Piecewise Functions

Vocabulary

piecewise function - function defined by multiple sub-functions

points of discontinuity - breaks in graph

step function - finite, linear combination of functions

extrema - min or max

average rate of change - slope

Evaluating Piecewise Functions

Example 1:
Evaluate the function when x = 3.

$$g(x)=\begin{cases} x+1, \text{ if } x \le 2 \\ 4x-1, \text{ if } x > 2 \end{cases}$$

$g(3) = 4(3) - 1$

$g(3) = 11$

Example 2:
Evaluate the function when x = –4 and x = 2.

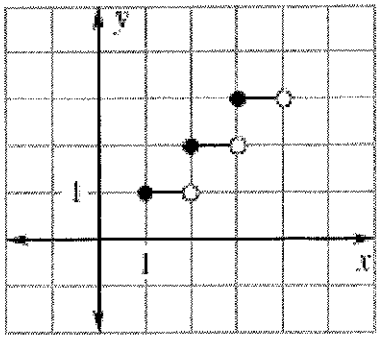
$$f(x)=\begin{cases} 3x-2, \text{ if } x \le 0 \\ \frac{1}{4}x+1, \text{ if } x > 0 \end{cases}$$

$f(-4) = 3(-4) - 2$
$= 14$

$f(2) = \frac{1}{4}(2) + 1 = 1.5$

Step Functions

Example 3:
Write the step functions as a piecewise function.



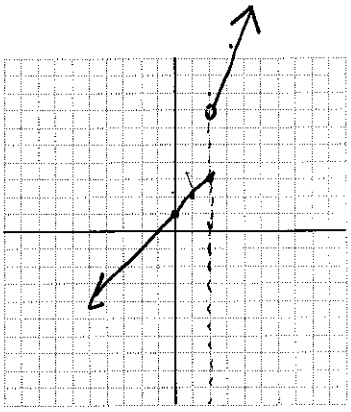
$$f(x) = \begin{cases} 1; & 1 \le x < 2 \\ 2; & 2 \le x < 3 \\ 3; & 3 \le x < 4 \end{cases}$$

Graphing Piecewise Functions

Example 4:

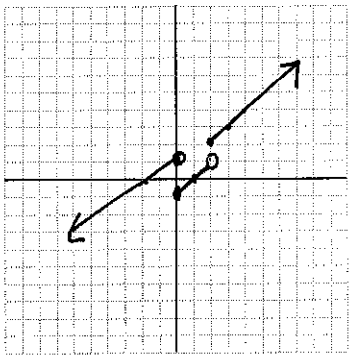
Graph the function.

$$g(x)=\begin{cases} x+1, & \text{if } x \le 2 \\ 4x-1, & \text{if } x > 2 \end{cases}$$



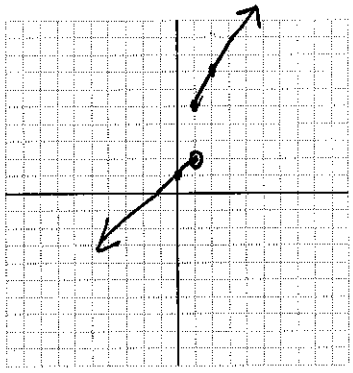
Example 5:
Graph the function.

$$f(x)=\begin{cases} \frac{1}{2}x+1, & \text{if } x < 0 \\ x-1, & \text{if } 0 \le x < 2 \\ x, & \text{if } x \ge 2 \end{cases}$$



Example 6:
Graph the function.

$$h(x)=\begin{cases} (x+1)^2, & \text{if } x < 1 \\ 2x+3, & \text{if } x \ge 1 \end{cases}$$



Absolute Value Functions as Piecewise Functions

Example 7:
Write the function $f(x) = 3|x + 1| - 2$ as a piecewise function.

a

(-1,-2) (-2,1)

$m = \frac{1--2}{-2+1} = \frac{3}{-1}$

$1 = -3(-2) + b$

$b = -5$

$y = -3x - 5$; $x \le -1$

b

(-1,-2) (0,1)

$m = \frac{1--2}{0-1} = \frac{3}{1}$

$m = 3$

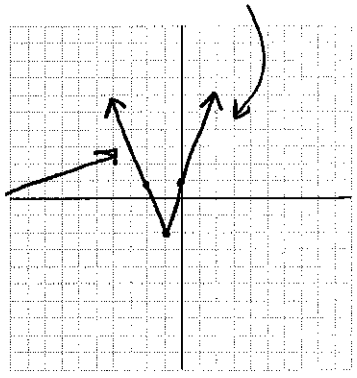
$1 = 3(0) + b$

$b = 1$

$y = 3x + 1$; $x \ge -1$

[a] write equation

[b] write equation



AC Math 2

Notes: Piecewise Functions

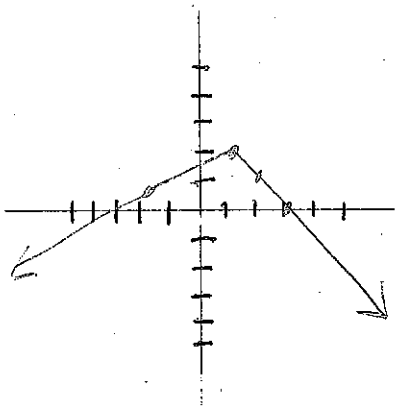
1. $f(x)=\begin{cases} x+2, \text{if } x<2 \\ 2x+1, \text{if } x\geq 2 \end{cases}$ Evaluate $f(x)$ when (a) x = 0, (b) x = 2, (c) x = 4.

a) $x=0$ $0+2=\boxed{2}$ b) $x=2$ $2(2)+1=\boxed{5}$ c) $x=4$ $2(4)+1=\boxed{9}$

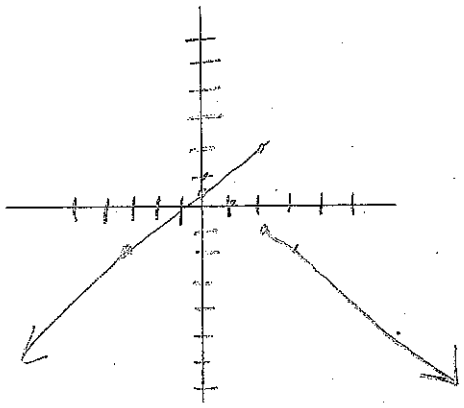
2. $f(x)=\begin{cases} 2x, \text{if } x<-1 \\ 2x+1, \text{if } x\geq -1 \end{cases}$ Evaluate $f(x)$ when (a) x = –3, (b) x = –1, (c) x = 5.

a) $x=-3$ $2(-3)=\boxed{-6}$ b) $x=-1$ $2(-1)+1=\boxed{-1}$ c) $x=5$ $2(5)+1=\boxed{11}$

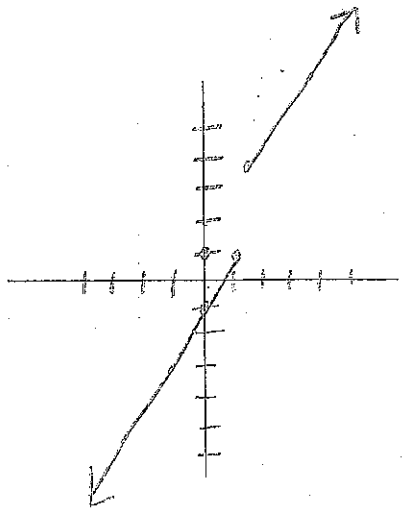
3. Graph: $f(x)=\begin{cases} \frac{1}{2}x+\frac{3}{2}, \text{if } x\leq 1 \\ -x+3, \text{if } x\geq 1 \end{cases}$



4. Graph this function: $f(x)=\begin{cases} \frac{2}{3}x+\frac{2}{3}, \text{if } x<2 \\ -x+1, \text{if } x\geq 2 \end{cases}$



5. $f(x)=\begin{cases} 2x-1, \text{if } x\leq 1 \\ 3x+1, \text{if } x>1 \end{cases}$



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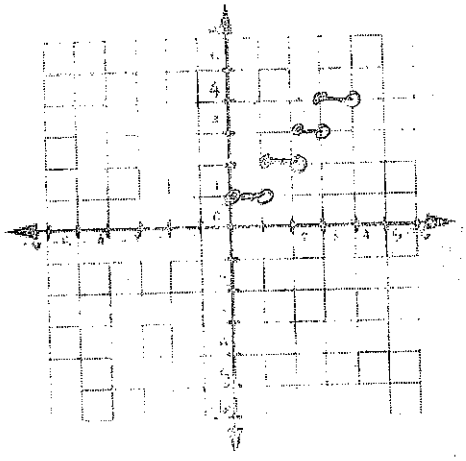
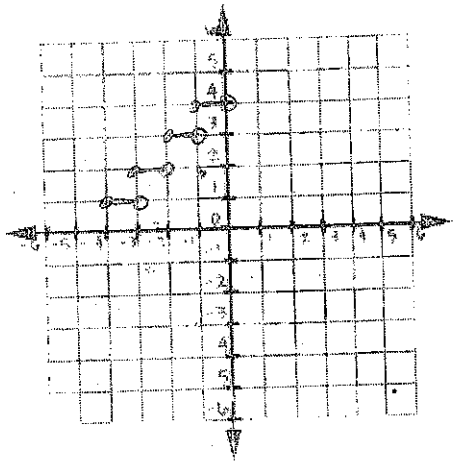
tes: Piecewise Functions

Step Functions:

4. Graph this function: $f(x)=\begin{cases}1, \text{if } 0 \le x<1\\ 2, \text{if } 1 \le x<2\\ 3, \text{if } 2 \le x<3\\ 4, \text{if } 3 \le x<4\end{cases}$



5. Graph this function: $f(x)=\begin{cases}1, \text{if } -4 \le x<-3\\ 2, \text{if } -3 \le x<-2\\ 3, \text{if } -2 \le x<-1\\ 4, \text{if } -1 \le x<0\end{cases}$



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Accelerated Math 2
Piecewise Functions Worksheet 2

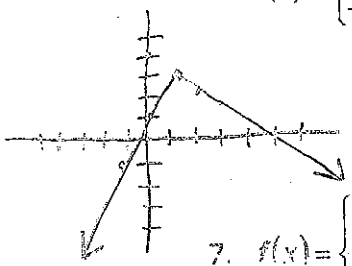
Name _____

Evaluate $f(x)=\begin{cases}3x-1, & x\le 4\\ 2x+7, & x>4\end{cases}$ for the given value of x.

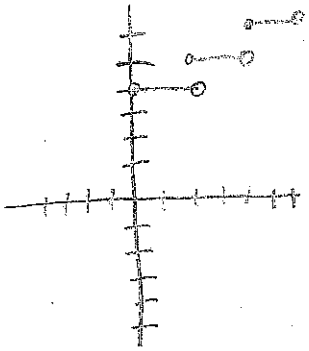
1. x=10 $2(10)+7=27$
2. $x=-\frac{1}{3}$ $3(-\frac{1}{3})-1=-2$
3. x=4 $3(4)-1=11$
4. x=-2 $3(-2)-1=-7$

Graph the function.

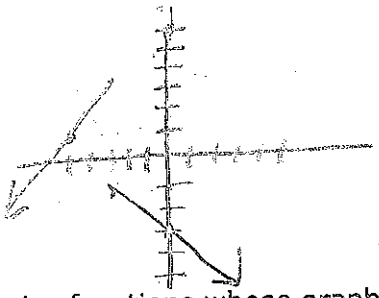
5. $f(x)=\begin{cases}2x+1, & x<1\\ -x+4, & x\ge 1\end{cases}$



6. $f(x)=\begin{cases}4, & 0\le x<2\\ 5, & 2\le x<4\\ 6, & 4\le x<6\end{cases}$

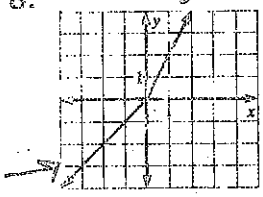


7. $f(x)=\begin{cases}x+6, & x\le -3\\ -\frac{2}{3}x-3, & x>-3\end{cases}$



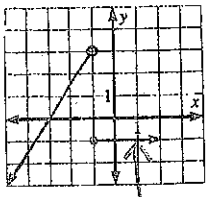
Write equations for the piecewise functions whose graphs are shown.

8. $y=2x;\ x\ge 0$



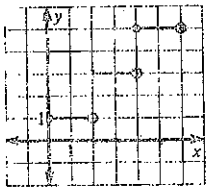
$y=-x;\ x\le 0$

9.



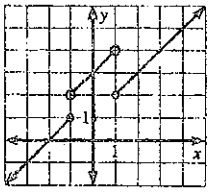
$y=-1;\ -1\le x\le 0$

10.



$\begin{cases}1;\ 0\le x<2\\ 3;\ 2\le x<4\\ 5;\ 4\le x\le 6\end{cases}$

11.



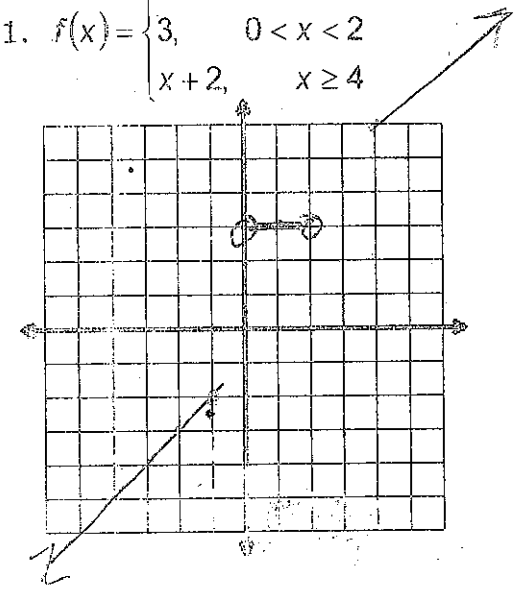
$\begin{cases}-x+2;\ x\le -1\\ x+3;\ -1<x<1\\ x+1;\ x\ge 1\end{cases}$

Accelerated Math 2
Piecewise Functions Worksheet

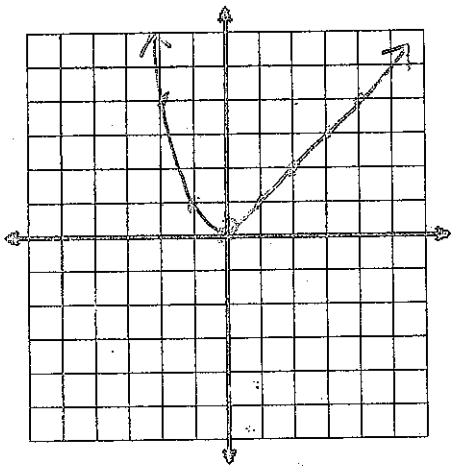
Name _____

For each of the following functions: (a) graph, (b) state the domain, (c) state the range, (d) Is it continuous? If not, what is(are) the point(s) of discontinuity?

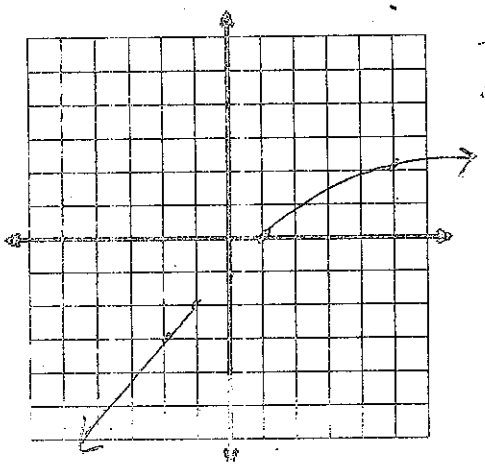
1. $f(x)=\begin{cases} x-1, & x<-1 \\ 3, & 0<x<2 \\ x+2, & x\geq 4 \end{cases}$



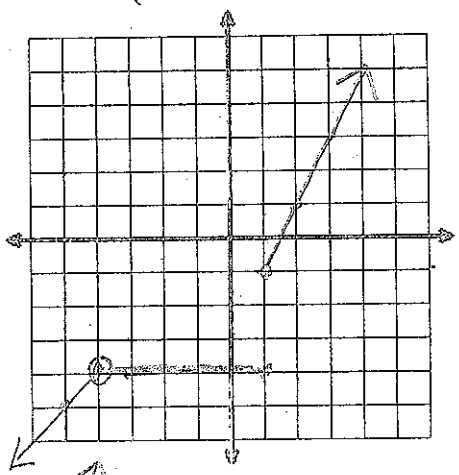
2. $f(x)=\begin{cases} x^2, & x<0 \\ |x|, & x>0 \end{cases}$



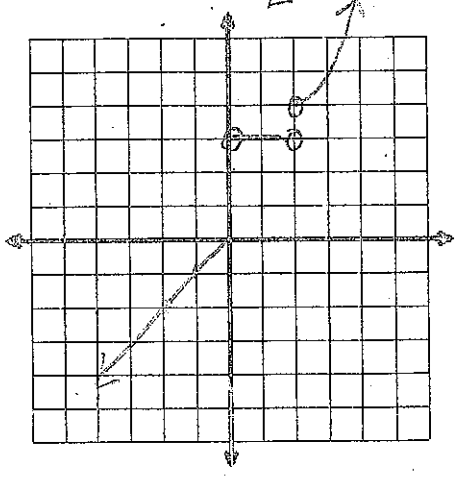
3. $f(x)=\begin{cases} \sqrt{x-1}, & x\geq 1 \\ x-1, & x\leq -1 \end{cases}$



4. $f(x)=\begin{cases} x, & x\leq -4 \\ -4, & -4<x<1 \\ 2x-3, & x\geq 1 \end{cases}$



5. $f(x)=\begin{cases} |x|, & x\leq 0 \\ 3, & 0<x<2 \\ x^2, & x>2 \end{cases}$



Solving Absolute Value Equations

By the end of this lesson you will be able to:
Solve absolute value equations analytically.
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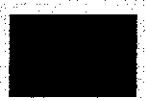
Solving Absolute Value Equations

- In order to solve absolute value equations, we must remember that $|x| = x$ *and* $|-x| = x$

So when we solve absolute value equations we need to split the equation up into a positive and negative expression. For example:



Find the positive solution. $|x+2| = 6$ Find the negative solution

$x+2=6$ $\quad$ $x+2=-6$

$x=4$ $\quad$ $x=-8$

Isolate the abs value!

$2|x+7|-6=12$

$2|x+7|=18$

$|x+7|=9$

Now you can split into two equations!

$x+7=9$ $\quad$ $x+7=-9$

$x=2$ $\quad$ $x=-16$

Review of absolute value

- Absolute value is defined as the distance from zero. (Can distance be negative??)

For example: $|-3|=3=|3|$

$|-y|=y=|y|$

$|-5r|=5r=|5r|$

What would be the answer to the following: $|-14|=?$ $\quad$ $|-8x|=?$

Solving Absolute Value Equations

What about solving this one? $|x-1|+2=5$

Much like solving quadratic equations, we need to isolate the variable part from the constants.

$|x-1|+2=5$

$-2 \quad -2$

$|x-1|=3$

$x-1=3$ $\quad$ $x-1=-3$

$x=4$ $\quad$ $x=-2$

Solving Absolute Value Equations with Variables on Both Sides

$|4x-8|=x+2$

$4x-8=x+2$ $\quad$ $4x-8=-(x+2)$

$3x=10$ $\quad$ $4x-8=-x-2$

$\boxed{x=\frac{10}{3}}$ $\quad$ $5x=6$

$\boxed{x=\frac{6}{5}}$

7

Less than/
Greater than

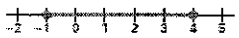
$<, >$

Represented with an open circle on a number line. Does not include the value

Less than or equal to/
Greater than or equal to

$\leq, \geq$

Represented with a closed circle on a number line. Includes the value.



Solving Absolute Value Inequalities

"GREAT OR THAN"

$|-2x+11|>5$

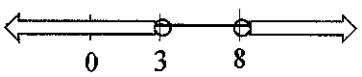
KEEP first equation the same

CHANGE to a negative FLIP the inequality

$-2x+11>5$ OR $-2x+11<-5$

$-2x>-6$ OR $-2x<-16$

$x<3$ OR $x>8$



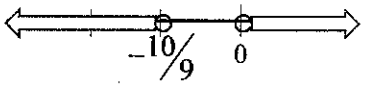
$3|9x+5|+1>16$

$|9x+5|>5$ **Isolate the absolute value first!**

$9x+5>5$ **Great OR than: 2 equations!** $9x+5<-5$

$9x>0$ **OR** $9x<-10$

$x>0$ **OR** $x<-10/9$



Solving Absolute Value Inequalities

"LESS THAN"

$|3x-4|\leq 8$

1. Rewrite with a (-) sign on the left. Use another "less than" sign.

$-8\leq 3x-4\leq 8$

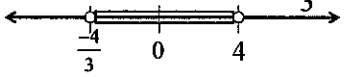
2. Solve for x on all three sides at once!

$+4 \quad +4 \quad +4$

$-4\leq 3x\leq 12$

3. Graph your answer on a number line.

$\frac{-4}{3}\leq x\leq 4$



X is "in between" the two numbers

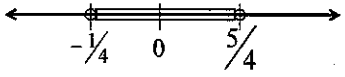
$|4x-2|+7<10$

$|4x-2|<3$ **Isolate the absolute value first!**

$-3<4x-2<3$ **Put -3 on the other side, draw a less than sign**

$-1<4x<5$ **Solve**

$-1/4<x<5/4$



X is "in between" the two numbers

When will an absolute value be BIGGER than a negative number?

$|-2x+3|>-4$ Every time! $\mathbb{R}$

When will an abs value be SMALLER than a negative number?

$|-2x+3|<-4$ Never $\varnothing$ no solution

When will an abs value EQUAL a negative number?

$|-2x+3|=-4$ Never! $\varnothing$ no solution

Decide whether the number is a solution to the equation.

$|2x-5|=9;-2$

So we take -2 and plug it in for x.

$|2(-2)-5|=9$

$|-4-5|=9$

$|-9|=9$

$9=9$ ✓

Practice Problems:

$|2x-1|=7$

$-3,4$

$|5x-3|=8$

$-1,\frac{11}{5}$

$|5-x|=5$

$0,10$

$3|(x-2)|-2=10$

$6,-2$

Decide whether the number is a solution to the equation.

$|5x+1|-11=0;4$

No; the two solutions are x=2 and x=-12/5

Classwork/Homework

- P. 30 (1-15) and p. 31 (1-9)

Advanced Algebra
Absolute Value Equations

Name _____

Rules:
1. Isolate bars
2. Check for special cases
3. Solve by setting up 2 cases
4. Check: Some answers may be extraneous.

1. $|2x + 3| = 4$

$2x+3=4$ $\quad$ $2x+3=-4$

$2x=+1$ $\quad$ $2x=-7$

~~$x=1/2$~~

$x=\frac{1}{2}$ $\quad$ $x=\frac{-7}{2}$

2. $|x - 3| = 3x + 5$

$x-3=3x+5$

$-2x=8$

$x=-4$

$x-3=-3x-5$

$4x=-2$

$x=-\frac{1}{2}$

3. $|4 - 3x| - 9 = 3$

$|4-3x|=12$

$4-3x=12$ $\quad$ $4-3x=-12$

$-3x=8$ $\quad$ $-3x=-16$

$x=-\frac{8}{3}$ $\quad$ $x=\frac{16}{3}$

4. $|10x + 2| - 18 = -12$

$|10x+2|=6$

$10x+2=6$ $\quad$ $10x+2=-6$

$\frac{10x}{10}=\frac{4}{10}$ $\quad$ $\frac{10x}{10}=\frac{-8}{10}$

$x=\frac{2}{5}$ $\quad$ $x=-\frac{4}{5}$

5. $|2x + 8| + 2 = 1$

$\emptyset$

6. $|x + 4| - 1 = 6x$

$|x+4|=6x+1$

$x+4=6x+1$ $\quad$ $x+4=-6x-1$

$-5x=-3$ $\quad$ $7x=-5$

$x=\frac{3}{5}$ $\quad$ $x=-\frac{5}{7}$

7. $\frac{1}{3}|3x + 6| - 2 = 2$

$|3x+6|=12$

$3x+6=12$ $\quad$ $3x+6=-12$

$3x=6$ $\quad$ $3x=-18$

$x=2$ $\quad$ $x=-6$

8. $|4 + x| = 1 - 2x$

$4+x=1-2x$ $\quad$ $4+x=-1+2x$

$3x=-3$ $\quad$ $5=x$

$x=-1$ $\quad$ $x=5$

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Absolute Value Equations

Date_____ Period____

Solve each equation.

1) $|6m| = 42$

$\{7, -7\}$

2) $|-6x| = 30$

$\{-5, 5\}$

3) $|k - 10| = 3$

$\{13, 7\}$

4) $\left|\frac{x}{7}\right| = 3$

$\{21, -21\}$

5) $|7 + p| = 7$

$\{0, -14\}$

6) $|-3p| = 15$

$\{-5, 5\}$

7) $7|n| = 56$

$\{8, -8\}$

8) $\frac{|m|}{5} = 3$

$\{15, -15\}$

9) $-3|p| = -12$

$\{4, -4\}$

10) $|m| + 2 = 11$

$\{9, -9\}$

11) $|n| + 1 = 2$

$\{1, -1\}$

12) $\frac{|x|}{7} = 5$

$\{35, -35\}$

13) $\frac{|a - 5|}{8} = 5$

$\{45, -35\}$

14) $4|n + 8| = 56$

$\{6, -22\}$

15) $|7m| + 3 = 73$

$\{10, -10\}$

16) $\left|\frac{x}{7}\right| - 8 = -7$

$\{7, -7\}$

17) $\frac{|-9 + v|}{8} = 3$

$\{33, -15\}$

18) $-10|v + 2| = -70$

$\{5, -9\}$

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