^x - 3$



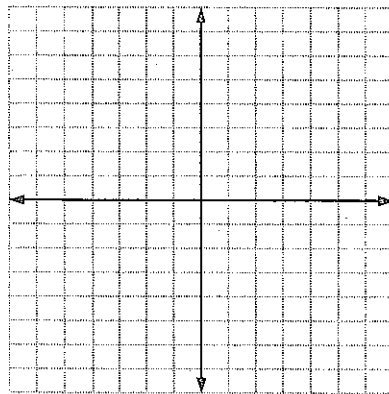
Domain \_\_\_\_\_  
 Range \_\_\_\_\_  
 Y intercept \_\_\_\_\_  
 X intercept \_\_\_\_\_  
 Asymptote \_\_\_\_\_

EXPONENTIAL  
FUNCTION

INVERSE  
FUNCTION

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

3.  $f(x) = \frac{1}{3}^x$



Domain \_\_\_\_\_  
 Range \_\_\_\_\_  
 Y intercept \_\_\_\_\_  
 X intercept \_\_\_\_\_  
 Asymptote \_\_\_\_\_

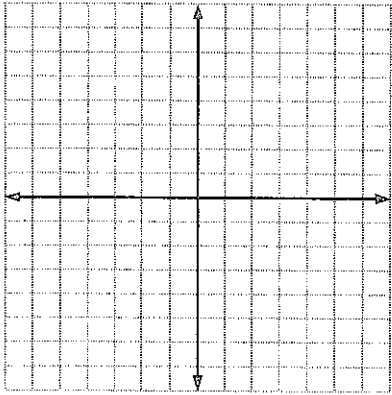
EXPONENTIAL  
FUNCTION

INVERSE  
FUNCTION

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

7.

$$f(x) = \left(\frac{1}{2}\right)^{x-3} + 2$$



Domain

Range

Y intercept

X intercept

Asymptote

EXPONENTIAL  
FUNCTION

INVERSE  
FUNCTION

\_\_\_\_\_

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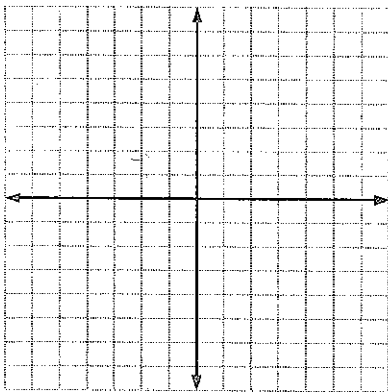
\_\_\_\_\_

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8.

$$f(x) = e^{x-1} + 3$$



Domain

Range

Y intercept

X intercept

Asymptote

EXPONENTIAL  
FUNCTION

INVERSE  
FUNCTION

\_\_\_\_\_

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# Intro to Logarithms

## Definition of Logarithm with base $b$

The **logarithm of  $y$  with base  $b$**  is denoted by  $\log_b y$  and is defined as follows:

$$\log_b y = x \quad \text{iff} \quad b^x = y$$

$$b > 0, y > 0, b \neq 1$$

$\log_b y$  is read as "log base  $b$  of  $y$ "

---

**Example 1:** Write the logarithmic equation in exponential form.

a.  $\log_3 9 = 2$

b.  $\log_8 1 = 0$

c.  $\log_5 \left( \frac{1}{25} \right) = -2$

**Special Logarithmic Values**

$$\log_b 1 = 0$$

$$\log_b b = 1$$

Example 2: Evaluate the expression.

a.  $\log_4 64$

b.  $\log_2 0.125$

c.  $\log_{\frac{1}{4}} 256$

d.  $\log_{32} 2$

Example 3: Solve for x.

a.  $\log_{\frac{1}{2}} x = -3$

b.  $\log_x \sqrt{8} = \frac{1}{2}$

Natural Logarithm:  $\ln x = \log_e x$

Common Logarithm:  $\log x = \log_{10} x$

Example 4: Evaluate using a calculator.

a.  $\log 7$

b.  $\ln 0.25$

# Intro to Logarithms

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Write the logarithmic equation in exponential form.

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b.  $\log_8 1 = 0$

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## Special Logarithmic Values

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Evaluate the expression.

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### Example 3:

Solve for  $x$ .

e.  $\log_{\frac{1}{2}} x = -3$

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## Natural Logarithm: $\ln x = \log_e x$

## Common Logarithm: $\log x = \log_{10} x$

### Example 4:

Evaluate using a calculator.

a.  $\log 7$

b.  $\ln 0.25$

Student Name: \_\_\_\_\_

Score: \_\_\_\_\_

**Logarithmic Form**

Write the following exponents in logarithmic form:

Exponent Form	Logarithmic Form
$2^5 = 32$	
$3^3 = 27$	
$5^3 = 125$	
$2^{-4} = \frac{1}{16}$	
$4^3 = 64$	
$3^2 = 9$	
$7^{-2} = \frac{1}{49}$	

Student Name: \_\_\_\_\_

Score: \_\_\_\_\_

**Exponent Form**

Write the following logarithmic form into exponent form:

Logarithmic Form	Exponent Form
$\log_3 9 = 2$	
$\log_4 64 = 3$	
$\log_5 25 = 2$	
$\log_2 128 = 7$	
$\log_3 \left(\frac{1}{27}\right) = -3$	
$\log_6 36 = 2$	
$\log_5 \left(\frac{1}{625}\right) = -4$	



## I. Change the expression to logarithmic form.

1. $3^4 = 81$	2. $10^3 = 1000$
3. $2^x = 32$	4. $2^3 = x$

## II. Change the expression to exponential form.

5. $\log_4 16 = 2$	6. $\log_5 125 = 3$
7. $\log_3 9 = 2$	8. $\log_6 6 = 1$

## III. Evaluate

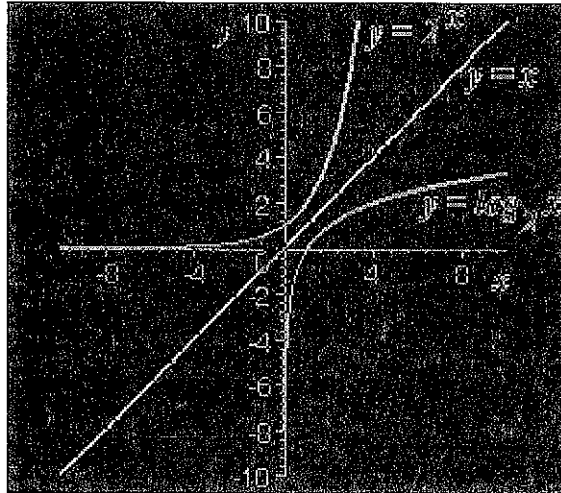
9. $\log_8 2$	10. $\log_7 1$	11. $\log 0.01$
12. $\log_3 \frac{1}{81}$	13. $\log_{\frac{1}{2}} 8$	14. $\log_4 2$
15. $\log_m m^3$	16. $\log_{27} 9$	17. $\log_3 243$
18. $\log_{\frac{1}{16}} 8$	19. $\log \sqrt{1000}$	20. $5^{\log_5 14}$
21. $\log_3 3^4$	22. $\log_{15} 1$	23. $\log_2 \frac{1}{16}$
24. $\log_{\frac{1}{3}} 27$	25. $\log_9 9$	26. $\log_8 4$

## IV. Solve

27. $\log_{\frac{1}{2}} 16 = x$	28. $\log_5 x = -2$
29. $\log_m \frac{1}{27} = -3$	30. $\log_x \sqrt[3]{7} = \frac{1}{3}$
31. $\log_{\frac{1}{2}} x = -6$	32. $\log_{64} 8 = x$

Odd Answers: 9)  $\frac{1}{3}$  11)  $-2$  13)  $-3$  15)  $3$  17)  $5$  19)  $\frac{3}{2}$  21)  $4$  23)  $-4$  25)  $1$  27)  $-4$  29)  $3$  31)  $64$

## Finding Inverses of Exponential and Log Functions Notes



\* The inverse of an exponential function is a log function.

\* The inverse of a log function is an exponential function.

~~~~~  
 We can convert forms following the rule:

$$y = b^x \text{ -----} > \log_b y = x$$

$$\quad \quad \quad \&$$

$$\log_b y = x \text{ -----} > y = b^x$$

To find the inverse of either of these functions, simply convert forms, then switch the 'x' and 'y'.

Example 1:

Find the inverse of the following.

$$y = 5^x$$

Example 2:

Find the inverse of the following.

$$y = 5^{x-2} + 1$$

Example 3:

Find the inverse of the following.

$$y = \log_5 x - 3$$

For each of the following, find the inverse.

|                                            |                         |
|--------------------------------------------|-------------------------|
| 1. $f(x) = 3^x$                            | 2. $f(x) = 3^x + 4$     |
| 3. $f(x) = 3^{x-2}$                        | 4. $f(x) = 3^{x+3} - 5$ |
| 5. $g(x) = \left(\frac{1}{2}\right)^x + 8$ | 6. $f(x) = e^{x+3}$     |
| 7. $f(x) = e^x - 4$                        | 8. $h(x) = e^{x+2} - 3$ |