# The Rational Root Theorem and The Fundamental Theorem of Algebra

**The Rational Root Theorem**

If $f(x) = a_n x^n + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of $f(x)$ has the form:

$$\frac{p}{q} = \pm \frac{\text{factors of } a_0}{\text{factors of } a_n}$$

**Example 1:**

List the possible rational zeros of $f(x) = x^3 - 4x^2 - 11x + 30$. Find the zeros.

degree 3

$\pm 1, 2, 3, 5, 6, 10, 15, 30$

$$\cancel{\begin{array}{r|rrrr} 1 & 1 & -4 & -11 & 30 \\ & & 1 & -3 & -14 \\ \hline & 1 & -3 & -14 & 16 \end{array}}$$

$$x = \begin{array}{r|rrrr} 5 & 1 & -4 & -11 & 30 \\ & & 5 & 5 & -30 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x^2 + x - 6 = 0$

$(x-5)(x+3)(x-2) = 0$

3 zeros: $\boxed{x=5 \quad x=-3 \quad x=2}$

**Example 2:**

List the possible rational zeros of $f(x) = 15x^4 - 68x^3 - 7x^2 + 24x - 4$. Find the zeros.

degree 4

$\pm \dfrac{1, 2, 4}{1, 3, 5, 15}$

$$\begin{array}{r|rrrrr} \frac{1}{5} & 15 & -68 & -7 & 24 & -4 \\ & & 3 & -13 & -4 & 4 \\ \hline & 15 & -65 & -20 & 20 & 0 \end{array}$$

$15x^3 - 65x^2 - 20x + 20 = 0$

$3x^3 - 13x^2 - 4x + 4 = 0$

$$\begin{array}{r|rrrr} -\frac{2}{3} & 3 & -13 & -4 & +4 \\ & & -2 & 10 & -4 \\ \hline & 3 & -15 & 6 & 0 \end{array}$$

$3x^2 - 15x + 6 = 0$

$x^2 - 5x + 2 = 0$

$x = \dfrac{5 \pm \sqrt{17}}{2}$

4 zeros:

$x = 1/5$

$x = -2/3$

$x = \dfrac{5 \pm \sqrt{17}}{2}$