

Geometric Sequences

Determine if the sequence is geometric. If it is, find the common ratio.

1) $-1, 6, -36, 216, \dots$

$r = -6$

2) $-1, 1, 4, 8, \dots$

Not geometric

3) $4, 16, 36, 64, \dots$

Not geometric

4) $-3, -15, -75, -375, \dots$

$r = 5$

5) $-2, -4, -8, -16, \dots$

$r = 2$

6) $1, -5, 25, -125, \dots$

$r = -5$

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

7) $a_n = 3^{n-1}$

First Five Terms: 1, 3, 9, 27, 81

$a_8 = 2187$

8) $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$

First Five Terms: $2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}$

$a_8 = \frac{1}{8192}$

9) $a_n = -2.5 \cdot 4^{n-1}$

First Five Terms: $-2.5, -10, -40, -160, -640$

$a_8 = -40960$

10) $a_n = -4 \cdot 3^{n-1}$

First Five Terms: $-4, -12, -36, -108, -324$

$a_8 = -8748$

Given the recursive formula for a geometric sequence find the common ratio, the first five terms, and the explicit formula.

11) $a_n = a_{n-1} \cdot 2$

$a_1 = 2$

Common Ratio: $r = 2$

First Five Terms: 2, 4, 8, 16, 32

Explicit: $a_n = 2 \cdot 2^{n-1}$

12) $a_n = a_{n-1} \cdot -3$

$a_1 = -3$

Common Ratio: $r = -3$ First Five Terms: $-3, 9, -27, 81, -243$

Explicit: $a_n = -3 \cdot (-3)^{n-1}$

13) $a_n = a_{n-1} \cdot 5$

$a_1 = 2$

Common Ratio: $r = 5$

First Five Terms: 2, 10, 50, 250, 1250

Explicit: $a_n = 2 \cdot 5^{n-1}$

14) $a_n = a_{n-1} \cdot 3$

$a_1 = -3$

Common Ratio: $r = 3$ First Five Terms: $-3, -9, -27, -81, -243$

Explicit: $a_n = -3 \cdot 3^{n-1}$

Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15) $a_1 = 0.8, r = -5$

First Five Terms: 0.8, -4, 20, -100, 500

Explicit: $a_n = 0.8 \cdot (-5)^{n-1}$

16) $a_1 = 1, r = 2$

First Five Terms: 1, 2, 4, 8, 16

Explicit: $a_n = 2^{n-1}$

Given the first term and the common ratio of a geometric sequence find the recursive formula and the three terms in the sequence after the last one given.

17) $a_1 = -4, r = 6$

Next 3 terms: -24, -144, -864

Recursive: $a_n = a_{n-1} \cdot 6$

$a_1 = -4$

18) $a_1 = 4, r = 6$

Next 3 terms: 24, 144, 864

Recursive: $a_n = a_{n-1} \cdot 6$

$a_1 = 4$

19) $a_1 = 2, r = 6$

Next 3 terms: 12, 72, 432

Recursive: $a_n = a_{n-1} \cdot 6$

$a_1 = 2$

20) $a_1 = -4, r = 4$

Next 3 terms: -16, -64, -256

Recursive: $a_n = a_{n-1} \cdot 4$

$a_1 = -4$

Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

21) $a_4 = 25, r = -5$

First Five Terms: -0.2, 1, -5, 25, -125

Explicit: $a_n = -0.2 \cdot (-5)^{n-1}$

Recursive: $a_n = a_{n-1} \cdot -5$

$a_1 = -0.2$

22) $a_1 = 4, r = 5$

First Five Terms: 4, 20, 100, 500, 2500

Explicit: $a_n = 4 \cdot 5^{n-1}$

Recursive: $a_n = a_{n-1} \cdot 5$

$a_1 = 4$

Given two terms in a geometric sequence find the 8th term and the recursive formula.

23) $a_4 = -12$ and $a_5 = -6$

$a_8 = -\frac{3}{4}$

Recursive: $a_n = a_{n-1} \cdot \frac{1}{2}$

$a_1 = -96$

24) $a_5 = 768$ and $a_2 = 12$

$a_8 = 49152$

Recursive: $a_n = a_{n-1} \cdot 4$

$a_1 = 3$

25) $a_1 = -2$ and $a_5 = -512$

$a_8 = 32768$

Recursive: $a_n = a_{n-1} \cdot -4$

$a_1 = -2$

26) $a_5 = 3888$ and $a_3 = 108$

$a_8 = 839808$

Recursive: $a_n = a_{n-1} \cdot 6$

$a_1 = 3$

Infinite Geometric Series

Determine if each geometric series converges or diverges.

1) $a_1 = -3, r = 4$

Diverges

2) $a_1 = 4, r = -\frac{3}{4}$

Converges

3) $a_1 = 5.5, r = 0.5$

Converges

4) $a_1 = -1, r = 3$

Diverges

5) $81 + 27 + 9 + 3 \dots$

Converges

6) $7.1 + 17.75 + 44.375 + 110.9375 \dots$

Diverges

7) $-3 + \frac{12}{5} - \frac{48}{25} + \frac{192}{125} \dots$

Converges

8) $\frac{128}{3125} - \frac{64}{625} + \frac{32}{125} - \frac{16}{25} \dots$

Diverges

9) $\sum_{k=1}^{\infty} -4^{k-1}$

Diverges

10) $\sum_{k=1}^{\infty} \frac{16}{9} \left(\frac{3}{2}\right)^{k-1}$

Diverges

11) $\sum_{i=1}^{\infty} 4.2 \cdot 0.2^{i-1}$

Converges

12) $\sum_{k=1}^{\infty} \frac{7}{6} \left(-\frac{1}{4}\right)^{k-1}$

Converges

Evaluate each infinite geometric series described.

13) $a_1 = 3, r = -\frac{1}{5}$

$$\frac{5}{2}$$

14) $a_1 = 1, r = -4$

No sum

15) $a_1 = 1, r = -3$

No sum

16) $a_1 = 1, r = \frac{1}{2}$

2

$$17) 1 + 0.5 + 0.25 + 0.125 \dots,$$

$$\frac{2}{2}$$

$$18) 3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64} \dots,$$

$$\frac{12}{7}$$

$$19) 81 - 27 + 9 - 3 \dots,$$

$$\frac{243}{4}$$

$$20) 1 - 0.6 + 0.36 - 0.216 \dots,$$

$$0.625$$

$$21) \sum_{k=1}^{\infty} 5 \cdot \left(-\frac{1}{5}\right)^{k-1}$$

$$\frac{25}{6}$$

$$22) \sum_{n=1}^{\infty} -6 \cdot \left(-\frac{1}{2}\right)^{n-1}$$

$$-4$$

$$23) \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i-1}$$

$$\frac{3}{2}$$

$$24) \sum_{k=1}^{\infty} 4^{k-1}$$

No sum

Determine the common ratio of the infinite geometric series.

$$25) a_1 = 1, S = 1.25$$

$$0.2$$

$$26) a_1 = 96, S = 64$$

$$-\frac{1}{2}$$

$$27) a_1 = -4, S = -\frac{16}{5}$$

$$-\frac{1}{4}$$

$$28) a_1 = 1, S = 2.5$$

$$0.6$$

Informal Algebra 2
Linear Programming 1

Name Key

Find the minimum and maximum values of the objective function subject to the given constraints.

1. Objective function:

$C = x + 3y$

Constraints:

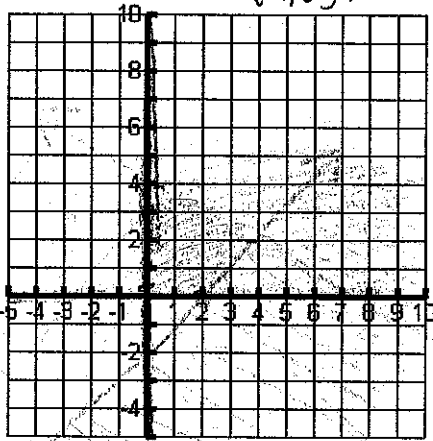
$x + 2y \leq 8$

$x - y \geq 2$

$y \geq 0$

$x \geq 0$

Max of 10 @ (4,2)
Min of 2 @ (2,0)
 $(2,0) = 2 + 0 = 2$
 $(8,0) = 8 + 0 = 8$
 $(4,2) = 4 + 3 \cdot 2 = 10$



2. Objective function:

$C = 3x + 2y$

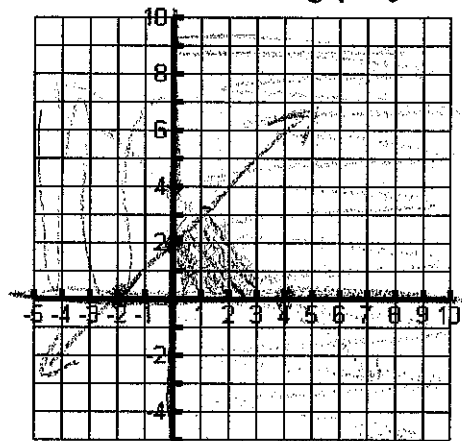
Constraints:

$x \geq 0, y \geq 0$

$x + y \leq 4$

$x - y \geq -2$

$(0,2) \quad 3 \cdot 0 + 2 \cdot 2 = 4$
 $(0,0) = 0$ Min
 $(4,0) \quad 3 \cdot 4 + 2 \cdot 0 = 12$ Max
 $(1,3) \quad 3 \cdot 1 + 2 \cdot 3 = 9$



3. Objective function:

$C = 5x - 2y$

Constraints:

$x \geq 0$

$y \geq 0$

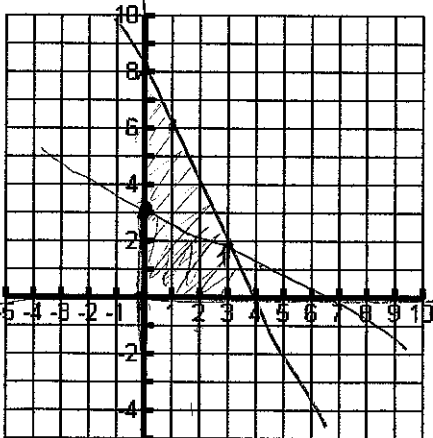
$2x + y \leq 8$

$x + 3y \leq 9$

$(0,0)$
 $(0,3)$
 $(3,0)$
 $(3,2)$
15
9

$y \leq -2x + 8$

$3y \leq -x + 9$
 $y \leq -\frac{1}{3}x + 3$



4. Objective function:

$C = 3x - y$

Constraints:

$y \leq 4$

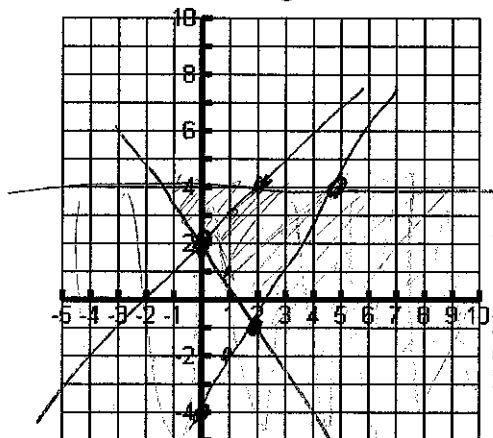
$x + y \geq 2$

$2x - y \leq 4$

$-x + y \leq 2$

$y \geq -x + 2$
 $-y \leq -2x + 4 \quad y \geq 2x - 4$
 $y \leq x + 2$

$(0,2)$
 $(2,-1)$
 $(5,4)$



Honors Algebra 2

Linear Programming

Key

Example 1:

A bakery produces cakes and cupcakes. It must produce at least 10 cakes per month. The company has the equipment to produce only 60 cakes. It also can produce only 120 cupcakes. The production of cakes and cupcakes cannot exceed 160. The profit on a cake is \$13 and on a cupcake is \$2. How many of each should be produced per month to maximize profit?

$$x = \text{cakes} \quad y = \text{cupcakes}$$

Objective function: $13x + 2y = P$

Constraints: $x \geq 10 \quad x \leq 60$

$0 \leq y \leq 120$

$x + y \leq 160$

Number of cakes: 60

Number of cupcakes: 100

Maximum profit: \$980

Test points

$(10, 0) \quad (60, 0)$

$(10, 120) \quad (40, 120)$

$(60, 100)$

$(10, 120)$

$13(10) + 2(120) = P$

$130 + 240 = P$

$P = 370$

$(40, 120)$

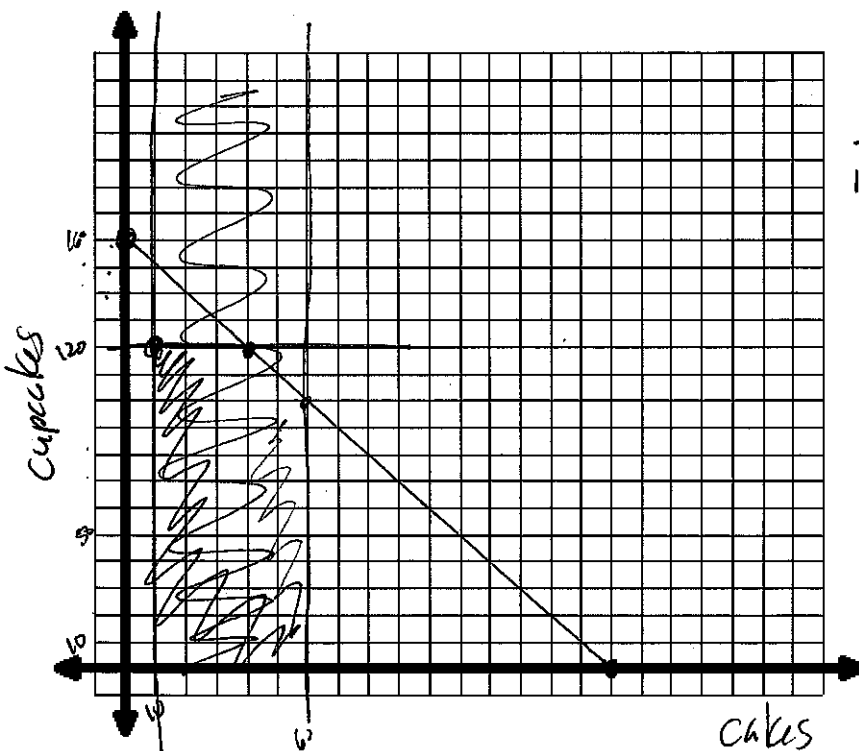
$13(40) + 2(120)$

$520 + 240 = 760$

$(60, 100)$

$13(60) + 2(100) = P$

$780 + 200 = 980$



A pizza shop makes \$1.50 on each small pizza and \$2.15 on each large pizza. On a typical Friday, it sells between 70 and 90 small pizzas and between 100 and 140 large pizzas. The total sales have never exceeded 210 pizzas. How many of each pizza must be sold to maximize profit?

$x = \text{small}$ $y = \text{large}$

Objective function: $1.50x + 2.15y = P$

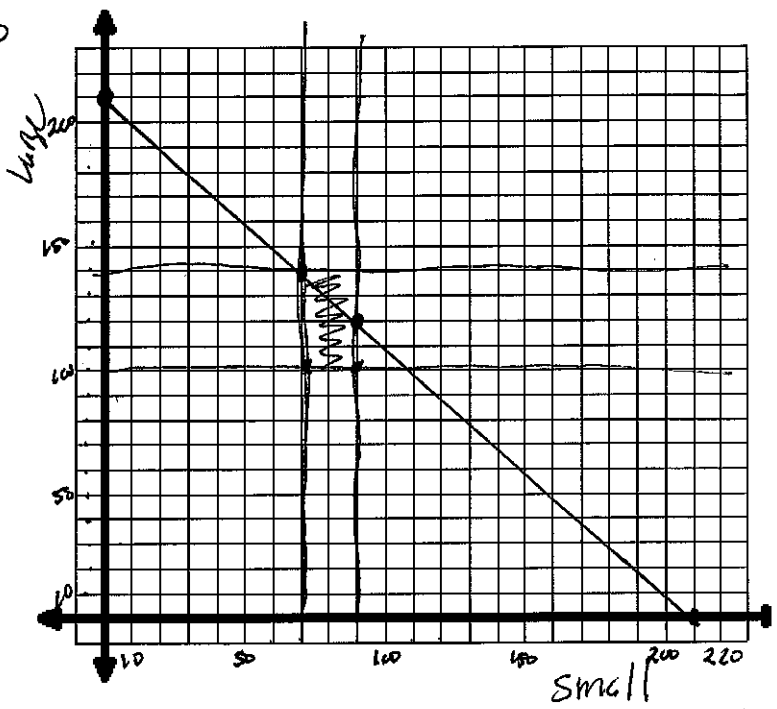
Constraints: $x + y \leq 210$

$70 \leq x \leq 90$
 $x \leq 90, x \geq 70$
 $100 \leq y \leq 140$
 $y \geq 100, y \leq 140$

Number of small pizza: 70

Number of large pizza: 140

Maximum profit: \$ 406



Test points

$(60, 100)$	$(90, 100)$
$1.50(60) + 2.15(100)$	$1.50(90) + 2.15(100)$
$= 305$	$= 350$
$(90, 120)$	$(70, 140)$
$1.50(90) + 2.15(120)$	$1.50(70) + 2.15(140)$
$= 393$	$= 406$

Informal Algebra 2
Linear Programming 2

Name _____

1. You are taking a test in which items of type A are worth 10 points and items of type B are worth 15 points. It takes 3 minutes to answer each item of type A and 6 minutes for each item of type B. The total time allowed is 60 minutes, and you may not answer more than 16 questions. Assuming all of your answers are correct, how many items of each type should you answer to get the highest score?
2. Mrs. Wood's Biscuit Factory makes two types of biscuits, Biscuit Jumbos and Mini Mint Biscuits. The oven can cook at most 200 biscuits per day. Jumbos each require 2 ounces of flour. Minis each require 1 ounce of flour. There are 300 ounces of flour available. The income from Jumbos is 10 cents each. The income from the Minis is 8 cents each. How many of each type should be baked to earn the greatest amount?
3. A company produces mopeds and bicycles. It must produce at least 10 mopeds per month. The company has the equipment to produce only 60 mopeds. It also can produce only 120 bicycles. The production of mopeds and bicycles cannot exceed 160. The profit on a moped is \$134 and on a bicycle \$20. How many of each should be manufactured per month to maximize profit?
4. There are 12 gallons of gas available to be distributed between the car and the scooter. The car's tank can hold at most 10 gallons. The scooter can hold at most 3 gallons. The car can go 20 miles on each gallon, and the scooter can go 100 miles on each gallon. How much should each vehicle get to achieve the greatest total miles?
5. You are about to take a test that contains questions of type A worth 4 points and of type B worth 7 points. You must answer at least 5 of type A and 3 of type B, but time restricts answering more than 10 of either type. In total, you can answer no more than 18. How many of each type of question must you answer, assuming all of your answers are correct, to maximize your score? What is the maximum score?
6. A man plans to invest up to \$22,000 in Bank X or Bank Y, or both. He will invest at least \$2,000, but no more than \$14,000, in Bank X. He will invest no more than \$15,000 in Bank Y. Bank X pays 6% simple interest and Bank Y pays 6.5%. How much should he invest in each to maximize income? What is the maximum income?

Linear Programming 2

1. $x = A$ questions
 $y = B$ questions

$10x + 15y = G$ obj function

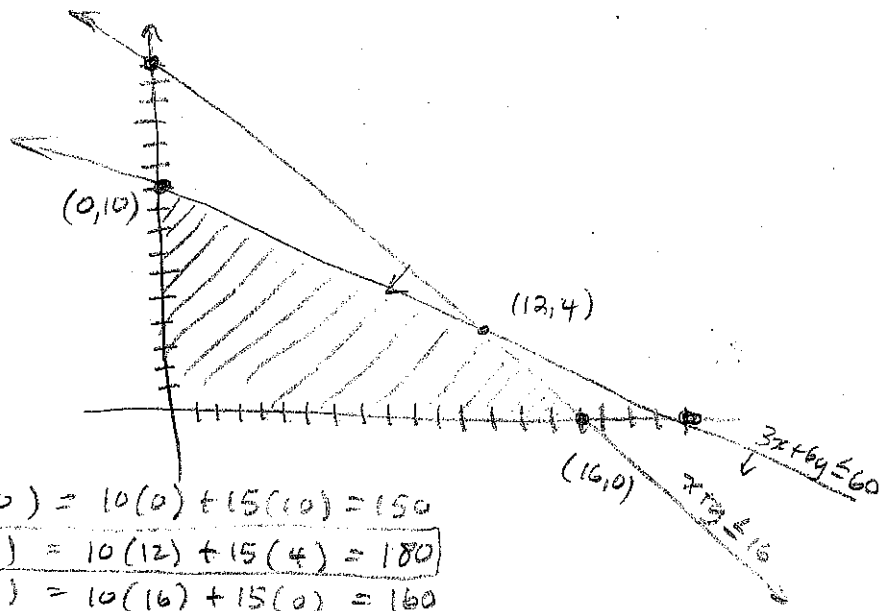
constraints

$3x + 6y \leq 60$

$x + y \leq 16$

$x \geq 0$

$y \geq 0$



$(0, 10) = 10(0) + 15(10) = 150$
 $(12, 4) = 10(12) + 15(4) = 180$
 $(16, 0) = 10(16) + 15(0) = 160$

You should answer 12 of type A and 4 of type B to get a score of 180.

2. $x =$ Jumbos
 $y =$ Minis

$.75x + .55y = I$

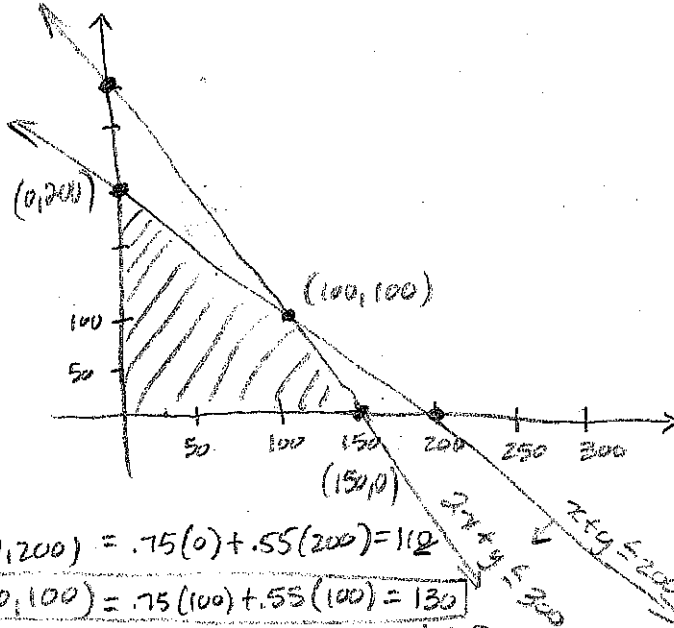
constraints

look at day $\rightarrow x + y \leq 200$

Flour $\rightarrow 2x + y \leq 300$

no negative biscuits $x \geq 0$

$y \geq 0$



$(0, 200) = .75(0) + .55(200) = 110$
 $(100, 100) = .75(100) + .55(100) = 130$
 $(150, 0) = .75(150) + .55(0) = 112.50$

You should produce 100 Jumbo biscuits and 100 Mini mint biscuits for an income of \$130.

3. $x =$ mopeds
 $y =$ bicycles

$134x + 20y = P$

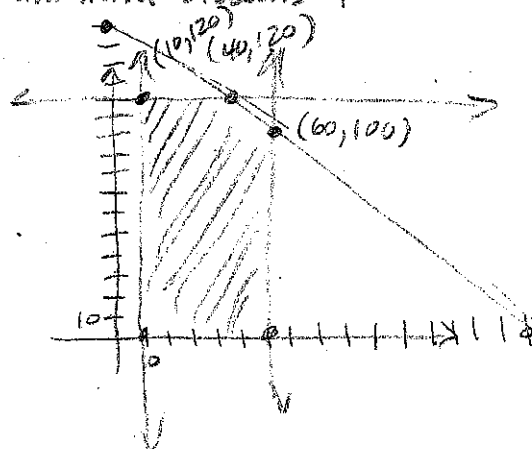
constraints

$x \geq 10$

$x \leq 60$

$0 \leq y \leq 120$

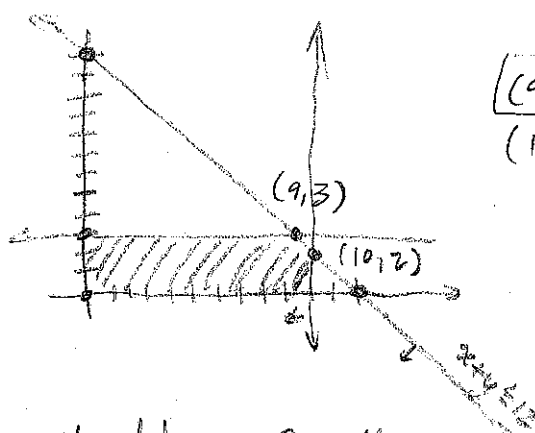
$x + y \leq 160$



$(40, 120) = 134(40) + 20(120) = 7760$
 $(60, 100) = 134(60) + 20(100) = 10,040$

You should produce 60 mopeds and 30 bicycles for a profit of \$10,040.

4. $x = \text{car}$
 $y = \text{scooter}$
 $20x + 100y = M$
 constraints
 $x + y \leq 12$
 $0 \leq x \leq 10$
 $0 \leq y \leq 3$

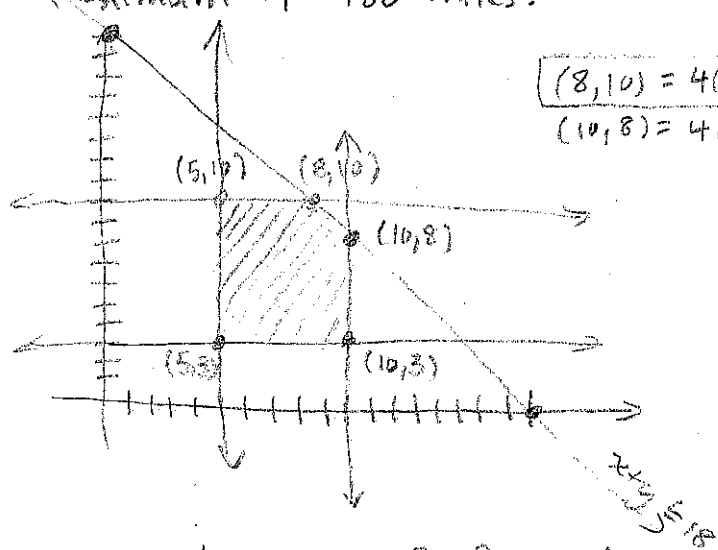


$$(9,3) = 20(9) + 100(3) = 480$$

$$(10,2) = 20(10) + 100(2) = 400$$

You should put 9 gallons in the car and 3 gallons in the scooter for a maximum of 480 miles.

5. $x = \text{A questions}$
 $y = \text{B questions}$
 $4x + 7y = S$
 constraints
 $5 \leq x \leq 10$
 $3 \leq y \leq 10$
 $x + y \leq 18$

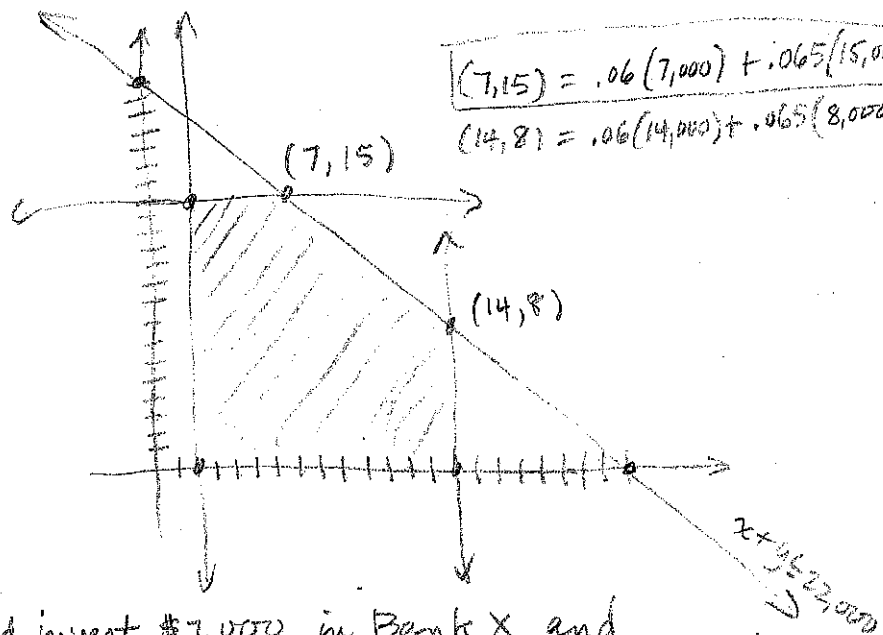


$$(8,10) = 4(8) + 7(10) = 102$$

$$(10,8) = 4(10) + 7(8) = 96$$

You should answer 8 of Type A and 10 of Type B for a score of 102.

6. $x = \text{Bank X}$
 $y = \text{Bank Y}$
 $.06x + .065y = I$
 constraints
 $x + y \leq 22,000$
 $2,000 \leq x \leq 14,000$
 $0 \leq y \leq 15,000$



$$(7,15) = .06(7,000) + .065(15,000) = 1395$$

$$(14,8) = .06(14,000) + .065(8,000) = 1360$$

You should invest \$7,000 in Bank X and 15,000 in Bank Y for an income of \$1395