

Simplifying Rational Expressions
Multiplying and Dividing Rational Expressions

Example 1:

*denominator $\neq 0$
 cannot \div by 0*

State the excluded value of x for each rational expression.

a. $\frac{x+2}{3x} \rightarrow 3x \neq 0$
 $x \neq 0$
 $D: (-\infty, 0) \cup (0, \infty)$

b. $\frac{4x}{(x+5)} \neq 0$
 $x \neq -5$
 $D: (-\infty, -5) \cup (-5, \infty)$

c. $\frac{x+3}{x^2+4x-12}$
 $(x+6)(x-2)$
 $x \neq -6$ $x \neq 2$
 $D: (-\infty, -6) \cup (-6, 2) \cup (2, \infty)$

Example 2:

This is like: $\frac{12}{18} = \frac{\cancel{3} \cdot 4}{\cancel{3} \cdot 6} = \frac{4}{6}$

Simplify.

a. $\frac{5(3-x)}{5x} = \frac{\cancel{5} \cdot (3-x)}{\cancel{5} \cdot x} = \frac{(3-x)}{x}$

b. $\frac{2x^2}{x(x+5)} = \frac{\cancel{2} \cdot \cancel{x} \cdot x}{\cancel{x} \cdot (x+5)} = \frac{2x}{(x+5)}$

c. $\frac{(5x+3)}{(x+3)}$ already simplified

Example 3:

FACTOR !!!

Simplify.

a. $\frac{15x}{(5-10x)} = \frac{3 \cdot \cancel{5} \cdot x}{\cancel{5} \cdot (1-2x)} = \frac{3x}{(1-2x)}$
 GCF

b. $\frac{(x^2+6x+9)}{(x^2-9)} = \frac{\cancel{(x+3)} \cdot (x+3)}{\cancel{(x+3)} \cdot (x-3)} = \frac{x+3}{x-3}$

For what values of the variable is the rational expression undefined?

1. $\frac{3}{4x} \quad x \neq 0$

2. $\frac{2x}{7}$

3. $\frac{5}{x+2} \quad x \neq -2$

4. $\frac{x+2}{x-5} \quad x \neq 5$

5. $\frac{14}{2x+8} \quad x \neq -4$

6. $\frac{x+5}{(x-2)(x+1)} \quad x \neq 2, -1$

7. $\frac{2x+9}{(x-5)(x+9)} \quad x \neq 5, -9$

8. $\frac{5}{x^2-1} \quad x \neq -1, 1$

9. $\frac{x+4}{x^2+x-6} \quad x \neq -3, 2$
 $(x+3)(x-2)$

Determine whether the rational expression has been simplified correctly.

10. $\frac{3x+7}{7} \stackrel{?}{=} 3x \quad \text{NO}$

11. $\frac{2x}{2x+10} \stackrel{?}{=} \frac{x}{x+5} \quad \text{yes}$

12. $\frac{5+x}{5+2x} \stackrel{?}{=} \frac{1}{2} \quad \text{NO}$

13. $\frac{x^2+7}{x+7} \stackrel{?}{=} x \quad \text{NO}$

14. $\frac{6x}{2x^2+x} \stackrel{?}{=} \frac{6}{2x+1} \quad \text{yes}$

15. $\frac{1+x}{1+x^2} \stackrel{?}{=} \frac{1}{x} \quad \text{NO}$

Simplify the expression if possible.

16. $\frac{4x}{12} = \frac{x}{3}$

17. $\frac{18x}{36} = \frac{x}{2}$

18. $\frac{3 \cancel{18}x^4}{2 \cancel{10}x} = \frac{3x}{2}$

19. $\frac{18x^2}{60x^3} = \frac{3}{10x}$

20. $\frac{3x}{10x+x^2} = \frac{3}{x+10}$

21. $\frac{2x^2+x}{2x+1} = \frac{x(2x+1)}{2x+1} = x$

22. $\frac{x^2-16}{3x+12} = \frac{(x-4)}{3}$

23. $\frac{x^2-25}{x-5} = \frac{x+5}{1}$

24. $\frac{x-3}{x^2-5x+6} = \frac{x-3}{(x-3)(x-2)} = \frac{1}{x-2}$

25. $\frac{9-x^2}{x+3} = \frac{(3+x)(3-x)}{x+3} = \frac{3-x}{1}$

26. $\frac{8-6x+x^2}{16-x^2} = \frac{(x-4)(x-2)}{(4-x)(4+x)} = \frac{-(x-2)}{x+4}$

27. $\frac{-(3-x)(5-x)}{(x-3)(x-5)} = \frac{(x-3)(x-5)}{(x-3)(x-5)} = 1$

For what values of the variable is the rational expression undefined?

1. $\frac{7}{14x} \quad x \neq 0$

2. $\frac{-5x}{10}$

3. $\frac{8}{x+4} \quad x \neq -4$

4. $\frac{x+3}{x-6} \quad x \neq 6$

5. $\frac{20}{5x+10} \quad x \neq -2$

6. $\frac{5x+2}{(x-6)(x+9)} \quad x \neq 6, -9$

7. $\frac{x-3}{x^2+5x-6} \quad x \neq -6, 1$
 $(x+6)(x-1)$

8. $\frac{x-7}{x^2-49} \quad x \neq 7$

9. $\frac{x^2-2x-3}{x^2-9} \quad x \neq -3$

Simplify the expression if possible.

10. $\frac{7x}{21} = \frac{x}{3}$

11. $\frac{20x}{28} = \frac{5x}{7}$

12. $\frac{18x^2}{12x} = \frac{3x}{2}$

13. $\frac{36x^4}{42x^7} = \frac{6}{7x^3}$

14. $\frac{5x}{x^2+3x} = \frac{5}{x+3}$

15. $\frac{2x^2+x}{4x} = \frac{2x+1}{4}$

16. $\frac{x^2-1}{6x+6} = \frac{(x-1)}{6}$

17. $\frac{4x-12}{x^2-9} = \frac{4}{x+3}$

18. $\frac{x^2-3x-10}{x^2+5x+6} = \frac{(x-5)(x+2)}{(x+5)(x+2)} = \frac{x-5}{x+3}$

19. $\frac{2x^2+5x+3}{4x^2+4x-3} = \frac{(2x+3)(x+1)}{(2x-1)(2x+3)} = \frac{x+1}{2x-1}$

20. $\frac{x^2+10x+24}{x^2-16} = \frac{(x+6)(x+4)}{(x-4)(x+4)} = \frac{x+6}{x-4}$

21. $\frac{x^3-x^2-12x}{x^3-9x} = \frac{x(x-4)(x+3)}{x(x-3)(x+3)} = \frac{x-4}{x-3}$

Example 4:

$$\text{This is like: } \frac{3}{5} \cdot \frac{12 \cancel{3}^1}{20 \cancel{3}^1} = \frac{36}{180} = \frac{1}{5}$$

Simplify.

$$\text{a. } \frac{(y-5) \cdot 2y^2}{(3y^2-3y)(y^2-6y+5)} = \frac{2 \cdot \cancel{y} \cdot \cancel{y} \cdot (y-5)}{3 \cdot \cancel{y} \cdot (y-1) \cdot \cancel{(y-5)} \cdot (y-1)} = \boxed{\frac{2y}{3(y-1)^2}}$$

$$\text{b. } \frac{(2x+1)(2x-3)}{(2x^2-x-3)(x+1)} = \frac{(2x+1) \cdot \cancel{(2x-3)}}{\cancel{(2x-3)} \cdot (x+1)} = \boxed{\frac{2x+1}{x+1}}$$

Example 5:

$$\text{This is like: } \frac{12}{5} \div \frac{3}{10} = \frac{12}{5} \cdot \frac{10}{3} = \frac{120}{15} = 8$$

Simplify.

$$\text{a. } \frac{(n-2)(n-2)}{2n(n+5)} = \frac{(n-2)}{2n} \cdot \frac{(n+5)}{\cancel{(n-2)}} = \frac{\cancel{(n-2)} \cdot (n+5)}{2 \cdot n \cdot \cancel{(n-2)}} = \boxed{\frac{n+5}{2n}}$$

$$\begin{aligned} \text{b. } \frac{(5x^2-20x)}{(x+5)} \div \frac{(x-4)}{1} &= \frac{(5x^2-20x)}{(x+5)} \cdot \frac{1}{(x-4)} \\ &= \frac{5x \cdot \cancel{(x-4)}}{(x+5) \cdot \cancel{(x-4)}} = \boxed{\frac{5x}{x+5}} \end{aligned}$$

Multiplying Rational Expressions

Name: _____
Date: _____

Simplify the expression.

1. $\frac{4x}{3} \cdot \frac{2}{x} = \boxed{\frac{8}{3}}$
2. $\frac{13x}{18} \cdot \frac{10^2}{4 \cdot 12x} = \frac{2}{4} = \boxed{\frac{1}{2}}$
3. $\frac{4x^2}{17} \cdot \frac{14^2}{5x} = \boxed{\frac{8x}{5}}$
4. $\frac{3x}{2x} \cdot \frac{34x}{5} = \boxed{\frac{6}{5}}$
5. $\frac{7x^2}{18} \cdot \frac{18^3}{2 \cdot 4x} = \boxed{\frac{3x}{2}}$
6. $\frac{1x^2}{12x} \cdot \frac{9 \cdot 8x^2}{38x} = \frac{9x^2}{3} = \boxed{3x^2}$
7. $\frac{x+4}{3} \cdot \frac{8^3}{2(x+4)} = \boxed{11}$
8. $\frac{x-3}{x+3} \cdot \frac{x+3}{x^2-9} = \boxed{\frac{1}{x+3}}$
9. $\frac{x+2}{3x+6} \cdot \frac{6}{x} = \boxed{\frac{2}{x}}$
10. $\frac{(x-5) \cdot 25}{1 \cdot 12} \cdot \frac{36^3}{x+5} = \boxed{3(x-5)}$
11. $\frac{x+4}{x^2+5x+4} \cdot \frac{(3x+4)^2}{(x+4)(x+1)} = \boxed{3}$
12. $\frac{x+4}{4} \cdot \frac{x-2}{x^2-4} = \boxed{\frac{x+4}{4(x+2)}}$

Simplify the expression.

1. $\frac{8x}{6} \cdot \frac{6}{x} = \boxed{8}$
2. $\frac{7x}{5} \cdot \frac{10}{x^2} = \boxed{\frac{14}{x}}$
3. $\frac{12x^3}{25} \cdot \frac{40}{9x^2} = \boxed{\frac{32x}{15}}$
4. $\frac{14x^5}{3x^2} \cdot \frac{9x^3}{28x^8} = \boxed{\frac{3}{2x^2}}$
5. $\frac{6-18x}{4x^2} \cdot \frac{x^3}{2-6x} = \boxed{\frac{3x}{4}}$
6. $\frac{6}{x^2-9x+20} \cdot (5x-25) = \boxed{\frac{30}{x-4}}$
7. $\frac{4x}{x+1} \cdot \frac{x^2-6x-7}{x^3+7x^2} = \boxed{\frac{4(x-7)}{x(x+7)}}$
8. $\frac{x}{2x^2-7x+3} \cdot (7x-21) = \boxed{\frac{7x}{2x-1}}$
9. $\frac{2x-6}{x^2-25} \cdot \frac{x^2+6x+5}{x^2-9} = \boxed{\frac{2(x+1)}{(x-5)(x+3)}}$
10. $\frac{x^2-16}{12} \cdot \frac{48}{x+4} = \boxed{4(x-4)}$
11. $\frac{x+3}{x^2+5x+6} \cdot (5x+10) = \boxed{5}$
12. $\frac{x+5}{7} \cdot \frac{x+7}{x^2-25} = \boxed{\frac{x+7}{7(x-5)}}$

Simplify the expression.

1. $\frac{6x}{5} \cdot \frac{1}{x}$
2. $\frac{8x^2}{3} \cdot \frac{9}{16x}$
3. $\frac{3x^2}{2x} \cdot \frac{12x^2}{6x}$
4. $\frac{5-4x}{4} \cdot \frac{48}{10-8x}$
5. $\frac{4x}{x^2-9} \cdot \frac{x-3}{8x^2+12x}$
6. $\frac{8}{2+3x} \cdot (8+12x)$
7. $\frac{3x}{x^2-2x-24} \cdot \frac{x-6}{6x^2+9x}$
8. $\frac{3x}{2x^2-9x+10} \cdot (2x-5)$
9. $\frac{x^2-3x}{x^2-5x+6} \cdot \frac{(x-2)^2}{2x}$
10. $\frac{16x}{25x^2-5} \cdot \frac{25x^2+30x+9}{8x}$
11. $\frac{1}{x^2+5x-24} \cdot \frac{x^2+6x-16}{3x}$
12. $\frac{x^2+x-6}{x^2-x-2} \cdot \frac{x^2+5x+4}{x^2+2x-3}$

Math 1

Name _____

Unit 2: Multiplying and Dividing Rational Expressions

$$1. \frac{a^2-b^2}{a-b} = \frac{(a+b)(\cancel{a-b})}{\cancel{a-b}} = \boxed{a+b}$$

$$2. \frac{4x-4}{4x+4} = \frac{4(\cancel{x-1})}{4(\cancel{x+1})} = \boxed{\frac{x-1}{x+1}}$$

$$2. \frac{3a+15}{a^2-25} = \frac{3(\cancel{a+5})}{(\cancel{a-5})(\cancel{a+5})} = \boxed{\frac{3}{a-5}}$$

$$4. \frac{3s^2-27}{s^2+7s+12} = \frac{3(\cancel{s-3})(\cancel{s+3})}{(s+3)(s+4)} = \boxed{\frac{3(s-3)}{(s+4)}}$$

$$5. \frac{5z^2+5z-30}{7z^2+7z-42} = \frac{5(\cancel{z^2+z-6})}{7(\cancel{z^2+z-6})} = \boxed{\frac{5}{7}}$$

$$6. \frac{5n+15}{8n+4} \cdot \frac{4n+2}{3n+9} = \frac{5(\cancel{n+3})}{24(\cancel{2n+1})} \cdot \frac{2(\cancel{2n+1})}{3(\cancel{n+3})} = \boxed{\frac{5}{6}}$$

$$6. \frac{k^2-4}{8k^2+3k} \cdot \frac{16k+6}{k-2} = \frac{(k+2)(\cancel{k-2})}{k(8k+3)} \cdot \frac{2(8k+3)}{\cancel{k-2}}$$

$$= \boxed{\frac{2(k+2)}{k}}$$

$$8. \frac{25-c^2}{12} \cdot \frac{4}{5-c} = \frac{(5-c)(5+c)}{12 \cdot 3} \cdot \frac{4}{\cancel{5-c}} = \boxed{\frac{5+c}{3}}$$

$$9. \frac{2c^2-5c-3}{c+d} \cdot \frac{c^2-d^2}{2c+1} = \frac{(2c+1)(c-3)(\cancel{c+d})}{(\cancel{c+d})(2c+1)} \cdot \frac{(c+d)(\cancel{c-d})}{(c+d)(2c+1)}$$

$$= \boxed{(c-3)(c-d)}$$

$$10. \frac{t^2+6t+9}{t^2-10t+25} \cdot \frac{t^2-t-20}{t^2+7t+12} = \frac{(t+3)(\cancel{t+3})}{(t-5)(\cancel{t-5})} \cdot \frac{(t-5)(\cancel{t+4})}{(t+3)(\cancel{t+4})}$$

$$= \boxed{\frac{t+3}{t-5}}$$

$$10. \frac{x^2-x-6}{x^2+2x-15} \div \frac{x^2-4x-5}{x^2-25}$$

$$\frac{(x-3)(\cancel{x+2})}{(\cancel{x+5})(x-3)} \cdot \frac{(x-5)(\cancel{x+5})}{(\cancel{x-5})(x+1)} = \boxed{\frac{x+2}{x+1}}$$

$$12. \frac{m^2+2m+1}{10m-10} \div \frac{m+1}{20}$$

$$\frac{(m+1)(\cancel{m+1})}{10(m-1)} \cdot \frac{20}{\cancel{m+1}} = \boxed{\frac{2(m+1)}{m-1}}$$

$$13. \frac{a^2+10a+25}{a^2-9} \div \frac{a+5}{a^2-3a}$$

$$\frac{(a+5)(\cancel{a+5})}{(a+3)(\cancel{a-3})} \cdot \frac{a(\cancel{a-3})}{a^2} = \boxed{\frac{a(a+5)}{a+3}}$$

$$14. \frac{x^2-1}{x^2-3x-10} \div \frac{x^2+3x+2}{x^2+4x+4}$$

$$\frac{(x-1)(\cancel{x+1})}{(x-5)(\cancel{x+2})} \cdot \frac{(x+2)(\cancel{x+2})}{(x+2)(x+2)}$$

$$= \boxed{\frac{x-1}{x-5}}$$

$$15. \frac{6a}{a-4} \div \frac{a^2-7a}{a^2-11a+28}$$

$$\frac{6\cancel{a}}{\cancel{a-4}} \cdot \frac{(a-4)(\cancel{a-7})}{\cancel{a}(a-7)} = \boxed{6}$$

⑥

Simplify the following problems as much as possible. Circle your final answer.

1. $\frac{-6}{\frac{1}{2}}$ $-\frac{6}{1} \div -\frac{1}{2}$
 $-6 \cdot \frac{-2}{1} = \boxed{12}$

2. $\frac{3}{\frac{5}{6}}$
 $\frac{3}{1} \cdot \frac{-6}{5} = \boxed{\frac{-18}{5}}$

3. $\frac{-18}{\frac{3}{8}}$
 $-\frac{18}{3} \cdot \frac{8}{1} = \boxed{-48}$

4. $\frac{\frac{1}{3}}{-3}$
 $\frac{1}{3} \div -3 = \boxed{-\frac{1}{9}}$

5. $\frac{\frac{8}{9}}{-2}$
 $\frac{8}{9} \cdot -\frac{1}{2} = \boxed{\frac{4}{9}}$

6. $\frac{\frac{21}{2}}{7}$
 $\frac{21}{2} \cdot \frac{1}{7} = \boxed{\frac{-3}{7}}$

$$7. \frac{\frac{12}{5}}{\frac{-8}{15}}$$

$$\frac{-3 \cdot \frac{12}{18}}{\frac{-8}{15} \cdot \frac{3}{2}} = \boxed{\frac{9}{2}}$$

$$8. \frac{\frac{x^3}{3xy}}{\frac{y^2}{3x}}$$

$$\frac{\frac{x^3}{3xy} \cdot \frac{3x}{y^2}}{\frac{y^2}{3x}} = \boxed{\frac{x^3}{y^3}}$$

$$9. \frac{\frac{x-2}{4}}{x^2-4}$$

$$\frac{\frac{x-2}{4} \cdot \frac{1}{(x-2)(x+2)}}{1} = \boxed{\frac{1}{4(x+2)}}$$

$$10. \frac{\frac{y^2-y-6}{y^2-5y-14}}{\frac{y^2+6y+5}{y^2-6y-7}}$$

$$\frac{(x-3)(y+2)}{(y-7)(y+2)} \cdot \frac{(y-7)(y+1)}{(y+5)(y+1)} = \boxed{\frac{y-3}{y+5}}$$

$$11. \frac{\frac{x}{x-y}}{x^2-x^2-y^2}$$

$$\frac{\frac{x}{x-y} \cdot \frac{(x+y)(x-y)}{x^2}}{x^2-y^2} = \boxed{\frac{x+y}{x}}$$

$$12. \frac{\frac{x^2-x-12}{x^2-2x-15}}{x^2+8x+12}$$

$$\frac{(x-4)(x+3)}{(x-5)(x+3)} \cdot \frac{(x-5)(x+2)}{(x+6)(x+2)} = \boxed{\frac{x-4}{x+6}}$$