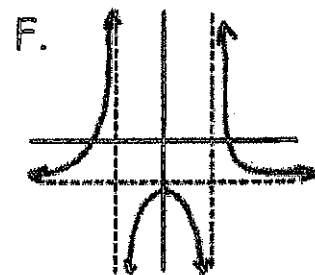
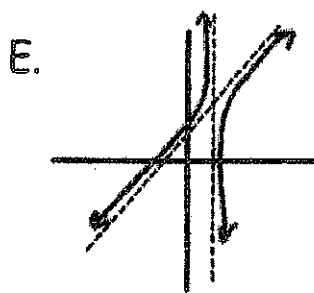
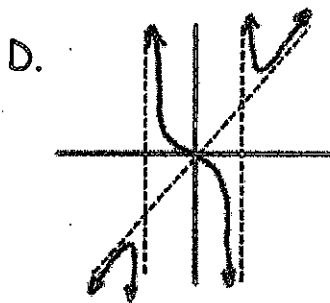
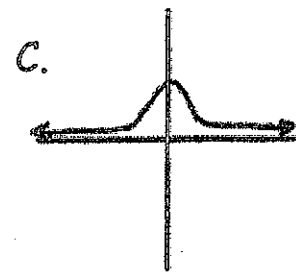
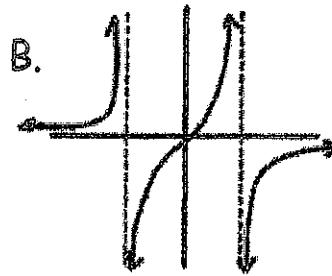
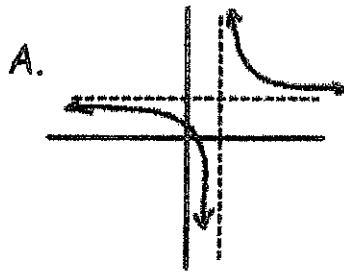


Intro to Graphing Rational Functions

Rational Functions come in a wide variety!

All of the following graphs are graphs of rational functions.



Looking at the graphs above, identify ...

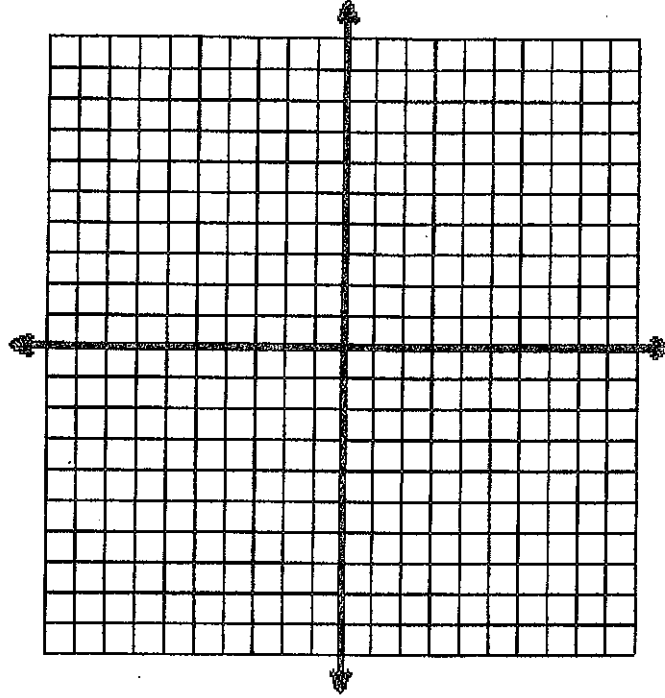
- 1) ... which have vertical asymptote(s)?
- 2) ... which have horizontal asymptote(s)?
- 3) ... which have BOTH vertical and horizontal asymptote(s)?
- 4) ... which have slanted asymptotes?
- 5) ... which have NO vertical asymptote?

Think about what you know about "excluded values" of a rational function ...

- 6) ... how many factors would the denominator of the function represented in graph A have?
- 7) ... which graphs represent a rational function that would have two different factors in the denominator?
- 8) ... what type of factor(s) would be in the denominator of the function represented in graph C?!

Graphic Organizer #6: RATIONAL FUNCTION
 Parent Function: $f(x) = 1/x$

| $f(x) = \frac{1}{x}$ | |
|----------------------|------|
| x | f(x) |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |



| |
|-----------|
| Describe: |
| |

| |
|------------------------|
| x - intercepts (zeros) |
| |

| |
|---------|
| Domain: |
| |

| |
|--------|
| Range: |
| |

| |
|---------------|
| y - intercept |
| |

| |
|--------------------------------|
| Intervals of Increase/Decrease |
| |

| |
|--------------|
| End Behavior |
| |

| |
|------------|
| Max or Min |
| |

Notes: Graphing Rational Functions

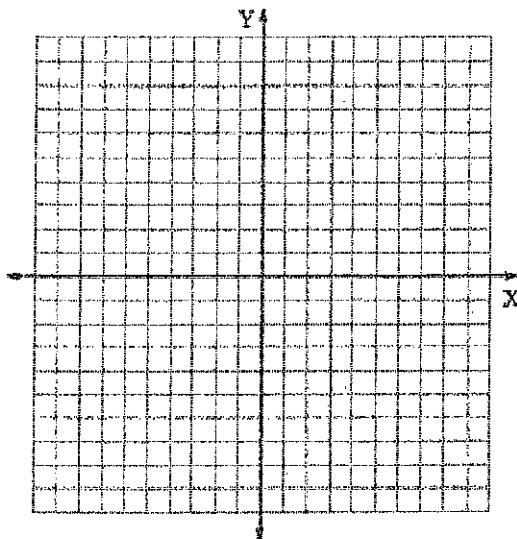
Example 1:

$$f(x) = \frac{2}{x-3} + 4$$

Vertical Asymptote: _____

Horizontal Asymptote: _____

| x | f(x) |
|---|------|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Domain: _____
Range: _____

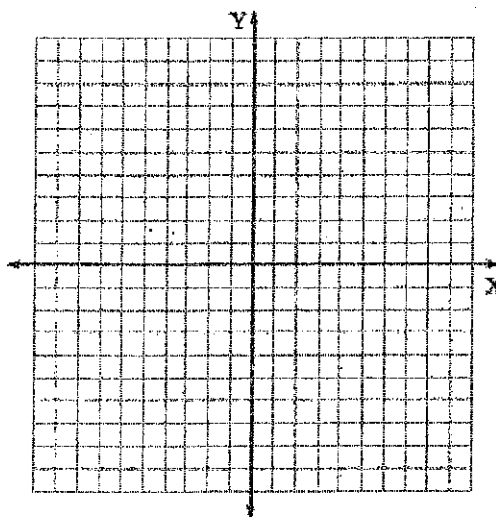
Example 2:

$$f(x) = \frac{-3}{x+2} + 4$$

Vertical Asymptote: _____

Horizontal Asymptote: _____

| x | f(x) |
|---|------|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Domain: _____
Range: _____

Example 3:

$$f(x) = -\frac{2}{2x-4} + 1$$

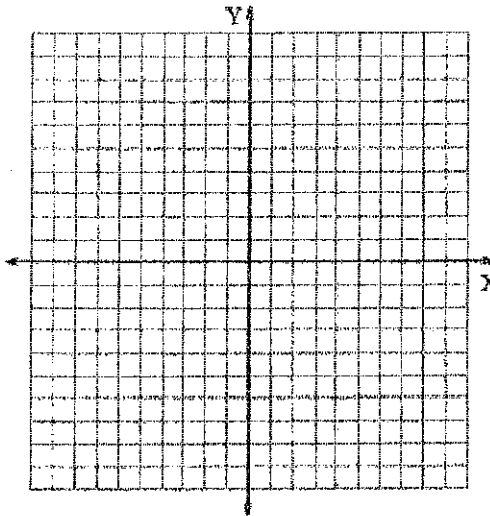
Vertical Asymptote: _____

Horizontal Asymptote: _____

| x | f(x) |
|---|------|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

Domain: _____

Range: _____



Example 4:

$$f(x) = \frac{1}{x} - 3$$

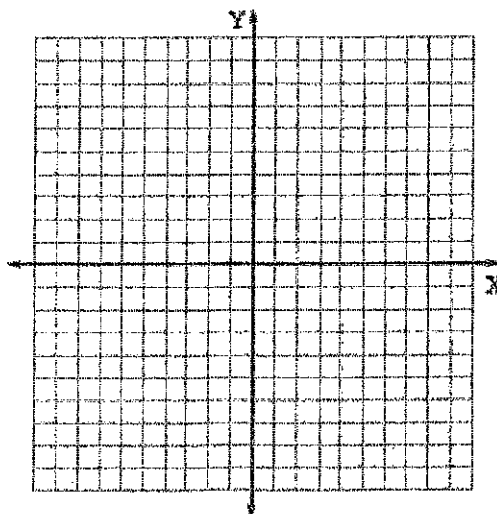
Vertical Asymptote: _____

Horizontal Asymptote: _____

| x | f(x) |
|---|------|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

Domain: _____

Range: _____



Graphing Simple Rational Functions

Identify the vertical asymptotes, horizontal asymptote, domain, and range of each.

1) $f(x) = -\frac{4}{x}$

2) $f(x) = \frac{4}{x-1} + 1$

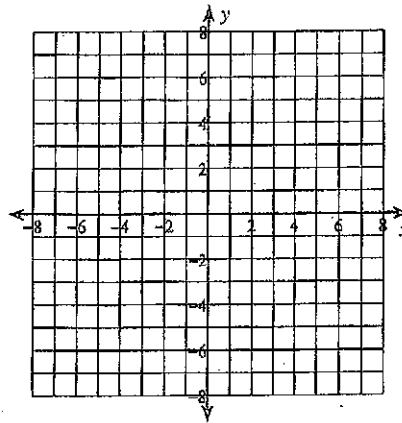
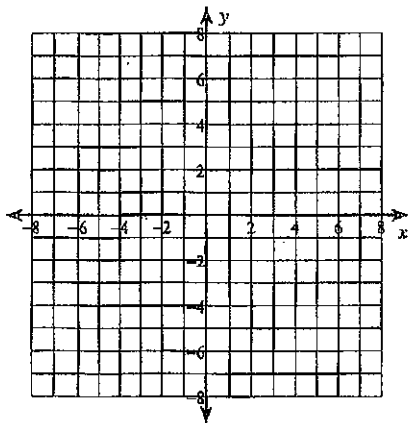
3) $f(x) = -\frac{3}{x-1} - 1$

4) $f(x) = -\frac{3}{x}$

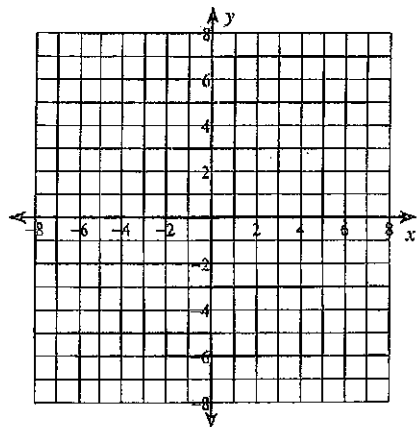
Identify the vertical asymptotes, horizontal asymptote, domain, and range of each. Then sketch the graph.

5) $f(x) = \frac{3}{x+1} - 2$

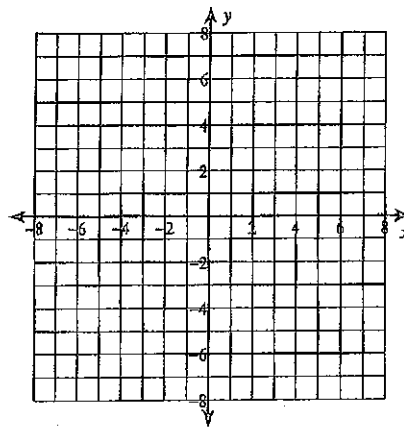
6) $f(x) = \frac{3}{x+1} + 2$



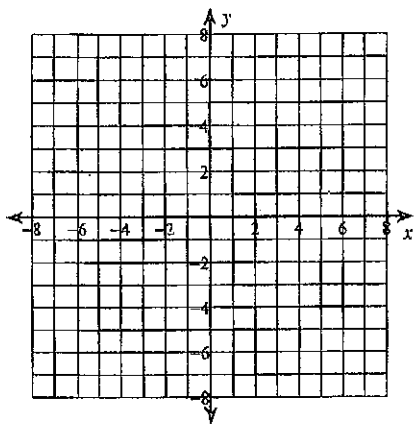
$$7) f(x) = \frac{3}{x} + 1$$



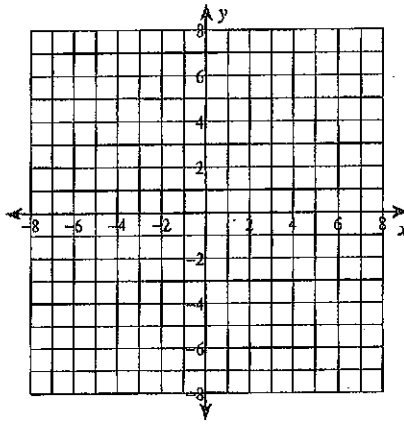
$$8) f(x) = \frac{2}{x-3} + 1$$



$$9) f(x) = -\frac{4}{x+1} + 1$$



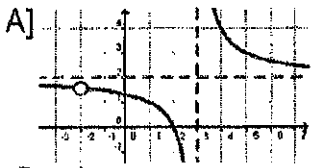
$$10) f(x) = \frac{4}{x} + 2$$



Critical thinking question:

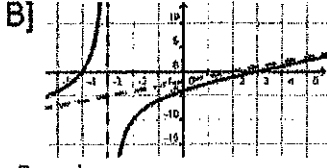
11) Write a function of the form $f(x) = \frac{a}{x-h} + k$ with a vertical asymptote at $x = 25$

Steps 6 - 7 Graph, Domain, and Range



Domain:

Range:



Domain:

Range:

- 1] Find any holes
- 2] Find the vertical asymptote
- 3] Find any zeros
- 4] Find the y-int (if any)
- 5] Find the horizontal or oblique asymptote
- 6] Sketch the graph
- 7] Identify the domain and range

Rational Functions

$$B] y = \frac{x^2 + x - 12}{x + 3}$$

$$A] y = \frac{2x^2 - 8}{x^2 - x - 6}$$

- 1] Factor completely and cancel common factors
- 2] Factors that cancel form holes in the graph. To find the x-coordinate of a hole, set the canceled factor equal to zero and solve for x.
- 3] Substitute that result back in to the simplified form to find the y-coordinate of the hole.

Step 1: Look for holes

Steps 2 - 4: Find vertical asymptotes, zeros, and y-intercept

- 1] Set the denominator equal to zero to find any vertical asymptotes
- 2] Set the numerator equal to zero to find any zeros
- 3] Substitute 0 for x and solve for y to find the y-intercept, if any

$$A] y = \frac{2x^2 - 8}{x^2 - x - 6} = \frac{2(x+2)(x-2)}{(x+2)(x-3)}$$

$$B] y = \frac{x^2 + x - 12}{x + 3} = \frac{(x+4)(x-3)}{x+3}$$

Step 5: Find horizontal OR oblique (SLANT) asymptote (can't have both)

- 1] If the degree of the numerator is LESS than the degree of the denominator, then $y=0$ is the horizontal asymptote
- 2] If the degree of the numerator EQUALS the degree of the denominator, then the ratio of leading coefficients is the horizontal asymptote.
- 3] If the degree of the numerator is MORE than the degree of the denominator, then divide to find the asymptote, ignoring the remainder. When the difference in degree is one, the asymptote will be oblique.

$$A] y = \frac{2x^2 - 8}{x^2 - x - 6} = \frac{2(x+2)(x-2)}{(x+2)(x-3)}$$

$$B] y = \frac{x^2 + x - 12}{x + 3} = \frac{(x+4)(x-3)}{x+3}$$

Graphing Rational Functions Worksheet

For the following function, find:

- the x -intercepts, y -intercepts
- the vertical asymptote(s)
- the horizontal/ slant asymptote
- the holes
- any additional points needed
- then, graph the function.

1. $f(x) = \frac{2x}{x^2 - 1}$

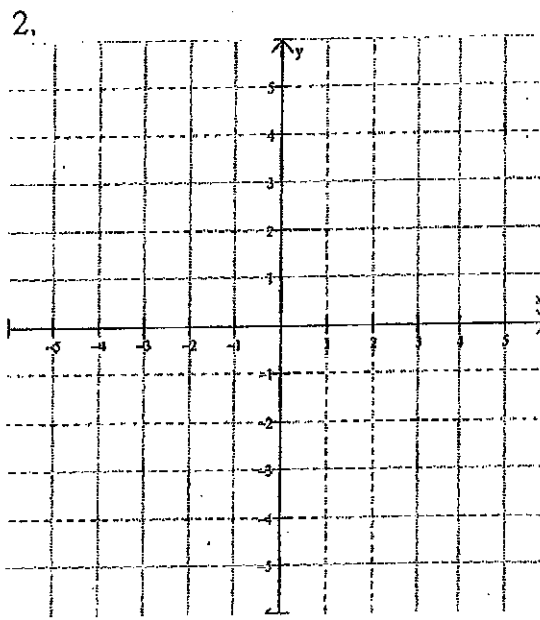
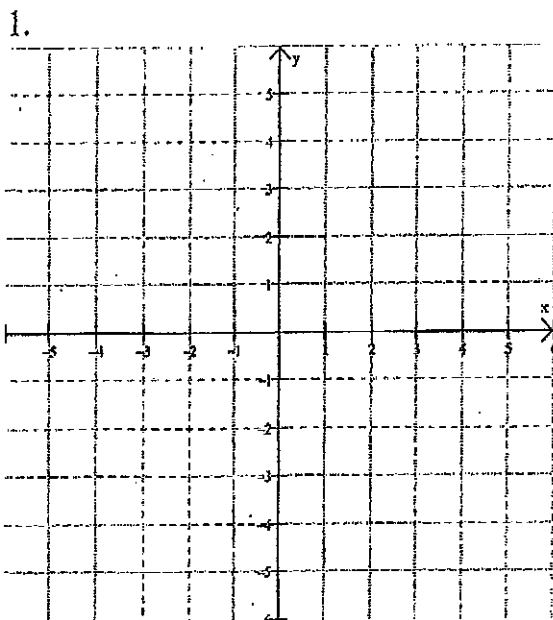
4. $y = \frac{x^2 - 5x + 6}{x^2 - 4x + 3}$

2. $y = \frac{8}{x^2 - x - 6}$

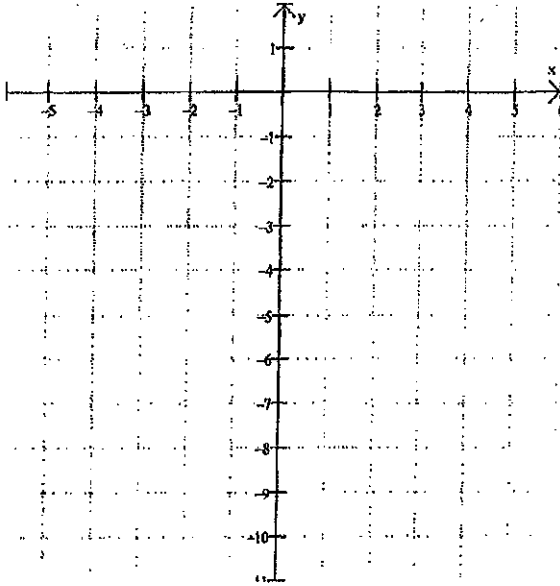
5. $y = \frac{x^2 + 11x + 18}{2x + 1}$

3. $f(x) = \frac{x^2 - 9}{2x^2 + 1}$

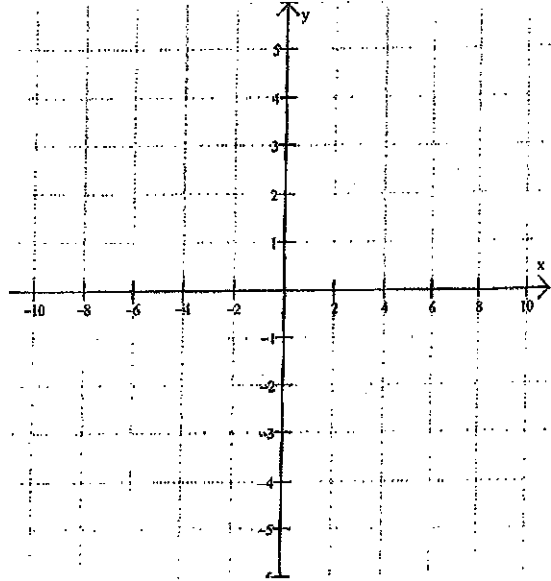
6. $g(x) = \frac{x - 4}{x^2 - 3x}$



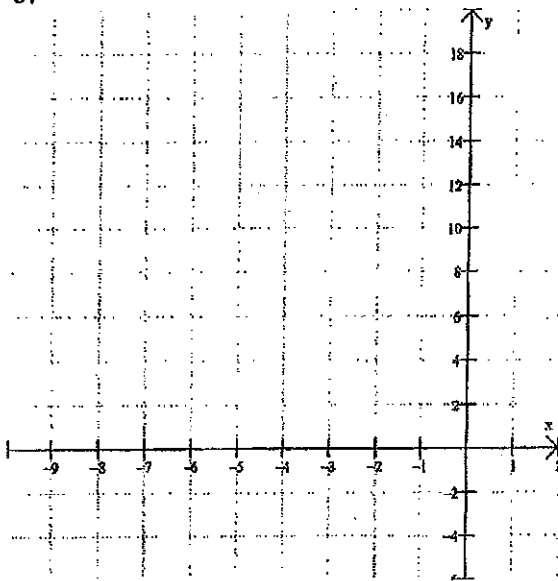
3.



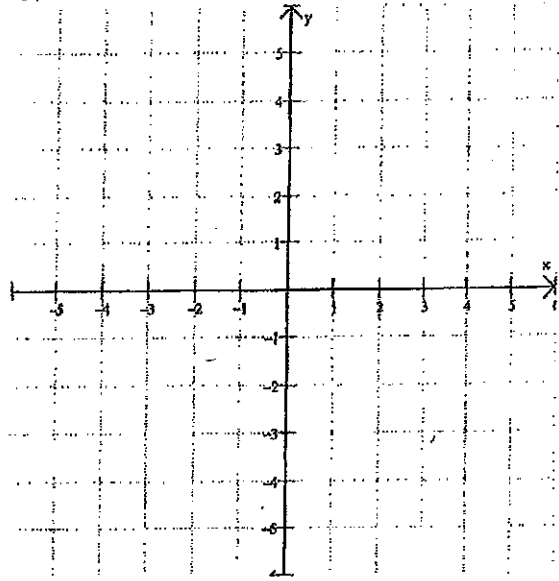
4.



5.



6.



$$9] f(x) = \frac{x^2+2x-3}{x-1}$$

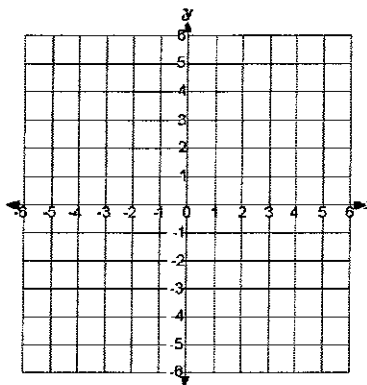
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



$$10] f(x) = \frac{x-3}{x^2+x-12}$$

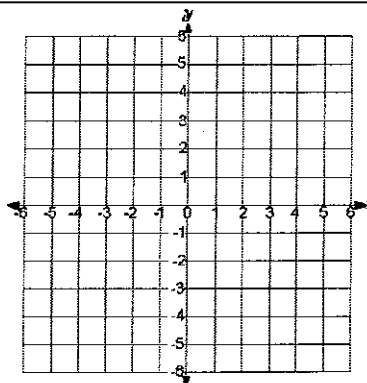
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



$$11] f(x) = \frac{x^2+2x-15}{x+3}$$

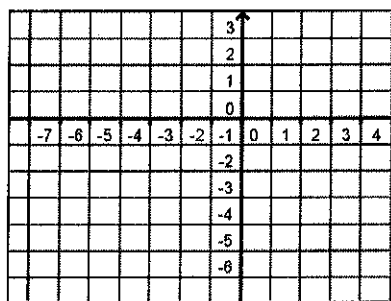
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



$$12] f(x) = \frac{(x+1)(x^2+2x+1)}{x^2-1}$$

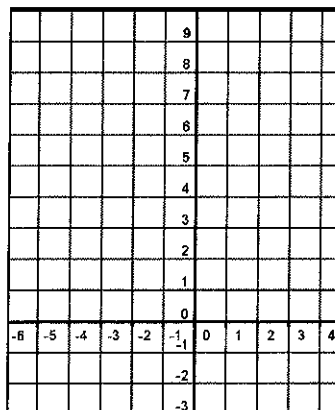
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



Hint: The point (3, 8) is on the graph and it is a rel. minimum.

WRITING EQUATIONS OF
RATIONAL FUNCTIONS

WRITE THE EQUATION OF THE
RATIONAL FUNCTION GIVEN:

- 1. Vertical Asymptote: $x = 3$
Horizontal Asymptote: $y = -2$
Zero of the function: $(-6, 0)$

- 2. Vertical Asymptote: $x = -4$
Slant Asymptote: $y = x - 2$
Zero of the function: $(3, 0)$

Rational Functions
WS Writing Equations 1

Name: _____

Write a rational function satisfying the following criteria.

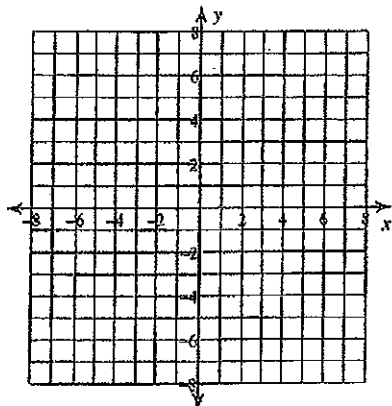
1. VA: $x = -2$ and $x = 3$, zero $x = -5$
2. Hole at $x = -1$, Graph resembles the line $y = 2x + 1$
3. VA: $x = -7$, HA: $y = 2$, zero: $x = 3$
4. VA: $x = 4$, no zero, y intercept $(0, -2)$
5. VA: $x = 0$, $x = -5$, HA: $y = 4$, zeros: $x = \pm \frac{1}{2}$
6. VA: $x = -3$, $x = 3$, HA: $y = 0$, zero: $x = 1$
7. VA: $x = -4$, $x = 1$, HA: $y = 0$, no zeros
8. Holes at $x = -3$ and $x = -7$, resembles $y = x$
9. Hole at $x = -1$, VA: $x = -5$, and $x = 1$
10. VA: $x = \frac{2}{3}$, HA: $y = \frac{2}{3}$, zero: $x = 0$

4.4: Graphing Rational Functions Practice

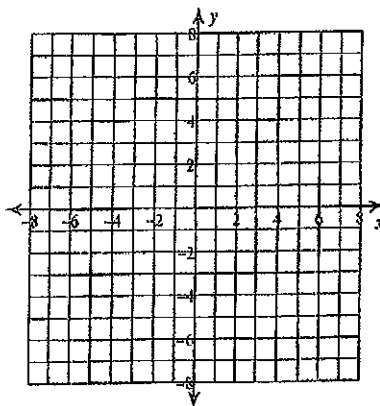
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Identify the holes, vertical asymptotes, x-intercepts, horizontal asymptote, and domain of each. Then sketch the graph.

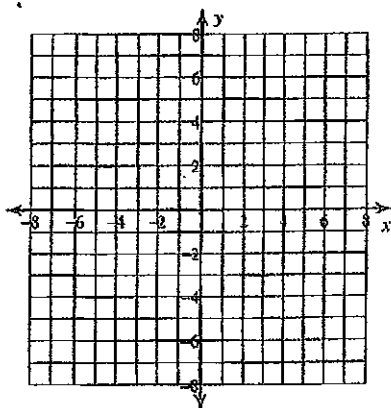
1) $f(x) = \frac{4}{x-3}$



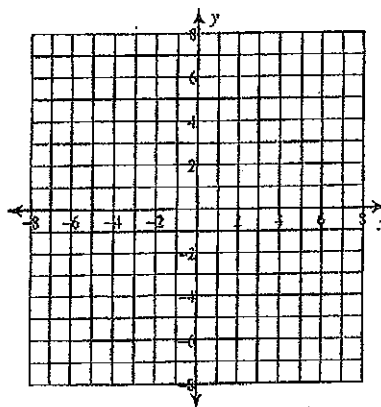
2) $f(x) = \frac{x^2 + 7x + 12}{-2x^2 - 2x + 12}$



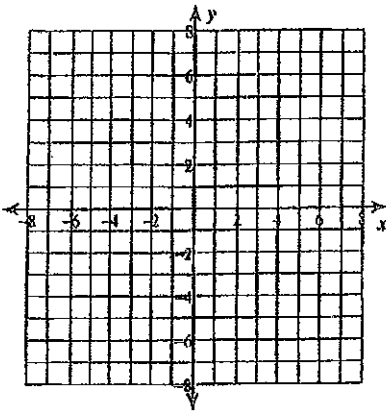
3) $f(x) = \frac{1}{-x+4}$



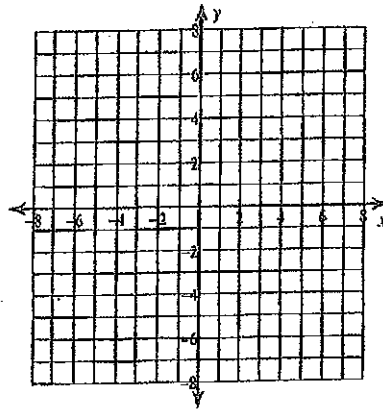
4) $f(x) = \frac{-3x+12}{x^2-3x-4}$



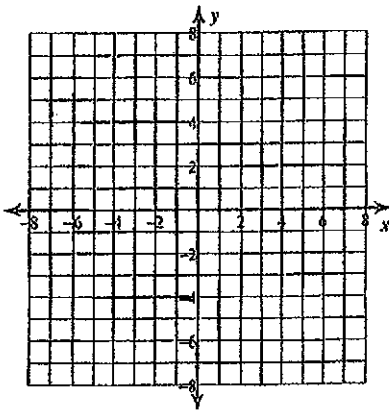
$$5) f(x) = \frac{-2x^2 + 4x + 16}{x^2 - 5x + 4}$$



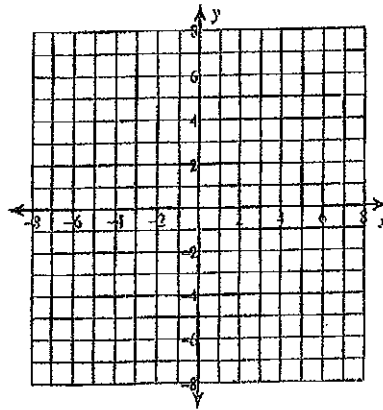
$$6) f(x) = \frac{x^2 - 3x}{2x^2 + 2x - 12}$$



$$7) f(x) = \frac{3x + 6}{x + 3}$$



$$8) f(x) = \frac{x^2 + 5x + 4}{x^2 - 1}$$



Name _____

Graphing Rational Functions

Graph and give the requested information.

1. $f(x) = \frac{x^2 - 4}{x - 2}$

VA: _____

HA: _____

x-int: _____

y-int: _____

SA: _____

holes: _____

2. $f(x) = \frac{x^2 - 9}{x + 1}$

VA: _____

HA: _____

x-int: _____

y-int: _____

SA: _____

holes: _____

3. $f(x) = \frac{x}{x^2 - 1}$

VA: _____

HA: _____

x-int: _____

y-int: _____

SA: _____

holes: _____

4. $f(x) = \frac{x^2}{x - 1}$

VA: _____

HA: _____

x-int: _____

y-int: _____

SA: _____

holes: _____

5. Write the equation of a rational function given:

a) VA: $x = 7$ HA: $y = -2$

b) VA: $x = 2, x = -1$ HA: $y = 0$ zero at -5

Graphing Rational Functions

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1) $f(x) = \frac{1}{3x^2 + 3x - 18}$

2) $f(x) = \frac{x-2}{x-4}$

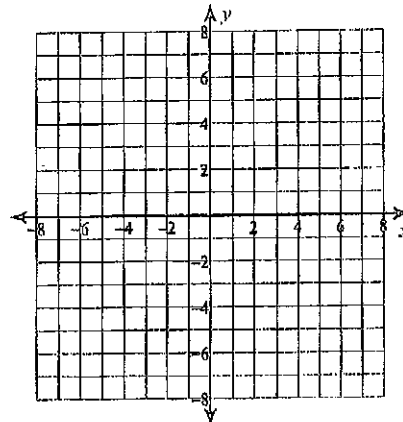
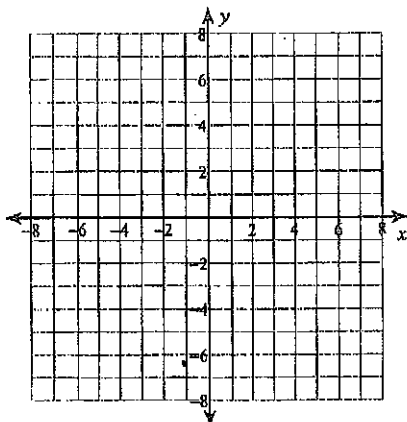
3) $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

4) $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

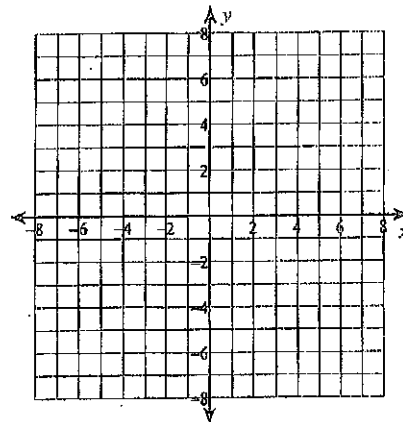
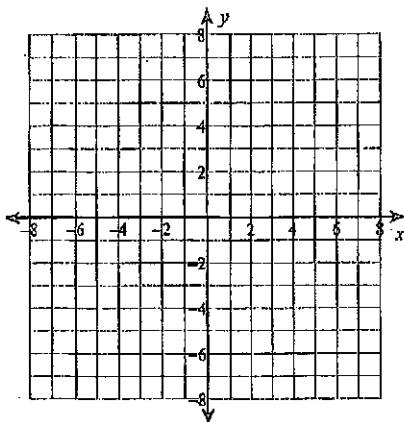
5) $f(x) = -\frac{4}{x^2 - 3x}$

6) $f(x) = \frac{x-4}{-4x-16}$

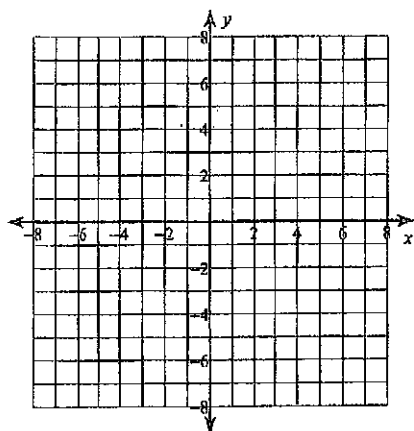


7) $f(x) = \frac{x+4}{-2x-6}$

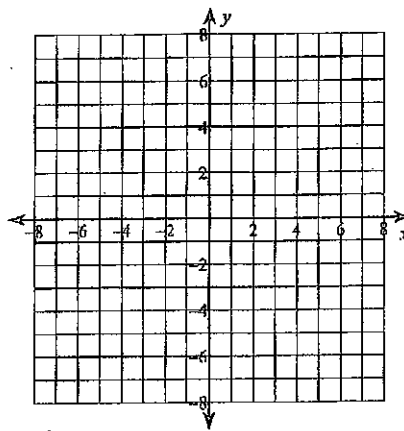
8) $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9}$



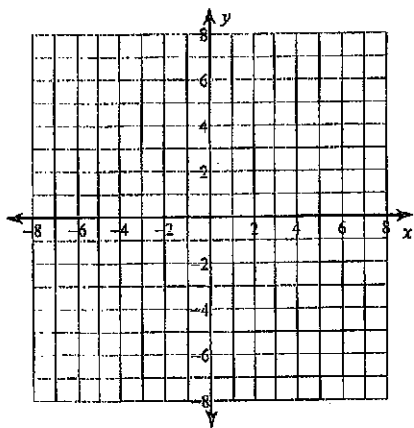
$$9) f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3}$$



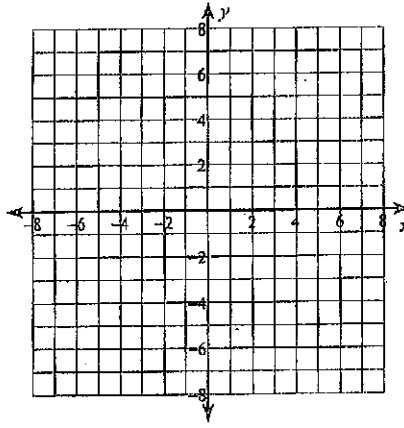
$$10) f(x) = \frac{x^3 - 16x}{-4x^2 + 4x + 24}$$



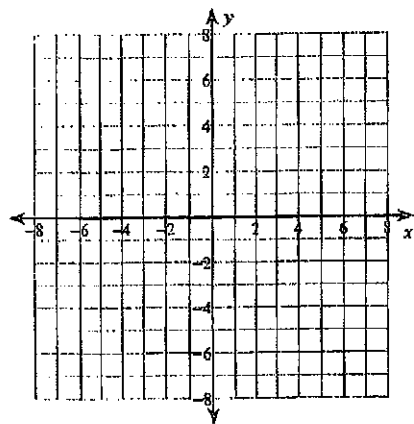
$$11) f(x) = \frac{x^2 + 2x}{-4x + 8}$$



$$12) f(x) = \frac{x + 2}{2x + 6}$$



$$13) f(x) = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$$



$$14) f(x) = \frac{3}{x - 2}$$

