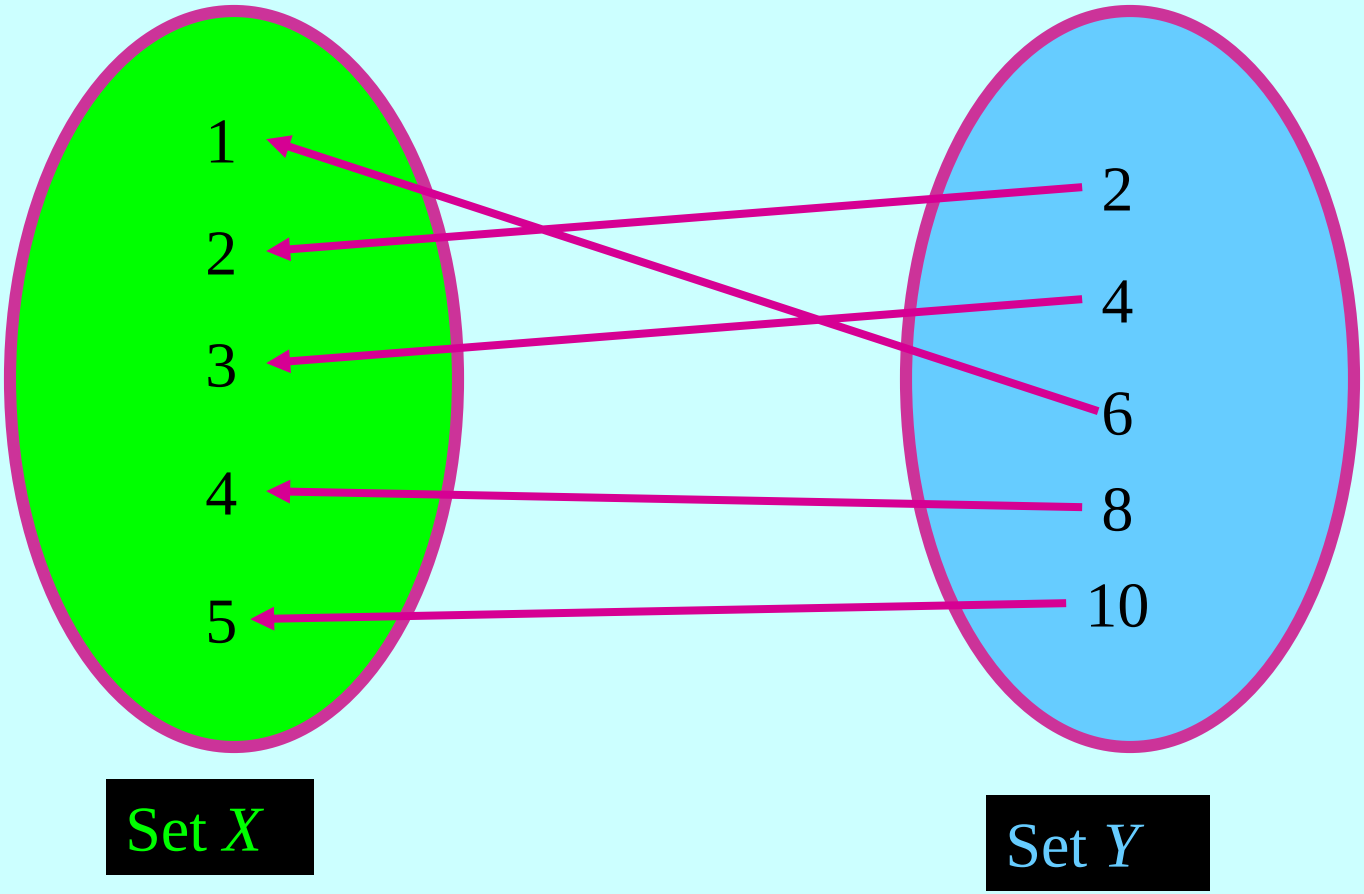
INVERSE

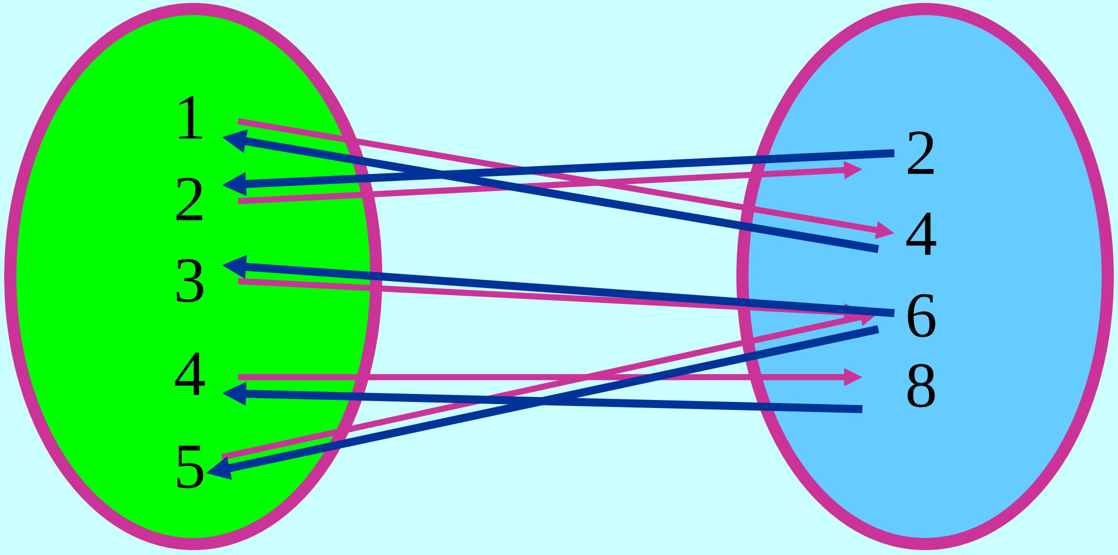
FUNCTIONS

Remember we talked about functions---taking a set *X* and mapping into a Set *Y*

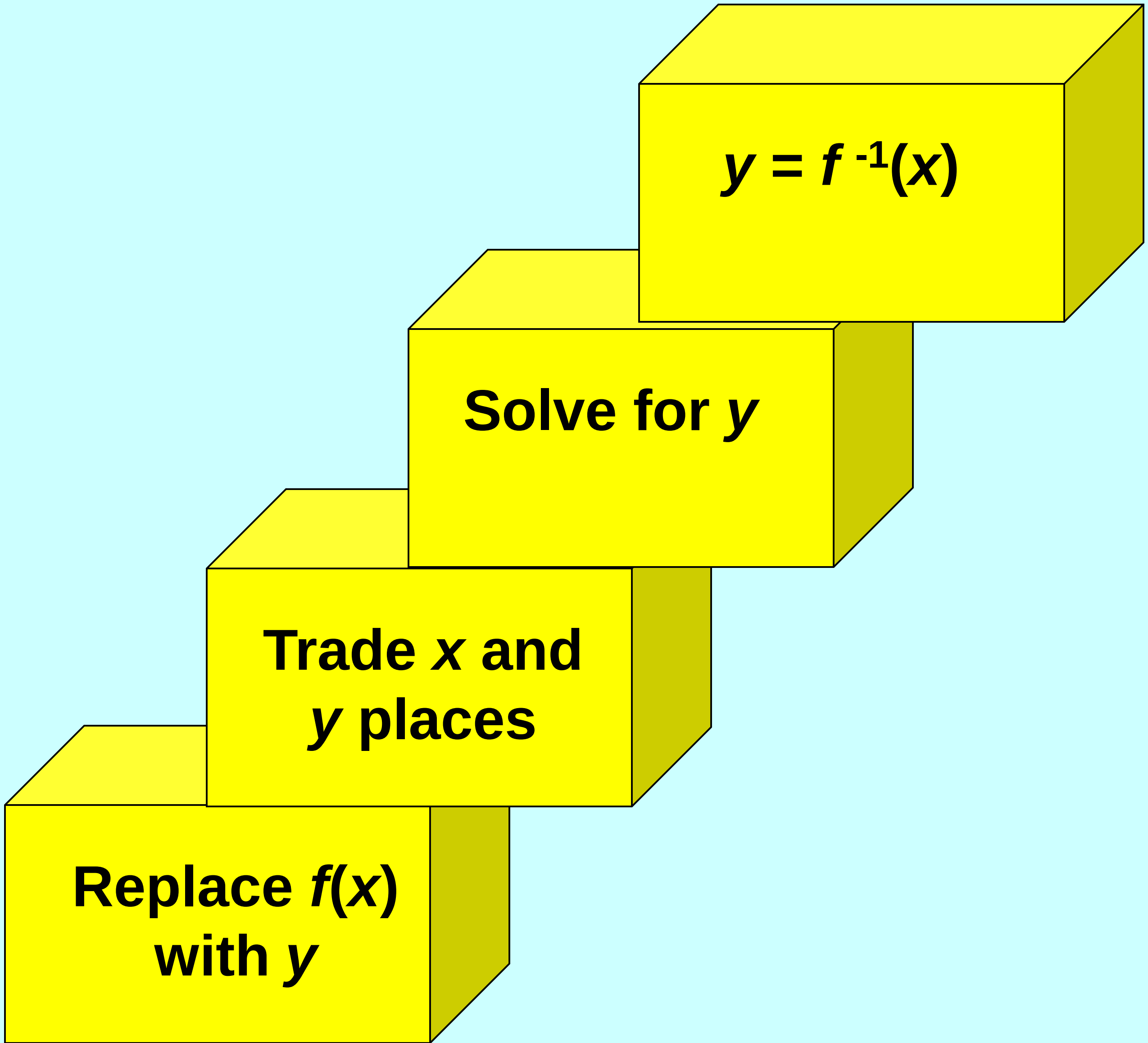


An inverse function would reverse that process and map from Set *Y* back into Set *X*

If we map what we get out of the function back, we won't always have a function going back!!!



Steps for Finding the Inverse of a One-to-One Function



Find the inverse of $f\left(x\right)=\dfrac{4}{2-x}$

Replace $f(x)$ with y

$$y=\frac{4}{2-x}$$

Trade x and y places

$$x=\frac{4}{2-y}$$

Solve for y

$$x\left(2-y\right)=4$$

$$2x-xy=4$$

$$-xy=4-2x$$

$$y=\frac{4-2x}{-x}$$

$y = f^{-1}(x)$

$$f^{-1}\left(x\right)=-\frac{4-2x}{x}$$

or

$$f^{-1}\left(x\right)=\frac{2x-4}{x}$$

Let's check this by doing

$$f\circ f^{-1}$$

$$f\circ f^{-1}=\frac{4}{2-\left(\dfrac{2x-4}{x}\right)}$$

$$=\frac{4}{\dfrac{2x-2x+4}{x}}$$

$$=x$$

Yes!

Ensure $f(x)$ is one to one first. Domain may need to be restricted.

Find the inverse of a function :

Example 1: y = 6x - 12

Step 1: Switch x and y: **x = 6y - 12**

Step 2: Solve for y:

$$x = 6y - 12$$

$$x + 12 = 6y$$

$$\frac{x+12}{6} = y$$

$$\frac{1}{6}x + 2 = y$$

Example 2:

Given the function : $y = 3x^2 + 2$ find the inverse:

Step 1: Switch x and y: $x = 3y^2 + 2$

Step 2: Solve for y:

$$x = 3y^2 + 2$$

$$x - 2 = 3y^2$$

$$\frac{x-2}{3} = y^2$$

$$\sqrt{\frac{x-2}{3}} = y$$

Ex: Find an inverse of y = -3x+6.

- Steps: -switch x & y
  -solve for y

y = -3x+6

x = -3y+6

x-6 = -3y

$$\frac{x-6}{-3} = y$$

$$y = \frac{-1}{3}x + 2$$

Finding the Inverse

Given $f(x) = -2x - 7$

then $y = -2x - 7$

solve for x $\quad x = \dfrac{-y-7}{2}$

$$f^{-1}(y) = \frac{-y-7}{2}$$

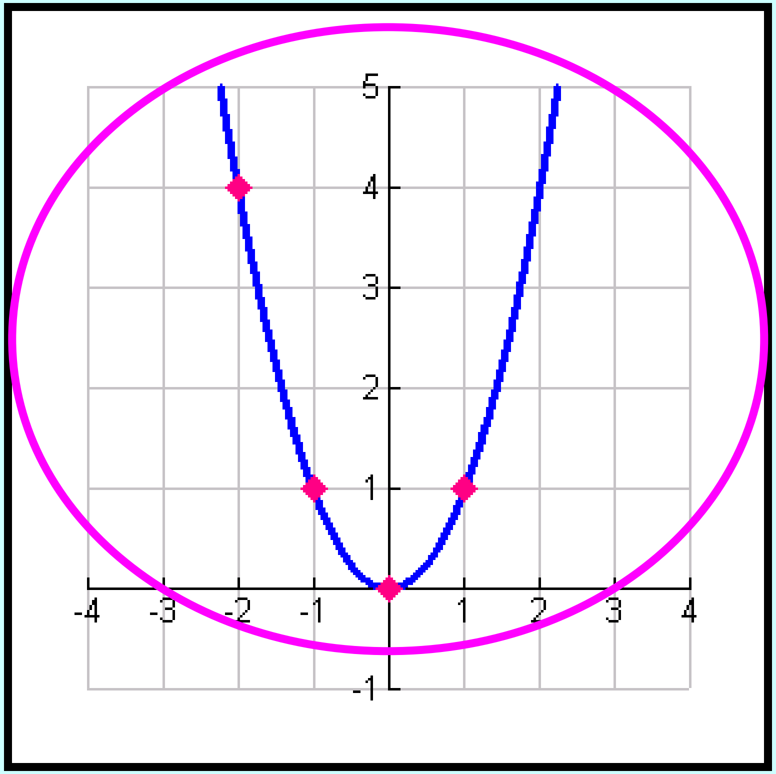
Try $\quad y = \dfrac{x+2}{x-2}$

Review from chapter 2

- Relation – a mapping of input values (x-values) onto output values (y-values).
- Here are 3 ways to show the same relation.

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1



Equation

Table of values

Graph

- <u>Inverse relation</u> – just think: switch the x & y-values.

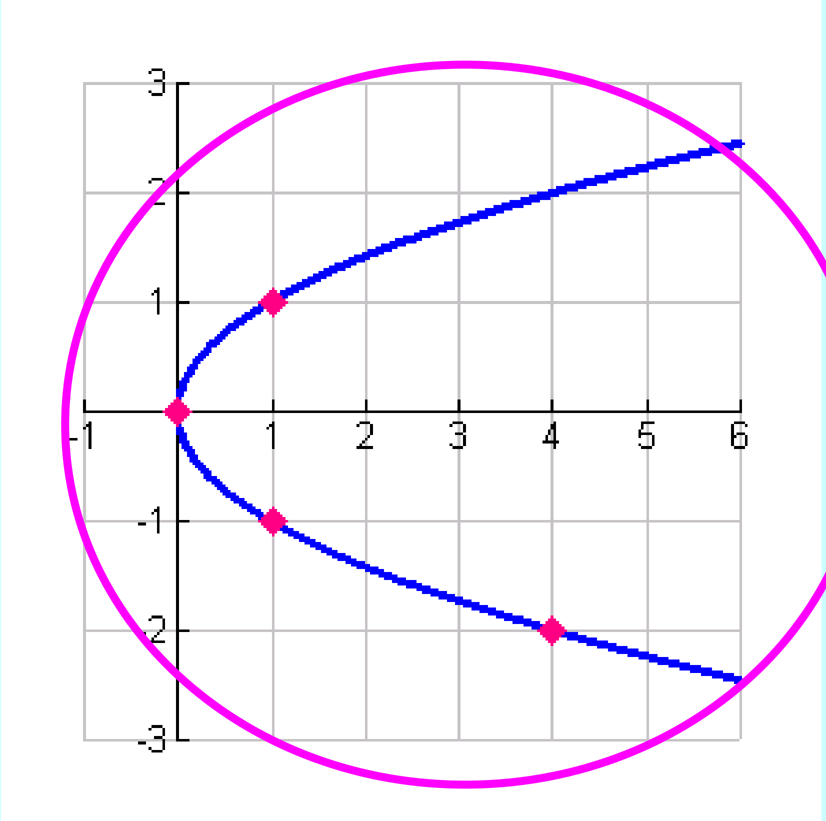
$x = y^2$

$y = \sqrt{x}$

**** the inverse of an equation: switch the x & y and solve for y.**

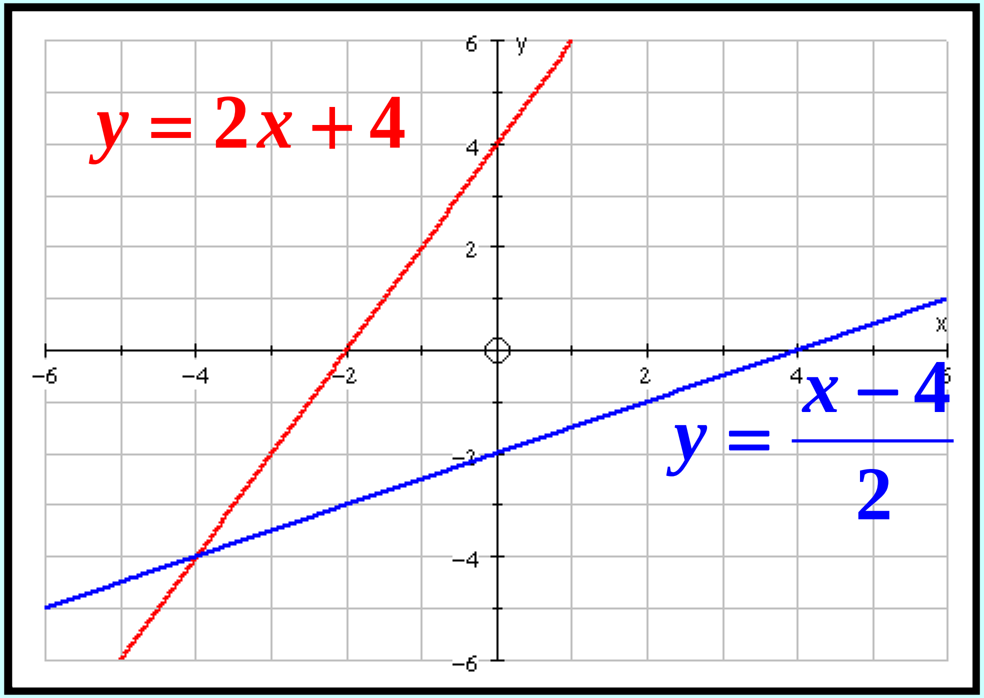
x	y
4	-2
1	-1
0	0
1	1

**** the inverse of a table: switch the x & y.**



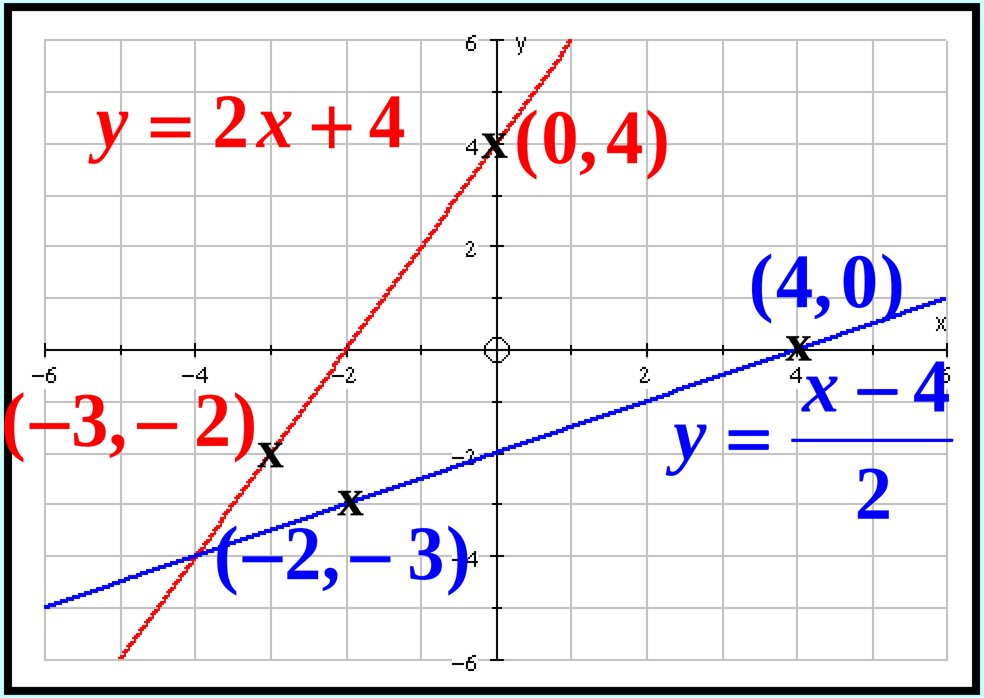
**** the inverse of a graph: the reflection of the original graph in the line y = x.**

Consider the graph of the function $f(x) = 2x + 4$



The inverse function is $f^{-1}(x) = \frac{x-4}{2}$

Consider the graph of the function $f(x) = 2x + 4$

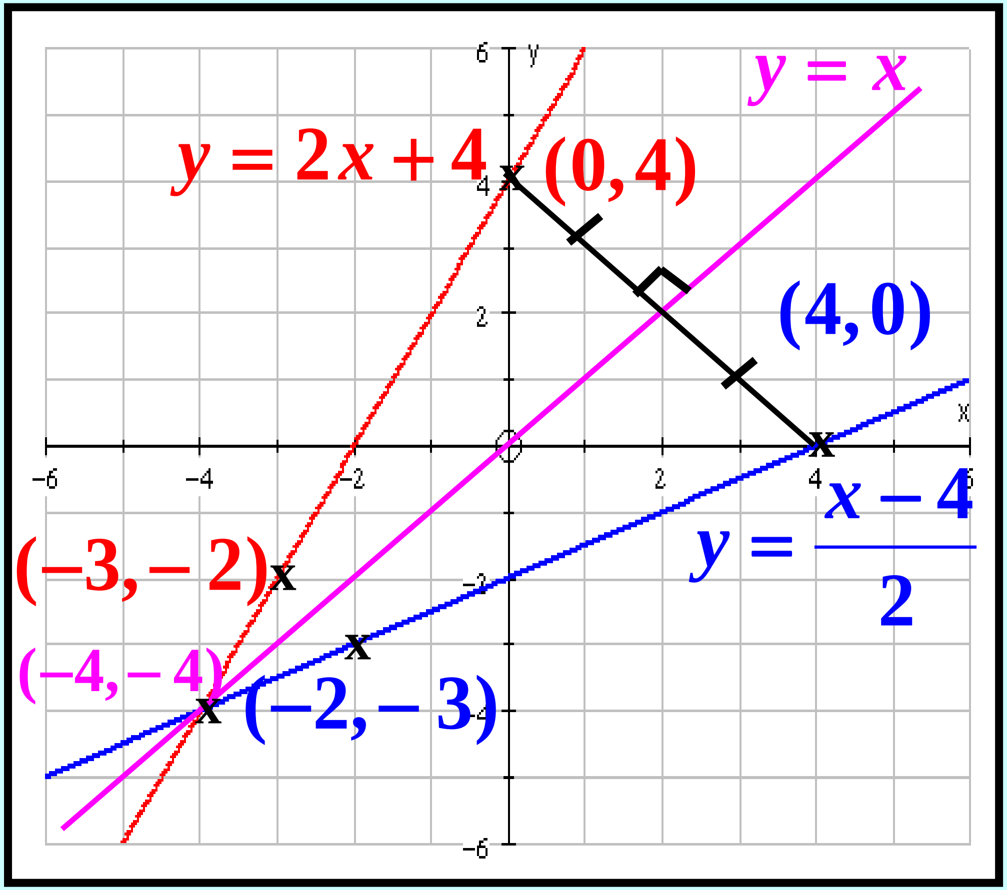


The inverse function is $f^{-1}(x) = \frac{x-4}{2}$

An inverse function is just a rearrangement with *x* and *y* swapped.

So the graphs just swap *x* and *y*!

What else do you notice about the graphs?



$f^{-1}(x)$ is a reflection of $f(x)$ in the line $y = x$

The function and its inverse must meet on $y = x$

Graph f(x) and $f^{-1}(x)$ on the same graph.

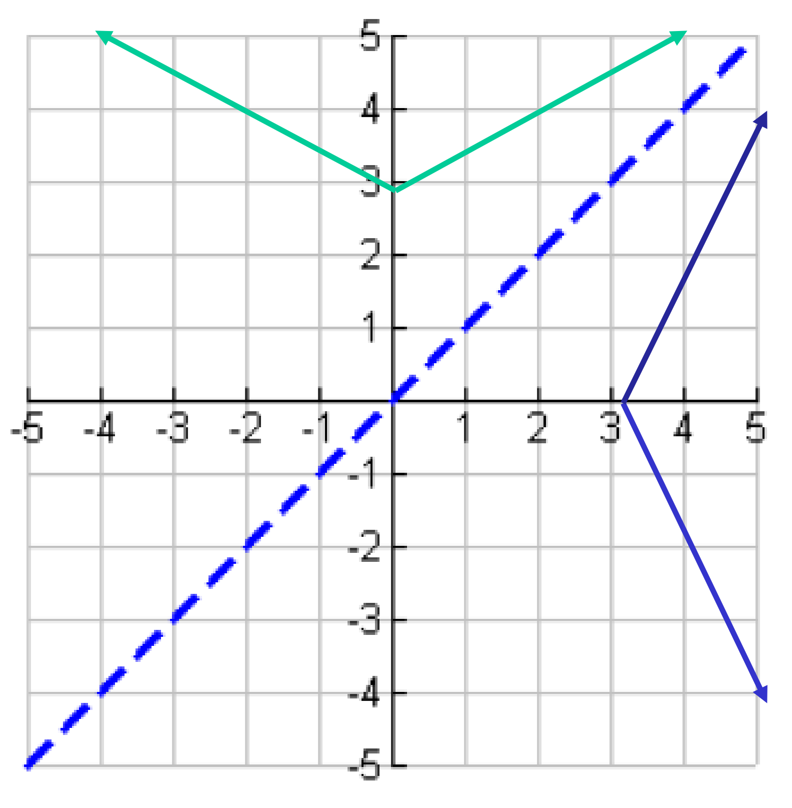
1.) $y = \frac{1}{2}|x| + 3$

x	y
-4	5
0	3
4	5

f (x)

x	y
5	-4
3	0
5	4

f^{-1} (x)



Graph f(x) and $f^{-1}(x)$ on the same graph.

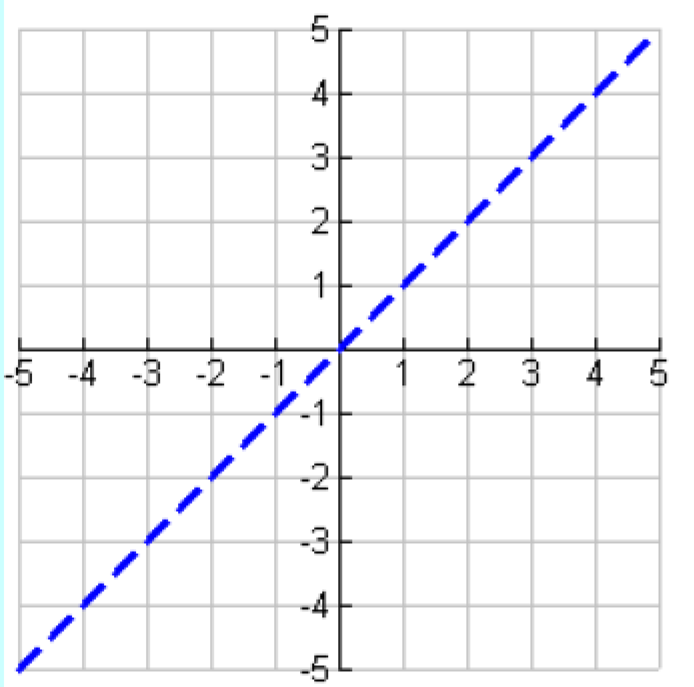
2.) $y = (x + 3)^2 - 5$

x	y
-4	-4
-3	-5
0	4

f (x)

x	y
-4	-4
-5	-3
4	0

f^{-1} (x)



Notice that the *x* and *y* values traded places for the function and its inverse.

$f(x) =$

These functions are reflections of each other about the line *y* = *x*

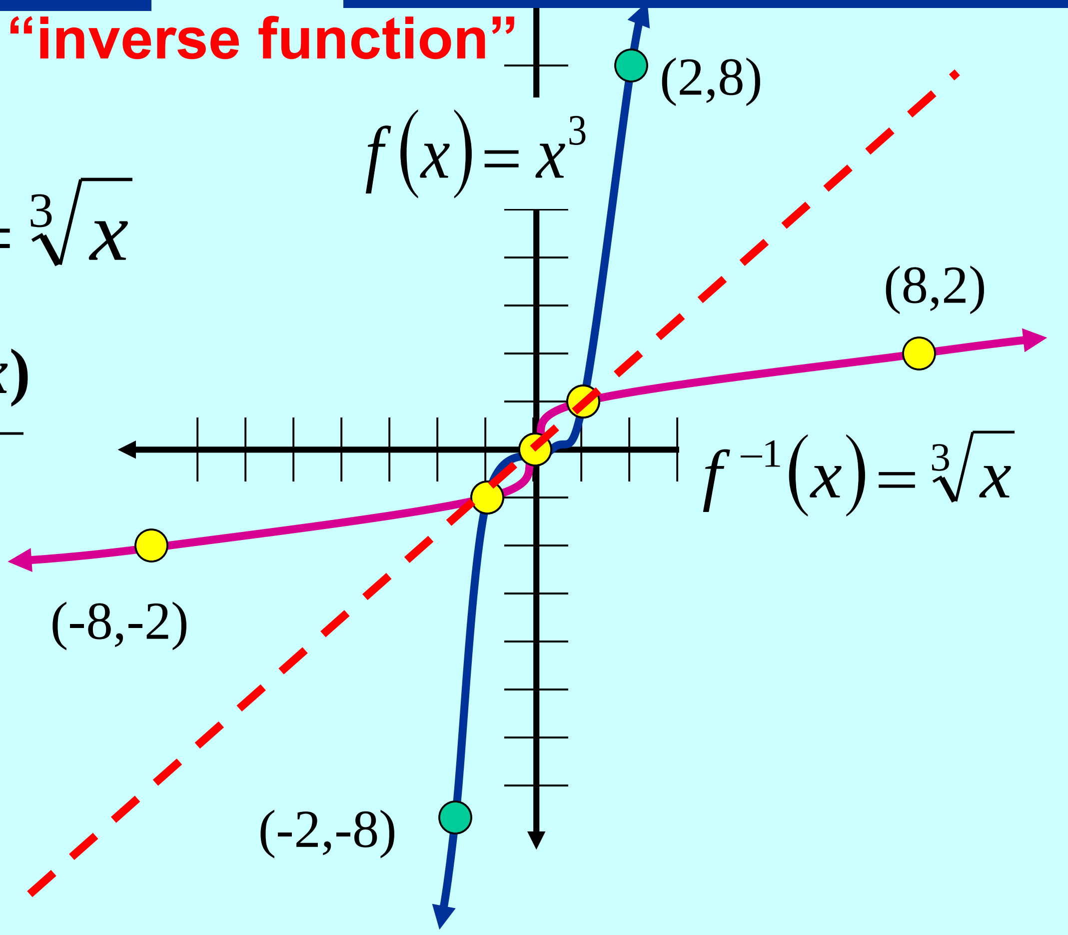
This means "inverse function"

x	*f* (*x*)
-2	-8
-1	-1
0	0
1	1
2	8

$$f^{-1}(x) = \sqrt[3]{x}$$

x	$f^{-1}(x)$
-8	-2
-1	-1
0	0
1	1
8	2

Let's take the values we got out of the function and put them into the inverse function and plot them



Is this a function?

Yes

What will "undo" a cube?

A cube root

Graph f(x) = 3x − 2 and $f^{-1} = \dfrac{x+2}{3}$
using the same set of axes.

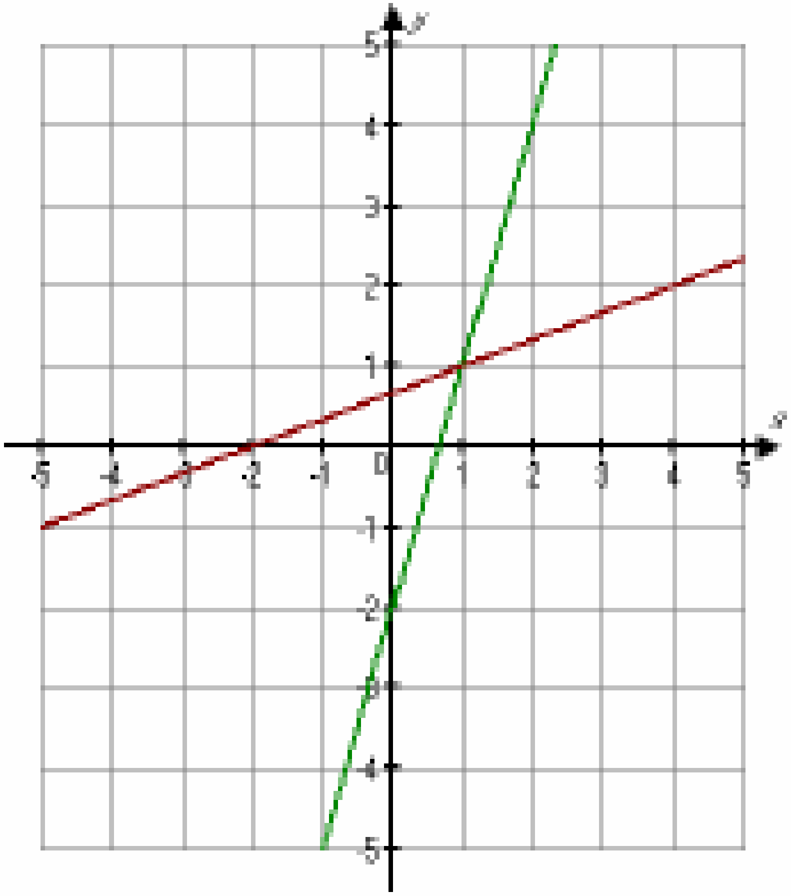
Then compare the two graphs.

Determine the domain and range of the function
and its inverse.

Solution

x	$f(x) = 3x - 2$
–1	–5
0	–2
2	4
3	7

x	$f^{-1}(x) = \frac{x+2}{3}$
–5	–1
–2	0
1	1
4	2



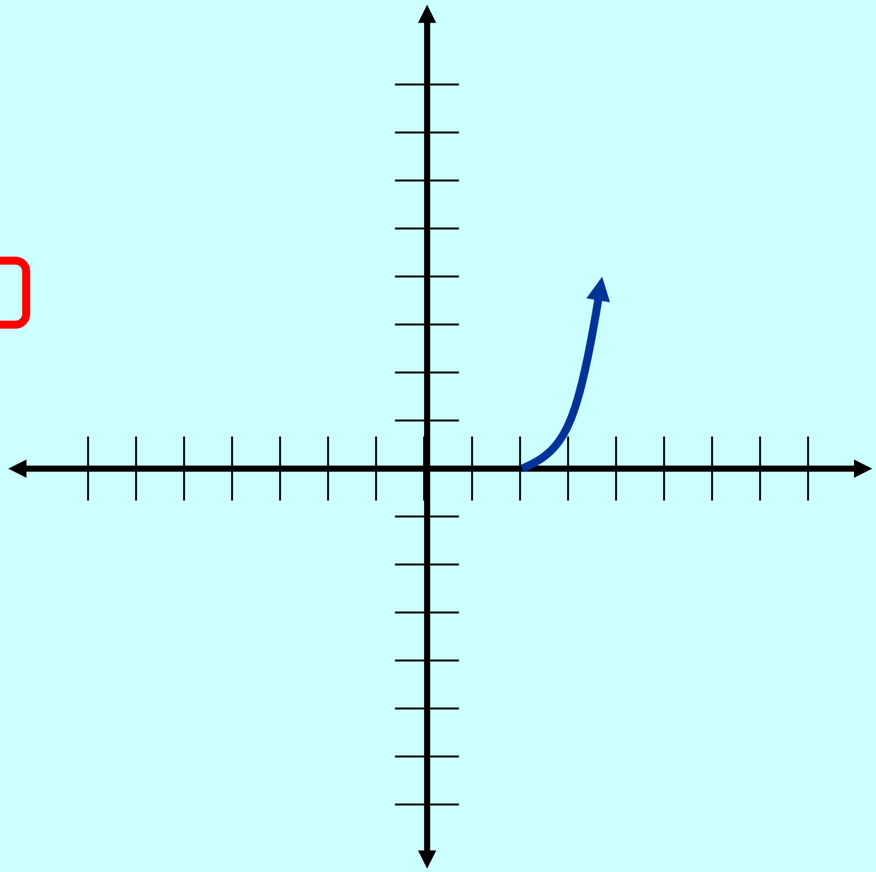
Verify that the functions *f* and *g* are inverses of each other.

$$f(x)=(x-2)^2,\ x\ge 2; \qquad g(x)=\sqrt{x}+2$$

If we graph $(x - 2)^2$ it is a parabola shifted right 2.

Is this a one-to-one function?

This would not be one-to-one but they restricted the domain and are only taking the function where *x* is greater than or equal to 2 so we will have a one-to-one function.

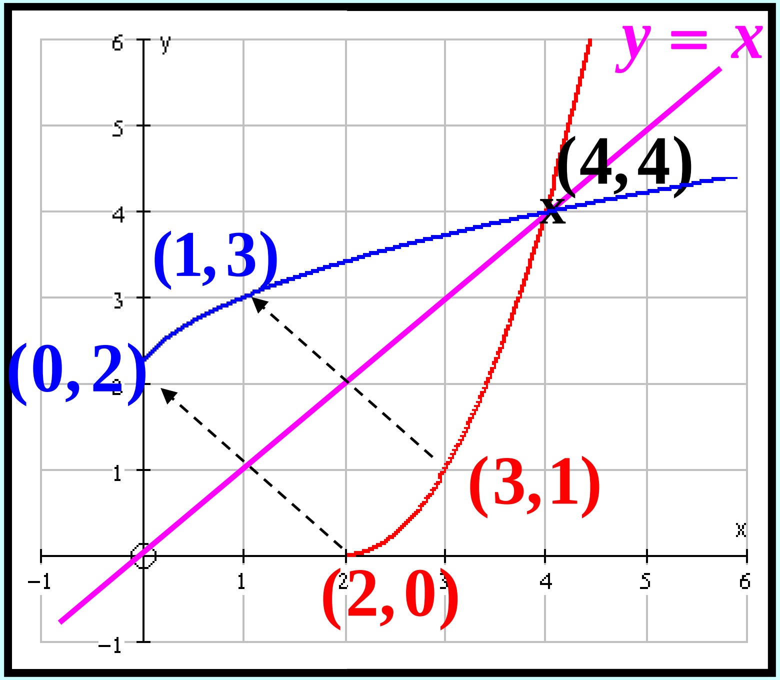


e.g. On the same axes, sketch the graph of

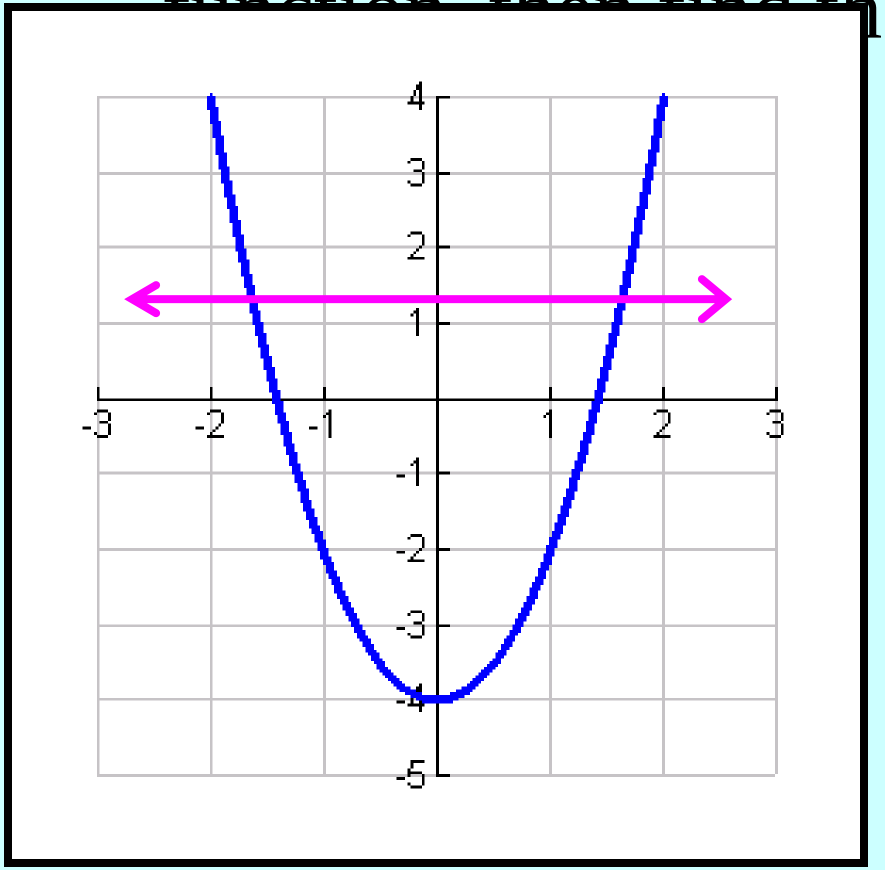
$$y = (x - 2)^2, \qquad x \geq 2$$

and its inverse.

Solution:



Ex: $f(x)=2x^2-4$ Determine whether $f^{-1}(x)$ is a function, then find the inverse equation.



$y = 2x^2-4$

$x = 2y^2-4$

$x+4 = 2y^2$

$$\frac{x+4}{2} = y^2$$

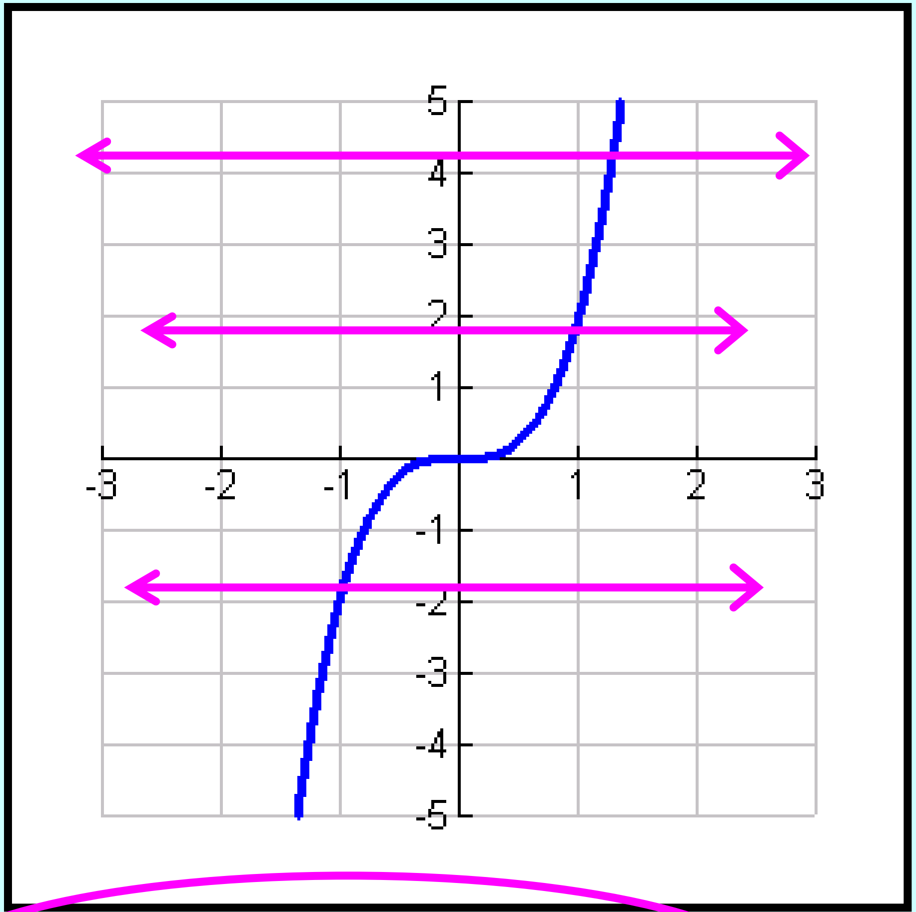
$$y = \pm\sqrt{\frac{x+4}{2}}$$

$f^{-1}(x)$ is not a function.

OR, if you fix the tent in the basement…

$$y = \pm\sqrt{\frac{1}{2}x+2}$$

Ex: $g(x)=2x^3$



$y=2x^3$

$x=2y^3$

$$\frac{x}{2}=y^3$$

$$\sqrt[3]{\frac{x}{2}}=y$$

$$y=\sqrt[3]{\frac{x}{2}}$$

Inverse is a function!

OR, if you fix the tent in the basement…

$$y=\frac{\sqrt[3]{4x}}{2}$$

Exercise

1 (a) Sketch the function $y = f(x)$ where $f(x) = x^2 - 1$.

(b) Write down the range of $f(x)$.

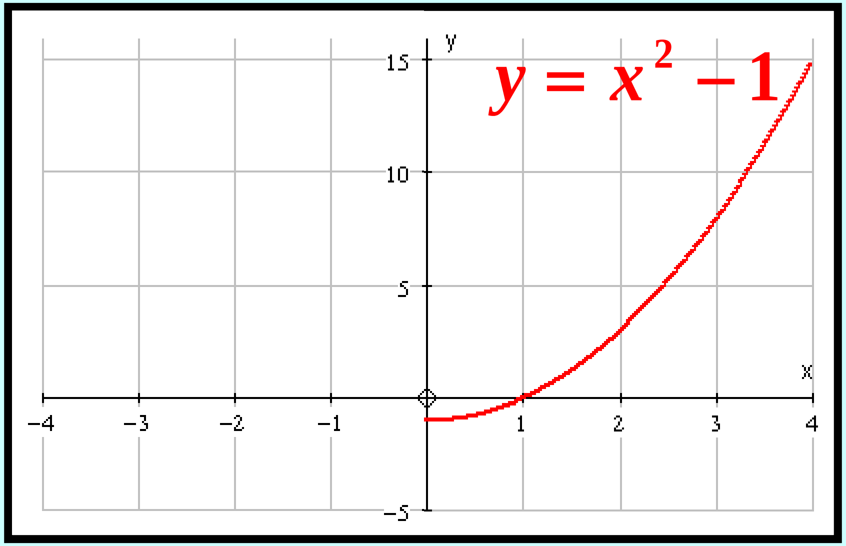
(c) Suggest a suitable domain for $f(x)$ so that the inverse function $f^{-1}(x)$ can be found.

(d) Find $f^{-1}(x)$ and write down its domain and range.

(e) On the same axes sketch $y = f^{-1}(x)$.

Solution:

(a)



(b) Range of $f(x)$:

$f(x) \geq -1$

(c) Restricted domain:

$x \geq 0$

(We'll look at the other possibility in a minute.)

(d) Inverse: **Let** $y = x^2 - 1$

Rearrange: $y + 1 = x^2$

$\sqrt{y+1} = x$

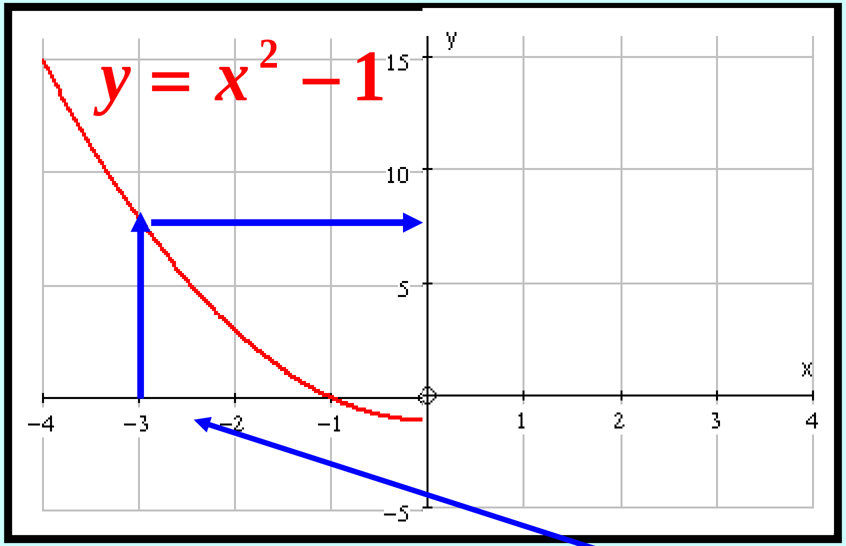
Swap: $\sqrt{x+1} = y$

$\Rightarrow \quad f^{-1}(x) = \sqrt{x+1}$

Domain: $x \geq -1$ **Range:** $y \geq 0$

Solution:

(a)



(b) Range of $f(x)$:

$$f(x) \geq -1$$

(c) Suppose you chose $x \leq 0$ for the domain

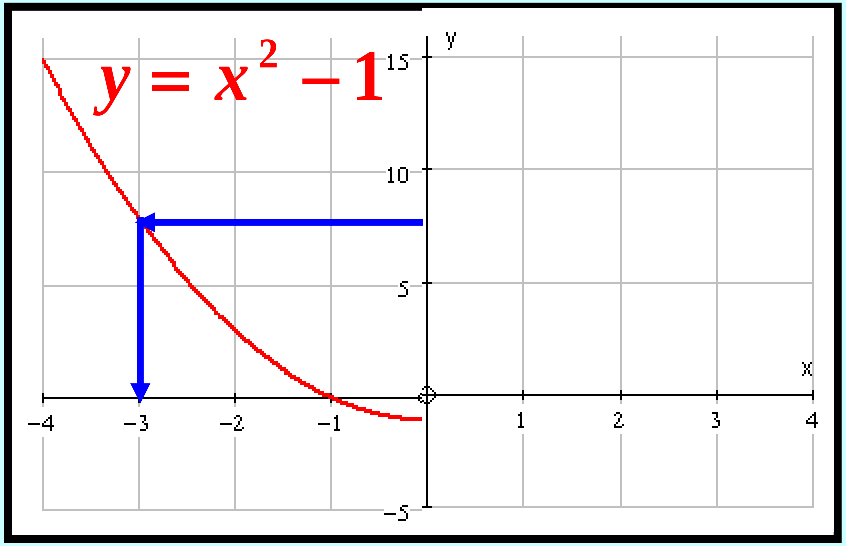
As before Let $y = x^2 - 1$

Rearrange: $y + 1 = x^2$

We now $-\sqrt{y+1} = x$ since $x \leq 0$

Solution:

(a)



(b) Range: $y \geq -1$

(c) Suppose you chose $x \leq 0$ for the domain

Choosing $x \geq 0$ is easier!

As before Let $y = x^2 - 1$

Rearrange: $y + 1 = x^2$

We now $-\sqrt{y+1} = x$ since $x \leq 0$

Swap: $-\sqrt{x+1} = y$

$$f^{-1}(x) = -\sqrt{x+1}$$

Domain: $x \geq -1$ Range: $y \leq 0$