

$$11. \frac{\frac{2y+3}{xy} \cdot \frac{xy}{3y-x^2}}{\frac{xy}{x^2y} \cdot \frac{xy}{yx^2}} = \frac{3+2y}{xy} \cdot \frac{xy}{3y-x^2}$$

$$\frac{2y+3}{xy} \cdot \frac{x^2y}{3y-x^2} = \frac{x(2y+3)}{3y-x^2}$$

$$13. \frac{x^2-y^2}{x^4-y^4} = \frac{y^2 \cdot \frac{1}{x^2} - \frac{1}{y^2} \cdot x^2}{y^4 \cdot \frac{1}{x^4} - \frac{1}{y^4} \cdot x^4}$$

$$\frac{y^2-x^2}{x^2y^2} \cdot \frac{y^4x^4}{y^4-x^4}$$

$$\frac{y^2-x^2}{x^2y^2} \cdot \frac{x^4y^4}{(y^2-x^2)(y^2+x^2)} = \frac{x^2y^2}{y^2+x^2}$$

$$15. \frac{\frac{2x}{x^2-9} + \frac{4}{(x+3)(x-3)}}{(x+3) \cdot \frac{2}{x-3} + \frac{4x}{x^2-9}}$$

$$\frac{2x+4x-12}{(x-3)(x+3)} \cdot \frac{(x+3)(x-3)}{2x+6+4x} = \frac{6x-12}{6x+6}$$

$$= \frac{6(x-2)}{6(x+1)} = \frac{x-2}{x+1}$$

$$17. \frac{x^3-xy^2}{x^4+2x^3y+x^2y^2} = \frac{x(x^2-y^2)}{x^2(x^2+2xy+y^2)} = \frac{x(x-y)(x+y)}{x^2(x+y)(x+y)} = \frac{(x-y)}{x(x+y)}$$

$$12. \frac{\frac{a^2}{ab} - \frac{ba}{ab} + \frac{6bb}{ab}}{\frac{a}{a} + \frac{4}{4} + \frac{4b}{a}} = \frac{\frac{a}{ab} - \frac{ab}{ab} - \frac{6b^2}{ab}}{1 + 1 + \frac{4b}{a}} = \frac{\frac{a-ab-6b^2}{ab}}{\frac{a^2+4ab+4b^2}{a^2}} = \frac{a-ab-6b^2}{a^2+4ab+4b^2}$$

$$14. \frac{x^2 \cdot \frac{6x}{x^2} + \frac{5}{x^2}}{\frac{1}{x^2} - \frac{3x}{x} + \frac{10}{x^2}} = \frac{x^2-6x+5}{x^2} \cdot \frac{x^3}{-3x-9} = \frac{(x-5)(x-1)}{1} \cdot \frac{1}{-3(x+3)} = \frac{(x-5)(x-1)}{-3(x+3)}$$

$$16. \frac{\frac{2}{x-1}}{\frac{4x}{x(x-1)} + \frac{1}{x(x-1)}} = \frac{2}{(x-1)} \cdot \frac{x(x-1)}{5x-1} = \frac{2x}{5x-1}$$

Simplify each of the following. Make sure your answers are in simplest form.

$$1. \frac{\frac{x^2+3x-10}{x^2-5x+6}}{\frac{x^2-25}{x^2-4x-5}}$$

$$= \frac{x+1}{x-3}$$

$$\frac{(x+5)(x-2)}{(x+3)(x-2)} \cdot \frac{(x-5)(x+1)}{(x+5)(x-5)}$$

$$2. \frac{a^2-b^2}{\frac{ab}{\frac{b \cdot 1 + 1 \cdot a}{b \cdot a} \cdot \frac{1 \cdot a}{b \cdot a}}} = \frac{(a-b)(a+b)}{ab} \cdot \frac{ab}{(a+b)}$$

$$= a-b$$

$$3. \frac{y \cdot \frac{1}{y} + \frac{1}{y} \cdot y}{\frac{1 \cdot y}{xy} \cdot \frac{1}{y} \cdot x} =$$

$$= \frac{x+y}{xy} \cdot \frac{xy}{y-x} = \frac{x+y}{y-x}$$

$$4. \frac{x^2-16}{\frac{x^2-6x+9}{x^2-3x-4}}$$

$$= \frac{(x+4)(x-4)}{(x-3)(x-4)} \cdot \frac{(x-3)(x+1)}{(x-4)(x+1)}$$

$$= \frac{x+4}{x-3}$$

$$5. \frac{\frac{x^2+5x-6}{x^2+8x+12}}{\frac{x^2+2x-15}{x^2+9x+20}}$$

$$\frac{(x+6)(x-1)}{(x+6)(x+2)} \cdot \frac{(x+4)(x+5)}{(x+5)(x-3)}$$

$$= \frac{(x-1)(x+4)}{(x+2)(x-3)}$$

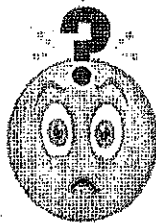
$$6. \frac{\frac{x^3+y^3}{x^2-y^2}}{x^2-2xy+y^2}$$

$$\frac{(x+y)(x^2+xy+y^2)}{(x+y)(x-y)} \cdot \frac{(x-y)(x-y)}{x^2-xy+y^2}$$

$$= x-y$$

## Solving Rational Equations

How do I solve rational equations?



1. Multiply WHOLE equation by LCD.
2. Solve the equation. (You might need to solve by factoring, completing the square or using the quadratic formula.)
3. Check for extraneous solutions.

Solve for x.

$$2 \cdot \frac{3x}{2} + \frac{1}{4}(x-2) = 10 \quad \text{LCD} = 4$$

$$6x + (x-2) = 40$$

$$7x - 2 = 40$$

$$7x = 42$$

$$\boxed{x = 6}$$

Solve for x.

$$\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2+x-6} \quad \text{LCD} = (x+3)(x-2)$$

$$x+3 + 3(x-2) = 4$$

$$x+3 + 3x-6 = 4$$

$$4x-3 = 4$$

$$4x = 7$$

$$\boxed{x = \frac{7}{4}}$$

~~(x+1)~~ Solve for x. LCD: (x+1)(x-1)

$$3 \cdot \frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3(x+1)(x-1)$$

~~(x+1)~~

$$(4x+1)(x-1) = 12 + 3(x+1)(x-1)$$

$$4x^2 - 4x + x - 1 = 12 + 3(x^2 - 1)$$

$$4x^2 - 3x - 1 = 3x^2 + 9$$

$$-3x^2 \quad -9 \quad -3x^2 \quad -9$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x-5=0$$

$$x+2=0$$

$$\boxed{x=5}$$

$$\boxed{x=-2}$$

Solving Rational Equations

Solve each equation. Remember to check for extraneous solutions.

LCD =  $6k^2$   
 1)  $\frac{1}{6k^2} = \frac{1}{3k^2} - \frac{1}{k}$

$1 = 2 - 6k$   
 $-2 -2$   
 $-1 = -6k$   
 $k = \frac{1}{6}$

LCD =  $6b^2$   
 3)  $\frac{1}{6b^2} + \frac{1}{6b} = \frac{1}{b^2}$

$1 + b = 6$   
 $b = 5$

LCD =  $5x$   
 5)  $\frac{1}{x} = \frac{6}{5x} + 1$

$5 = 6 + 5x$   
 $\frac{-1}{5} = \frac{5x}{5}$   
 $x = -\frac{1}{5}$

LCD =  $v(v-5)$   
 7)  $\frac{1}{v} + \frac{3v+12}{v^2-5v} = \frac{7v-56}{v^2-5v}$

$v-5 + 3v+12 = 7v-56$   
 $4v+7 = 7v-56$   
 $-7v+56 -4v+56$   
 $63 = 3v$   
 $v = 21$

LCD =  $n-8$   
 9)  $\frac{1}{n-8} - \frac{1}{1} = \frac{7}{n-8}$

$1 - (n-8) = 7$   
 $1 - n + 8 = 7$   
 $-n + 9 = 7$   
 $9 - 9$   
 $-n = -2$   
 $n = 2$

LCD =  $2n^2$

2)  $\frac{1}{n} + \frac{1}{2n^2} = \frac{1}{2n^2}$

$2 + 2n = 1$   
 $2n = -1$   
 $n = -\frac{1}{2}$

LCD =  $4b^2$   
 4)  $\frac{b+6}{4b^2} + \frac{3}{2b} = \frac{b+4}{2b^2}$

$b+6 + 6 = 2(b+4)$   
 $b+12 = 2b+8$   
 $-b-8 -b-8$   
 $-16 = b$   
 $b = -16$

LCD =  $6x^2$   
 6)  $\frac{1}{6x^2} = \frac{1}{2x} + \frac{7}{6x^2}$

$1 = 3x + 7$   
 $-6 = 3x$   
 $x = -2$

LCD =  $m(m-1)$   
 8)  $\frac{1}{m^2-m} + \frac{1}{m} = \frac{5}{m^2-m}$

$1 + m - 1 = 5$   
 $m = 5$

LCD =  $(r-2)(r-5)$   
 10)  $\frac{1}{r-2} + \frac{1}{r^2-7r+10} = \frac{6}{r-2}$

$r-5 + 1 = 6(r-5)$   
 $r-4 = 6r-30$   
 $-6r+4 -6r+4$   
 $-12r = -26$   
 $r = \frac{26}{12} = \frac{13}{6}$   
 (16)

$$(v-4) \quad (v+2) \quad (v-4)$$

$$11) 1 = \frac{v+2}{v-4} + \frac{7v-42}{v-4}$$

$$v-4 = v+2 + 7v-42$$

$$v-4 = 8v-40$$

$$-8v + 4 = -8v + 4$$

$$-7v = -36$$

$$\boxed{v = \frac{36}{7}}$$

$$(3x) \quad (2x)$$

$$13) 1 + \frac{(x^2 - 5x - 24)}{3x} = \frac{(x-6)(3x)}{3x}$$

$$3x + x^2 - 5x - 24 = x - 6$$

$$x^2 - 2x - 24 = x - 6$$

$$x^2 - 3x - 18 = 0 \quad (x-6)(x+3) = 0$$

$$\boxed{x=6}$$

$$\boxed{x=-3}$$

$$(nt+5) \quad (nt+5) \quad (nt+5)$$

$$15) \frac{n+5}{n+8} = 1 + \frac{6}{n+1}$$

$$(nt+5)(n+1) = (nt+5)(n+8) + 6(n+8)$$

$$n^2 + 6n + 5 = n^2 + 9n + 8 + 6n + 48$$

$$6n + 5 = 15n + 56$$

$$-9n = 51$$

$$\boxed{n = -\frac{17}{3}}$$

$$(x(x-5)) \quad (x(x-5)) \quad (x(x-5))$$

$$17) \frac{1}{x^2 - 5x} = \frac{(x+7)}{x} - 1 \times \frac{(x-5)}{(x-5)}$$

$$1 = (x+7)(x-5) - x^2 + 5x$$

$$1 = x^2 + 2x - 35 - x^2 + 5x$$

$$1 = 7x - 35 \quad 36 = 7x$$

$$\boxed{x = \frac{36}{7}}$$

$$19) \frac{p+5}{p^2+p} = \frac{1}{p^2+p} - \frac{p-6}{p+1}$$

$$p+5 = 1 - p(p-6)$$

$$p+5 = 1 - p^2 + 6p$$

$$p^2 - 5p + 4 = 0$$

$$(p-4)(p-1) = 0$$

$$\boxed{p=1}$$

$$\boxed{p=4}$$

$$(5r) \quad (5r)$$

$$12) \frac{r-4}{r} = \frac{1}{5r} + \frac{1}{r}$$

$$r-4 = 1 + 5r$$

$$-r = 4$$

$$-5 = 4r$$

$$\boxed{r = -\frac{5}{4}}$$

$$(x(x+2)) \quad (x(x+2)) \quad (x(x+2))$$

$$14) 1 = \frac{1}{x^2+2x} + \frac{x-1}{x} \times \frac{(x+2)}{(x+2)}$$

$$\text{LCD: } x(x+2)$$

$$x^2 + 2x = 1 + (x-1)(x+2)$$

$$x^2 + 2x = 1 + x^2 + x - 2$$

$$2x = x - 1$$

$$\boxed{x = -1}$$

$$(r^2-2r) \quad (r^2-2r) \quad (r^2-2r)$$

$$16) \frac{r+5}{r^2-2r} - 1 = \frac{1}{r^2-2r} \times \frac{(r^2-2r)}{(r^2-2r)}$$

$$\text{LCD: } r(r-2)$$

$$r+5 - r^2 + 2r = 1$$

$$-r^2 + 3r + 5 = 1$$

$$-r^2 + 3r + 4 = 0$$

$$-1(r^2 - 3r - 4) = 0$$

$$-(r-4)(r+1) = 0$$

$$r=4 \quad r=-1$$

$$(a(x)) \quad (a(x)) \quad (a(x))$$

$$18) \frac{a-2}{a+3} - 1 = \frac{3}{a+2} \times \frac{(a+3)}{(a+3)}$$

$$(a-2)(a+2) - (a+3)(a+2) = 3(a+3)$$

$$a^2 - 4 - (a^2 + 5a + 6) = 3a + 9$$

$$a^2 - 4 - a^2 - 5a - 6 = 3a + 9$$

$$-5a - 10 = 3a + 9 \quad -8a = 19$$

$$\boxed{a = -\frac{19}{8}}$$

$$20) \frac{5}{n^3+5n^2} = \frac{4}{n+5} + \frac{1}{n^2}$$

$$5 = 4n^2 + (nt+5)$$

$$5 = 4n^2 + nt + 5$$

$$0 = 4n^2 + nt$$

$$0 = n(4n+t)$$

$$\boxed{n=0} \quad \boxed{n=-\frac{1}{4}}$$

Solve each equation. Check your solutions.

1.  $\left[ \frac{x-4}{x-2} = \frac{x-2}{x+2} + \frac{1}{x-2} \right] (x-2)(x+2)$  2.  $\left[ \frac{4}{x-2} - \frac{x+6}{x+1} = 1 \right] (x-2)(x+1)$

3.  $\left[ \frac{x-3}{2x} = \frac{x-2}{2x+1} - \frac{1}{2} \right] 2x(2x+1)$  4.  $\left[ \frac{12}{x^2-16} - \frac{24}{x-4} = 3 \right] (x+4)(x-4)$   
 $(x+4)(x-4)$

5.  $\left[ \frac{6}{x-7} = \frac{x-49}{x^2-7x} + \frac{1}{x} \right] x(x-7)$  6.  $\left( \frac{x}{x+2} - \frac{x+2}{x-2} = \frac{x+3}{x-2} \right) (x+2)(x-2)$

7.  $\left( x^2 + \frac{17x}{6} = \frac{1}{2} \right) \phi$  8.  $\left[ \frac{2}{y+2} - \frac{y}{2-y} = \frac{y^2+4}{y^2-4} \right] (y+2)(y-2)$   
 $-(y-2) (y+2)(y-2)$

9.  $\left[ \frac{x+4}{x} + \frac{3}{x-4} = \frac{-16}{x^2-4x} \right] x(x-4)$  10.  $\left[ \frac{y+3}{y+2} = 2 - \frac{3}{y^2+5y+6} \right] (y+2)(y+3)$   
 $(y+2)(y+3)$

11.  $\left[ \frac{t}{t^2-1} + \frac{2}{t+1} = \frac{1}{2t-2} \right] 2(t+1)(t-1)$  12.  $\frac{\overset{\textcircled{1}}{a+2} + \frac{2}{\overset{\textcircled{2}}{a+5}}}{\overset{\textcircled{2}}{a+6} + \frac{6}{\overset{\textcircled{2}}{a+1}}} = \frac{\overset{\textcircled{3}}{a+5} + \frac{3}{\overset{\textcircled{3}}{a+1}}}{\overset{\textcircled{4}}{a-1} - \frac{3}{\overset{\textcircled{4}}{a+1}}}$



$$1. (x-4)(x+2) = (x-2)(x-2) + 1(x+2)$$

$$x^2 - 2x - 8 = x^2 - 4x + 4 + x + 2$$

$$x^2 - 2x - 8 = x^2 - 3x + 6$$

$$\boxed{x = 14}$$

$$2. 4(x+1) - (x+6)(x-2) = (x-2)(x+1)$$

$$4x + 4 + (-x^2 + 4x + 12) = x^2 - x - 2$$

$$-x^2 + 16 = x^2 - x - 2$$

$$0 = 2x^2 - x - 18$$

$$\frac{1 \pm \sqrt{1 - 4(2)(-18)}}{2(2)} = \boxed{\frac{1 \pm \sqrt{145}}{4}}$$

$$3. (x-3)(2x+1) = 2x(x-2) - x(2x+1)$$

$$2x^2 - 5x - 3 = 2x^2 - 4x - 2x^2 - x$$

$$2x^2 - 5x - 3 = -5x$$

$$2x^2 - 3 = 0$$

$$2x^2 = 3$$

$$x^2 = \frac{3}{2}$$

$$x = \pm \sqrt{\frac{3}{2}} = \frac{\pm \sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \boxed{\frac{\pm \sqrt{6}}{2}}$$

$$4. 12 - 24(x+4) = 3(x^2 - 16)$$

$$12 - 24x - 96 = 3x^2 - 48$$

$$-24x - 84 = 3x^2 - 48$$

$$0 = 3x^2 + 24x + 36$$

$$0 = 3(x^2 + 8x + 12)$$

$$0 = 3(x+6)(x+2)$$

$$\boxed{x = -6, -2}$$

$$5. \quad 6x = x - 49 + x - 7$$

$$6x = 2x - 56$$

$$4x = -56$$

$$x = -14$$

$$6. \quad x(x-2) - (x+2)(x+2) = (x+3)(x+2)$$

$$x^2 - 2x + (-x^2 - 4x + 4) = x^2 + 5x + 6$$

$$-6x - 4 = x^2 + 5x + 6$$

$$0 = x^2 + 11x + 10$$

$$0 = (x+10)(x+1)$$

$$x = -10, -1$$

$$7. \quad 6x^2 + 17x = 3$$

$$6x^2 + 17x - 3 = 0$$

$$(6x-1)(x+3) = 0$$

$$x = \frac{1}{6}, -3$$

$$8. \quad 2(y-2) + y(y+2) = y^2 + 4$$

$$2y - 4 + y^2 + 2y = y^2 + 4$$

$$y^2 + 4y - 4 = y^2 + 4$$

$$4y = 8$$

$$y = 2$$

$$\boxed{\emptyset}$$

$$9. \quad (x+4)(x-4) + 3x = -16$$

$$x^2 - 16 + 3x = -16$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = 0, -3$$

$$x = -3$$

$$\begin{aligned}
 10. (y+3)(y+3) &= 2(y+2)(y+3) - 3 \\
 y^2 + 6y + 9 &= 2(y^2 + 5y + 6) - 3 \\
 &= 2y^2 + 10y + 12 - 3 \\
 y^2 + 6y + 9 &= 2y^2 + 10y + 9 \\
 0 &= y^2 + 4y \\
 0 &= y(y+4) \\
 \boxed{y=0, -4}
 \end{aligned}$$

$$\begin{aligned}
 11. 2t + 4(t-1) &= t+1 \\
 2t + 4t - 4 &= t+1 \\
 6t - 4 &= t+1 \\
 5t &= 5 \\
 t &= 1 \quad \boxed{\emptyset}
 \end{aligned}$$

$$12. \text{Sec ① } \frac{(a+2)(a+5)}{a+5} + \frac{2}{a+5} = \frac{a^2 + 7a + 10 + 2}{a+5} = \frac{a^2 + 7a + 12}{a+5}$$

$$\text{Sec ② } \frac{(a+6)(a+1)}{a+1} + \frac{6}{a+1} = \frac{a^2 + 7a + 6 + 6}{a+1} = \frac{a^2 + 7a + 12}{a+1}$$

$$\text{Sec ③ } \frac{(a+5)(a+1)}{a+1} + \frac{3}{a+1} = \frac{a^2 + 6a + 5 + 3}{a+1} = \frac{a^2 + 6a + 8}{a+1}$$

$$\text{Sec ④ } \frac{(a-1)(a+1)}{a+1} - \frac{3}{a+1} = \frac{a^2 - 1 - 3}{a+1} = \frac{a^2 - 4}{a+1}$$

$$\text{Left: } \frac{a^2 + 7a + 12}{a+5} \cdot \frac{a+1}{a^2 + 7a + 12} = \frac{a+1}{a+5}$$

$$\text{Right: } \frac{a^2 + 6a + 8}{a+1} \cdot \frac{a+1}{a^2 - 4} = \frac{a^2 + 6a + 8}{a^2 - 4} \cdot \frac{(a+4)(a+2)}{(a+2)(a-2)} = \frac{a+4}{a-2}$$

$$\text{Left} = \text{right} \quad \left[ \frac{a+1}{a+5} = \frac{a+4}{a-2} \right] (a+5)(a-2)$$

$$(a+1)(a-2) = (a+4)(a+5)$$

$$a^2 - a - 2 = a^2 + 9a + 20$$

$$-22 = 10a$$

$$\boxed{-\frac{11}{5} = a}$$

Solve.

$$7-x \quad 1. \left[ \frac{7}{7-x} = \frac{7}{7-x} + 1 \right]$$

$$7 = 7 + 1(7-x)$$

$$7 = 7 + 7 - x$$

$$7 = 14 - x$$

$$-7 = -x$$

$$\boxed{x=7}$$

NO solution

$$(x+2) \quad 3. \left[ \frac{x}{x+2} + x = \frac{5x+8}{x+2} \right]$$

$$x + x^2 + 2x = 5x + 8$$

$$x^2 + 3x = 5x + 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$\boxed{x=4 \quad x=-2}$$

$$(x-1)(x+1) \quad 2. \left[ \frac{x+3}{x-1} + 2 = \frac{(x+3)^2}{(x-1)^2} \right]$$

$$(x-1)(x+3) + 2(x-1)(x-1) = (x+3)(x+3)$$

$$x^2 + 2x - 3 + 2(x^2 - 2x + 1) = x^2 + 6x + 9$$

$$x^2 + 2x - 3 + 2x^2 - 4x + 2 = x^2 + 6x + 9$$

$$3x^2 - 2x - 1 = x^2 + 6x + 9$$

$$2x^2 - 8x - 10 = 0$$

$$2(x^2 - 4x - 5) = 0$$

$$2(x-5)(x+1) = 0$$

$$\boxed{x=5, -1}$$

$$2(a-1)(a+1) \quad 4. \left[ \frac{1}{2a-2} = \frac{a}{a^2-1} + \frac{2}{a+1} \right]$$

$$a+1 = 2a + 4(a-1)$$

$$a+1 = 2a + 4a - 4$$

$$a+1 = 6a - 4$$

$$-5a = -5$$

$$\boxed{a=1}$$

NO solution

$$6. \left[ \frac{x+4}{x} + \frac{3}{x-4} = \frac{-16}{x^2-4x} \right] \quad x(x-4)$$

$$(x+4)(x-4) + 3x = -16$$

$$x^2 - 16 + 3x = -16$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$\boxed{x=0 \quad x=-3}$$

$$-1(y-2)(y+2) \quad 5. \left[ \frac{3}{y-2} + \frac{2y}{4-y^2} = \frac{5}{y+2} \right]$$

$$-3(y+2) + -2y = -5(y-2)$$

$$-3y - 6 - 2y = -5y + 10$$

$$-5y - 6 = -5y + 10 \quad -6 \neq 10$$

NO SOLUTION

$$24 \quad 7. \left[ \frac{k-3}{8} - \frac{k+2}{6} = \frac{5}{12} \right]$$

$$3(k-3) - 4(k+2) = 10$$

$$3k - 9 - 4k - 8 = 10$$

$$-k - 17 = 10$$

$$-k = 27$$

$$\boxed{k=-27}$$

$$8. \left[ \frac{x}{x+1} + \frac{3}{x-3} + 1 = 0 \right] \quad (x+1)(x-3)$$

$$x(x-3) + 3(x+1) + (x+1)(x-3) = 0$$

$$x^2 - 3x + 3x + 3 + x^2 - 2x - 3 = 0$$

$$2x^2 - 2x = 0$$

$$2x(x-1) = 0$$

$$\boxed{x=0, x=1}$$

# Solving Rational Inequalities WS 1

1.

$$\sqrt{0}: \frac{5}{0-2} = -\frac{5}{2} \quad \sqrt{3}: \frac{5}{3-2} = \frac{5}{1} = 5 \quad \text{pos}$$

neg ☹

$$(-\infty, 2)$$

2.

$$\frac{x-5}{x+3} - 1 > 0$$

$$\frac{x-5}{x+3} - \frac{x+3}{x+3} > 0$$

$$\frac{x-5-x-3}{x+3} > 0$$

$$\frac{-8}{x+3} > 0$$

$$\sqrt{-4}: \frac{-9}{-1} = 9 \quad \sqrt{4}: \frac{-8}{1} = -8 = -1 = \text{neg}$$

$$(-\infty, -3)$$

3.

$$\frac{x^2-3x+2}{x-3} - \frac{x(x-3)}{1(x-3)} < 0$$

$$\frac{\cancel{x^2}-3x+2-\cancel{x^2}+\cancel{3x}}{x-3} < 0$$

$$\frac{2}{x-3} < 0$$

$$\sqrt{0}: \frac{0-0+2}{0-3} = -\frac{2}{3} \quad \sqrt{4}: \frac{16-12+2}{4-3} = \frac{4+2}{1} = 6$$

$$-\frac{2}{3} = 0 = \text{neg} \quad \text{pos} \quad \text{pos}$$

$$6-4=2 \text{ pos}$$

$$(-\infty, 3)$$

4.

$$\sqrt{-3}: \frac{10}{-1} = -10 \quad \sqrt{3}: \frac{10}{5} = 2 \quad \text{pos}$$

$$-10 = \text{neg}$$

$$(-2, \infty)$$

5.

$$-2x-3=0$$

$$-2x=3$$

$$x = -\frac{3}{2}$$

$$\sqrt{-2}: \frac{4-3}{-6} = -\frac{1}{6} \quad \sqrt{2}: -\frac{3}{2}$$

$$-\frac{1}{6} = \text{neg}$$

$$\sqrt{0}: -\frac{3}{4} \quad \sqrt{4}: 4$$

$$-\frac{3}{4} = \text{pos}$$

$$\sqrt{5}: \frac{-10-3}{1} = -13 = \text{neg}$$

$$(-\frac{3}{2}, 4)$$

$$6. \frac{x^2 - 4x + 8}{x-1} - \frac{x(x-1)}{x-1} < 0$$

$$\frac{x^2 - 4x + 8 - x^2 + x}{x-1} < 0$$

$$\frac{-3x + 8}{x-1} < 0$$

$$x = \frac{8}{3}, 1$$

$(-\infty, 1)$

√0: 0-0+8  
0-1  
-8-0=-8

neg

√2: 2-2+8  
4-8+8  
2-1

4-0=4  
pos

√3: 9-12+8

2  
2+3=pos

$$7. \frac{x+5}{x-4} > 0 \quad (-5, \infty)$$

√-6: -5  
-4  
-1 = 4 pos

√0: -4  
5 neg

$$8. \frac{x-4}{x+3} < 0 \quad (-\infty, 3)$$

√0: 4  
-3 neg

√4: 4  
1 = 4 pos

$$9. \frac{8}{x^2+1} - \frac{4(x^2+1)}{x^2+1} \geq 0$$

$$\frac{8-4x^2-4}{x^2+1} \geq 0$$

$$\frac{-4x^2+4}{x^2+1} \geq 0$$

$$\frac{-4(x^2-1)}{x^2+1} \geq 0$$

√-2: -1  
8  
5 = 4 neg

√0: 1  
8-4 pos

check 2: 8  
5 - 4 = neg

$(-1, 1)$

$$10. \frac{20}{x^2+1} - 2 \frac{(x^2+1)}{x^2+1} < 0$$

$$\frac{20}{x^2+1} - 2 \frac{(x^2+1)}{x^2+1} < 0$$

$$20 - 2x^2 - 2 < 0$$

$$x^2 + 1$$

$$-2x^2 + 18 < 0$$

$$x^2 + 1$$

$$-2(x^2 - 9) < 0$$

$$x^2 + 1$$

$$\begin{array}{c|c|c|c|c} & -4 & -3 & 10 & 3 & 4 \end{array}$$

$$\frac{20}{17} - 2 \quad \frac{20}{1} - 2 \quad \frac{20}{17} - 2$$

$$\boxed{\text{neg } \smile} \quad \text{pos } \smile \quad \boxed{\text{neg } \smile}$$

$$\boxed{(-\infty, -3) \cup (3, \infty)}$$

$$11. \frac{3x+2}{x-1} + \frac{2(x-1)}{x-1} < 0$$

$$\frac{3x+2}{x-1} + \frac{2(x-1)}{x-1} < 0$$

$$3x+2 + 2x-2 < 0$$

$$x-1$$

$$5x < 0$$

$$x-1$$

$$\begin{array}{c|c|c|c|c} & -1 & 0 & \frac{1}{3} & 1 & 2 \end{array}$$

$$\frac{-1}{-2} + 2 \quad \frac{1+2}{-7/3} + 2 \quad \frac{8}{1} + 2$$

$$\text{pos } \smile \quad -7/3 \quad \text{pos } \smile$$

$$3 - \frac{3}{2} + 2$$

$$-\frac{9}{2} + 2 \quad \boxed{\text{neg } \smile}$$

$$\boxed{(0, 1)}$$

$$12. \frac{3x+2}{x-1} - \frac{x(x-1)}{x-1} > 0$$

$$\frac{3x+2}{x-1} - \frac{x(x-1)}{x-1} > 0$$

$$3x+2 - x^2 + x > 0$$

$$x-1$$

$$-x^2 + 4x + 2 > 0$$

$$x-1$$

$$-(x^2 - 4x - 2) > 0$$

$$x-1$$

$$\begin{array}{c|c|c|c|c|c} & -1 & 2-\sqrt{6} & 1 & 2+\sqrt{6} & 5 \end{array}$$

$$\frac{-1}{-2} + 1 \quad \frac{2}{-1} - 0 \quad \frac{8}{1} - 8 \quad \frac{17}{1} - 5(x)$$

$$\text{pos } \smile \quad \text{neg } \smile \quad \boxed{\text{pos } \smile} \quad \text{neg } \smile$$

$$\boxed{(-\infty, 2-\sqrt{6}) \cup (1, 2+\sqrt{6})}$$

$$x^2 - 4x - 2$$

$$4 \cdot 1 = 4$$

$$-b \pm \sqrt{b^2 - 4ac} = 4 \pm \sqrt{16 + 8}$$

$$2a$$

$$2$$

$$= \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6} \approx 4.4, -0.4$$

13.

$$\frac{-1}{x+5} - \frac{2(x+5)}{x+5} \leq 0$$

$$\frac{-1 + 2x + 10}{x+5} \leq 0$$

$$\frac{2x+9}{x+5} \leq 0$$

$$(-2, \infty)$$

14.

$$\frac{-1}{x+5} - \frac{2(x+5)}{x+5} \leq 0$$

$$\frac{-1 + 2x + 10}{x+5} \leq 0$$

$$\frac{2x+9}{x+5} \leq 0$$

$$\begin{array}{c} | \\ \hline \sqrt{-6}: -5 \quad -4.75 \quad -9/2 \quad \sqrt{0}: \\ \frac{-1}{-1} + 2 \quad \frac{-1}{-0.25} + 2 \quad \frac{-1}{5} + 2 \\ \text{Pos} \quad \text{neg} \quad \text{Pos} \\ \text{":} \quad \text{":} \quad \text{":} \end{array}$$

$$(-5, -\frac{9}{2})$$

15.

$$\frac{2(x+3)}{x+2} - \frac{1(x+2)}{x+3} > 0$$

$$\frac{2x+6-x-2}{(x+2)(x+3)} > 0$$

$$\frac{x+4}{(x+2)(x+3)} > 0$$

$$\begin{array}{c} | \quad | \quad | \\ \hline \sqrt{-5}: -4 \quad \sqrt{-3.5}: -3 \quad \sqrt{-2.5}: -2 \quad \sqrt{0}: \\ \frac{2}{-3} + \frac{1}{12} \quad \frac{2}{-1.5} + \frac{1}{1.5} \quad \frac{2}{-1.5} - \frac{1}{1.5} \quad \frac{2}{2} - \frac{1}{3} \\ \text{neg} \quad \text{pos} \quad \text{pos} \quad \text{pos} \\ \text{":} \quad \text{":} \quad \text{":} \quad \text{":} \end{array}$$

$$(-4, \infty)$$

16.

$$\frac{5(x+4)}{x-4} - \frac{1(x-4)}{x+4} < 0$$

$$\frac{5x+20-x+4}{(x+4)(x-4)} < 0$$

$$\frac{4x+24}{(x+4)(x-4)} < 0$$

$$\frac{4(x+6)}{(x+4)(x-4)} < 0$$

$$\begin{array}{c} | \quad | \quad | \quad | \\ \hline \sqrt{-7}: -6 \quad \sqrt{-5}: -4 \quad \sqrt{0}: 4 \quad \sqrt{5} \\ \frac{5}{-11} + \frac{1}{13} \quad \frac{5}{-9} + \frac{1}{11} \quad \frac{5}{-4} - \frac{1}{4} \quad \frac{5}{1} - \frac{1}{4} \\ \text{neg} \quad \text{pos} \quad \text{neg} \quad \text{pos} \\ \text{":} \quad \text{":} \quad \text{":} \quad \text{":} \end{array}$$

$$(-\infty, -6) \cup (-4, 4)$$



17.

$$\frac{5(x+2)}{x+3} - \frac{4(x+3)}{x+2} \geq 0$$

$$\frac{5x+10-4x-12}{(x+3)(x+2)} \geq 0$$

$$\frac{x+2}{(x+3)(x+2)} \geq 0$$

√-4:	-3	√-2.5:	-2	√0:
-2		-5		2
(-1)(-2)		(1.5)(-5)		(3)(2)
neg ∴		pos ∴		pos ∴

$$(-3, \infty)$$

18.

$$\frac{2(x^3)}{x+6} + \frac{3(x+6)}{x-3} > 0$$

$$\frac{2x-6+3x+18}{(x+6)(x-3)} > 0$$

$$\frac{5x+12}{(x+6)(x-3)} > 0$$

√-7:	-6	√-3:	-12/5	√0:	3	√4:
2/-1 + 3/-10		2/3 + 3/-6		2/6 + 3/-3		2/10 + 3/1
neg ∴		pos ∴		neg ∴		pos ∴

$$(-6, -\frac{12}{5})(3, \infty)$$

# Solving Rational Inequalities 2

WS 2

1.  $\frac{(x-5)}{(x-3)(x-2)} > 0$

$(x-3)(x-2)$

$(2,3) \cup (5,8)$

√0:  
-5  
(-3)(2)  
-5  
Neg:  $\ddot{}$

2 1/25  
-2.5  
(-5.5)(.5)  
Pos  $\smile$

3 1/4  
-1  
(1)(2)  
Neg  $\ddot{}$

5 1/6  
1  
3(4)  
Pos  $\smile$

2.  $\frac{4}{x} + \frac{3x}{x} \geq 0$

$\frac{4+3x}{x} \geq 0$

$(-\infty, -\frac{4}{3}] \cup [0, \infty)$

√-2:  
-2  
-2  
Pos  $\smile$

-4  
3  
4-3  
1  
Neg  $\ddot{}$

0  
4+3  
Pos  $\smile$

3.  $(2x+1)(3x+1) < (x-1)(3x+1)$

$(2x+1)(3x+1) - (x-1)(3x+1) < 0$

$(6x^2+5x+1) - (3x^2+2x+1) < 0$

$3x^2+7x+2 < 0$

$(3x+1)(x+2) < 0$

√-3:  
-8(-)  
Pos  $\ddot{}$

-2  
-2(1)  
Neg  $\ddot{}$

√3:  
10(5)  
Pos  $\ddot{}$

$(-2, -\frac{1}{3})$

4.  $\frac{3x}{x-1} - 1 > 0$

$\frac{3x-x-1}{x-1} > 0$

$\frac{2x+1}{x-1} > 0$

√-1:  
-1  
-2  
Pos  $\smile$

-1/2  
1  
1  
Neg  $\ddot{}$

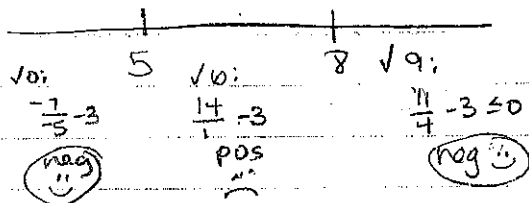
√2:  
5/2  
Pos  $\smile$

$(-\infty, -\frac{1}{2}) \cup (1, \infty)$

$$5. \frac{2x-7}{x-5} - 3 \leq 0 \quad (x-5)$$

$$\frac{2x-7}{x-5} + \frac{-3x+15}{x-5} \leq 0$$

$$\frac{-x+8}{x-5} \leq 0$$



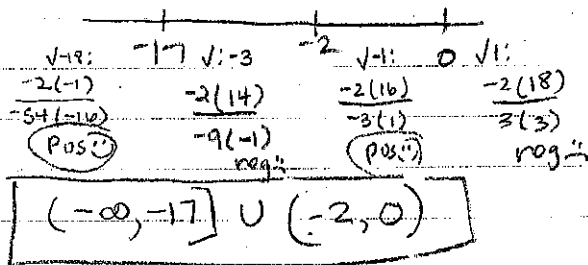
$$(-\infty, 5) \cup [8, \infty)$$

$$6. \frac{5^{3x}}{(x+2)} - \frac{5^{3(x+2)}}{x} - \frac{2^{(x+2)}}{3x} \geq 0$$

$$\frac{5^x}{3x(x+2)} + \frac{-5^x+30}{3x(x+2)} + \frac{-2^x+4}{3x(x+2)} \geq 0$$

$$\frac{-2x-34}{3x(x+2)} \geq 0$$

$$\frac{-2(x+17)}{3x(x+2)} \geq 0$$



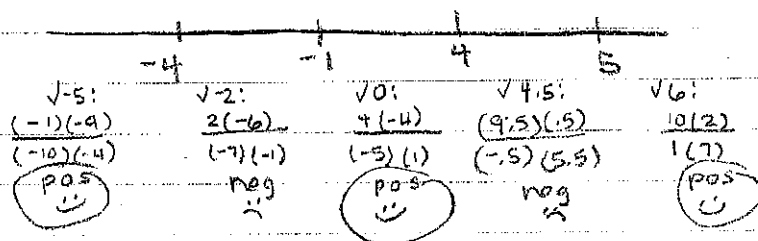
$$(-\infty, -17] \cup (-2, 0)$$

$$7. \frac{(x+4)(x-4)}{(x-5)(x+1)} > 0$$

$$(-\infty, -4) \cup$$

$$(-1, 4) \cup$$

$$(5, \infty)$$



$$8. \quad \frac{1-2x}{x-3} - \frac{1}{x+3} \leq 0$$

$$\frac{(x+3)(x-3) - 2x(x+3) - (x-3)}{(x+3)(x-3)(x+3)(x-3)} \leq 0$$

$$\frac{x^2 - 9 - 2x^2 - 6x - x + 3}{(x+3)(x-3)} \leq 0$$

$$\frac{-x^2 - 7x - 6}{(x+3)(x-3)} \leq 0$$

$$\frac{-(x^2 + 7x + 6)}{(x+3)(x-3)} \leq 0$$

$$\frac{-(x+6)(x+1)}{(x+3)(x-3)} \leq 0$$

$\sqrt{-4}$	$-6$	$\sqrt{-1}$	$-3$	$\sqrt{-2}$	$-1$	$\sqrt{0}$	$\frac{1}{3}$	$\sqrt{4}$
$\frac{-(-1)(-6)}{(-4)(-10)}$		$\frac{-(-2)(-3)}{(-1)(-7)}$		$\frac{-(-4)(-2)}{(1)(-5)}$		$\frac{-(-6)(1)}{(3)(-3)}$		$\frac{-(-10)(5)}{7(1)}$
neg		pos		neg		pos		neg

$$(-\infty, -6] \cup (-3, -1] \cup (3, \infty)$$

Unit 2C Review Sheet

**Factor Completely.**

$$1. \frac{2x^2 - 8}{2(x^2 - 4)} = \frac{2(x+2)(x-2)}{2(x+2)(x-2)}$$

$$2. \frac{-3x^2 + 21x^3}{-3x^3(x^2 - 7)}$$

$$3. \frac{18x^3 - 6x^2 + 3x}{3x(6x^2 - 2x + 1)}$$

$$4. \frac{x^2 + 4x - 12}{(x+6)(x-2)}$$

$$5. \frac{x^2 - 10x + 25}{(x-5)(x-5)}$$

$$6. \frac{x^2 - 3x - 10}{(x-5)(x+2)}$$

$$7. \frac{2x^2 + 32x + 30}{2(x^2 + 16x + 15)} = \frac{2(x+1)(x+15)}{2(x+1)(x+15)}$$

$$8. \frac{3y^2 - 9y - 54}{3(y^2 - 3y - 18)} = \frac{3(y-6)(y+3)}{3(y-6)(y+3)}$$

$$9. \frac{2t^2 - 20t + 42}{2(t^2 - 10t + 21)} = \frac{2(t-7)(t-3)}{2(t-7)(t-3)}$$

$$10. \frac{4x^2 + 32x - 36}{4(x^2 + 8x - 9)} = \frac{4(x+9)(x-1)}{4(x+9)(x-1)}$$

**Simplify.**

$$1. \frac{3x}{10x + x^2} = \frac{3x}{x(10+x)} = \frac{3}{10+x}$$

$$2. \frac{3x^2 - 6xy}{x^2 - 4y^2} = \frac{3x(x-2y)}{(x+2y)(x-2y)} = \frac{3x}{x+2y}$$

$$3. \frac{8(6x+x^2)}{16+x^2} = \frac{(x+4)(x+2)}{x^2+16}$$

$$4. \frac{x^3 - x^2 - 12x}{x^2 - 9x} = \frac{x(x^2 - x - 12)}{x(x^2 - 9)} = \frac{(x-4)(x+3)}{(x+3)(x-3)} = \frac{x-4}{x-3}$$

**Perform the indicated operation and simplify.**

$$5. \frac{x+4}{x} \cdot \frac{x}{2x+8} = \frac{x+4}{1} \cdot \frac{1}{2(x+4)} = \frac{1}{2}$$

$$6. \frac{x^2 + 5x - 24}{x^2 + 9x + 8} \div \frac{x^2}{6x - 18} = \frac{(x+8)(x-3)}{(x+8)(x+1)} \cdot \frac{6(x-3)}{x^2} = \frac{6(x-3)^2}{x^2(x+1)}$$

$$9. \frac{4x}{x^2 - x - 12} \div \frac{1}{x-4} = \frac{4x}{(x-4)(x+3)} \cdot \frac{x-4}{1} = \frac{4x}{x+3}$$

$$10. \frac{x+2}{x-3} \cdot \frac{x^2 - 4x + 3}{x^2 + 6x + 8} = \frac{x+2}{x-3} \cdot \frac{(x-3)(x-1)}{(x+4)(x+2)} = \frac{x-1}{x+4}$$

$$11. \frac{2x+9}{x+1} \cdot \frac{7}{x+1} = \frac{2x+2}{x+1} = \frac{2(x+1)}{x+1} = 2$$

$$12. \frac{(x+1)}{x^2 + 5x + 6} \cdot \frac{x-4}{x^2 - 9} \cdot \frac{x+2}{x+2} = \frac{(x+1)(x-4)}{(x+2)(x+3)(x-3)(x+3)} = \frac{(x+1)(x-4)}{(x+2)(x+3)(x-3)}$$

see other sheet

$$13. \frac{x-1}{x^2 - 2x - 24} \div \frac{4}{x^2 - 5x - 6} = \frac{(x-6)(x+4)}{(x-6)(x+6)} \cdot \frac{(x-6)(x+6)}{4} = \frac{(x-6)(x+6)}{4}$$

see other sheet

$$14. \frac{x+2}{x^2 + 2x - 15} \div \frac{x-6}{x^2 + 4x - 21} = \frac{(x+2)(x-3)}{(x+7)(x-5)} \cdot \frac{(x-5)(x+7)}{(x-6)(x+7)} = \frac{(x+2)(x-3)}{(x-6)(x+7)}$$

see other sheet

$$15. \frac{x+6}{x^2+10x+24} \cdot \frac{x+1}{x^2+2x-15}$$

$$16. \frac{x-2}{x^2+10x+24} + \frac{4x}{x+1} \cdot \frac{5}{x+6}$$

17. John is having grass put in his backyard. The landscape company needs to know the area and perimeter of John's yard. The length of his yard is  $\frac{x^4}{x^4+5x^3}$  and the width is  $x+5$ .

Find the area and perimeter for the landscape company.

$$A = \frac{x^4}{x^4+5x^3} \cdot \frac{x+5}{1} = \frac{x^4}{x^3(x+5)} \cdot \frac{x+5}{1} = \frac{x^4}{x^3} = x$$

$$P = 2\left(\frac{x^4}{x^4+5x^3}\right) + 2(x+5)$$

$$= 2\left(\frac{x^4}{x^3(x+5)}\right) + 2x+10$$

For what values is the rational expression undefined?

$$= \frac{2x}{x+5} + \frac{2x+10}{x+5}$$

$$31. \frac{x+3}{x-6} \quad x \neq 6$$

$$32. \frac{x-3}{x^2+5x-6} \cdot \frac{-2x+2x^2+10x+10x+50}{(x+6)(x-1)}$$

$$x \neq -6 \quad x \neq 1$$

$$= \frac{2x^2+22x+50}{(x+5)}$$

$$33. \frac{x^2-1}{6x+6} \quad x \neq -\frac{1}{6}$$

$$34. \frac{x^2-3x-10}{x^2+5x+6}$$

$$(x+3)(x+2)$$

$$x \neq -3 \quad x \neq -2$$

Simplify the following complex fractions.

$$35. \frac{2x-14}{x^2-4x-21} \cdot \frac{2x-14}{x^2-4x-21} \cdot \frac{1}{x+3}$$

$$\frac{2(x-7)}{(x-7)(x+3)} \cdot \frac{1}{x+3}$$

$$\frac{2}{(x+3)^2}$$

$$36. \frac{x^3}{3xy} \div \frac{y^2}{3x}$$

$$\frac{x^3}{3xy} \cdot \frac{3x}{y^2} = \frac{x^3}{y^3}$$

$$37. \frac{x+2}{3x-3} \cdot \frac{x^2+11x+18}{x-1}$$

$$38. \frac{x^2+8x+15}{x^2+7x+10} \cdot \frac{3x(x+3)}{3x^2+9x}$$

$$\frac{x^2-2x-8}{x^2-2x-8}$$

$$\frac{x+2}{3(x-1)} \cdot \frac{x^2+11x+18}{x-1}$$

$$\frac{x+2}{3(x-1)} \cdot \frac{(x+9)(x+2)}{(x-1)(x+2)}$$

$$\frac{1}{3(x+9)}$$

$$\frac{(x+5)(x+3)}{(x+5)(x+2)} \cdot \frac{3x(x+3)}{(x-4)(x+2)}$$

$$\frac{(x+3)}{(x+2)} \cdot \frac{3x(x+3)}{(x-4)(x+2)}$$

$$\frac{x-4}{3x}$$