Geometric Sequences

Examples of Geometric Sequences:

a) 12, 6, 3, 1.5.....
Geometric $r = 1/2$

b) 1, 3, 9, 27.....
$r = 3$

So what is a Geometric Sequence anyways?
A geometric sequence is one that has a common ratio to get from one term to the next.

Different type of equations:

Recursive:	Explicit:
$a_1 =$ $a_n = a_{n-1}(r)$ *finds next few terms	$a_1 =$ $a_n = a_1(r)^{n-1}$ *finds any term

Examples:

a) 1, 2, 4, 8.... Recursive: $a_1 = 1$ $a_n = a_{n-1}(2)$ Explicit: $a_1 = 1 \quad a_n = 1(2)^{n-1}$	b) 27, 9, 3, 1..... Recursive: $a_1 = 27$ $a_n = a_{n-1}(1/3)$ Explicit: $a_1 = 27 \quad a_n = 27(1/3)^{n-1}$
c) 40, 10, 10/4..... Recursive: $a_1 = 40$ $a_n = a_{n-1}(1/4)$ Explicit: $a_1 = 40 \quad a_n = 40(1/4)^{n-1}$	d) -1, -2, -4, -8..... Recursive: $a_1 = -1$ $a_n = a_{n-1}(2)$ Explicit: $a_1 = -1$ $a_n = -1(2)^{n-1}$

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Geometric Sequences

Date _____ Period ____

Determine if the sequence is geometric. If it is, find the common ratio.

1) 2, 12, 72, 432, ...

$r = 6$

2) $-2, 10, -50, 250, \ldots$

$r = -5$

3) 4, −20, 100, −500, ...

$r = -5$

4) 1, 4, 16, 64, ...

$r = 4$

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

5) $a_n = 2 \cdot 6^{n-1}$

$a_1 = 2$ $a_2 = 12$ $a_3 = 72$ $a_4 = 432$ $a_5 = 864$

6) $a_n = 2 \cdot 4^{n-1}$

$a_1 = 2$ $a_2 = 8$ $a_3 = 32$ $a_4 = 128$ $a_5 = 512$

7) $a_n = 3 \cdot (-3)^{n-1}$

$a_1 = 3$ $a_2 = -9$ $a_3 = 27$ $a_4 = -81$ $a_5 = 243$

8) $a_n = 3 \cdot (-2)^{n-1}$

$a_1 = 3$ $a_2 = -6$ $a_3 = 12$ $a_4 = -24$ $a_5 = 48$

Given two terms in a geometric sequence find both the recursive and explicit formulas.

9) $a_1 = 4$ and $a_4 = -32$ $r = -2$

rec: $a_1 = 4$, $a_n = a_{n-1}(-2)$

expl: $a_n = 4(-2)^{n-1}$

10) $a_6 = -\frac{3}{64}$ and $a_5 = \frac{3}{16}$ $r = -1/4$

rec: $a_1 = 48$, $a_n = a_{n-1}(-1/4)$

expl: $a_n = 48(-1/4)^{n-1}$

11) $a_4 = -\frac{1}{4}$ and $a_3 = -\frac{1}{2}$ $r = \frac{1}{2}$

$a_1 = -2$, $a_n = a_{n-1}(1/2)$

expl: $a_n = -2(1/2)^{n-1}$

12) $a_4 = 432$ and $a_5 = -2592$

recur: $a_1 = -2$, $a_n = a_{n-1}(-6)$

expl: $a_n = -2(-6)^{n-1}$

Evaluate each geometric series described.

13) $-2 - 8 - 32 - 128\ldots$, $n = 7$

-10922

14) $-1 - 5 - 25 - 125\ldots$, $n = 8$

-97656

15) $4 + 12 + 36 + 108\ldots$, $n = 9$

39364

16) $-1 - 2 - 4 - 8\ldots$, $n = 9$

-511

Find the missing term or terms in each geometric sequence.

17) ..., −1, ___, ___, −27, ...

$-3, -9$

18) ..., −2, ___, ___, ___, −162, ...

$-6, -18, -54$

Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15) $a_1 = 0.8,\ r = -5$

First Five Terms: 0.8, −4, 20, −100, 500
Explicit: $a_n = 0.8 \cdot (-5)^{n-1}$

16) $a_1 = 1,\ r = 2$

First Five Terms: 1, 2, 4, 8, 16
Explicit: $a_n = 2^{n-1}$

Given the first term and the common ratio of a geometric sequence find the recursive formula and the three terms in the sequence after the last one given.

17) $a_1 = -4,\ r = 6$

Next 3 terms: −24, −144, −864
Recursive: $a_n = a_{n-1} \cdot 6$
$a_1 = -4$

18) $a_1 = 4,\ r = 6$

Next 3 terms: 24, 144, 864
Recursive: $a_n = a_{n-1} \cdot 6$
$a_1 = 4$

19) $a_1 = 2,\ r = 6$

Next 3 terms: 12, 72, 432
Recursive: $a_n = a_{n-1} \cdot 6$
$a_1 = 2$

20) $a_1 = -4,\ r = 4$

Next 3 terms: −16, −64, −256
Recursive: $a_n = a_{n-1} \cdot 4$
$a_1 = -4$

Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

21) $a_4 = 25,\ r = -5$

First Five Terms: −0.2, 1, −5, 25, −125
Explicit: $a_n = -0.2 \cdot (-5)^{n-1}$
Recursive: $a_n = a_{n-1} \cdot -5$
$a_1 = -0.2$

22) $a_1 = 4,\ r = 5$

First Five Terms: 4, 20, 100, 500, 2500
Explicit: $a_n = 4 \cdot 5^{n-1}$
Recursive: $a_n = a_{n-1} \cdot 5$
$a_1 = 4$

Given two terms in a geometric sequence find the 8th term and the recursive formula.

23) $a_4 = -12$ and $a_5 = -6$

$a_8 = -\frac{3}{4}$

Recursive: $a_n = a_{n-1} \cdot \frac{1}{2}$
$a_1 = -96$

24) $a_5 = 768$ and $a_2 = 12$

$a_8 = 49152$
Recursive: $a_n = a_{n-1} \cdot 4$
$a_1 = 3$

25) $a_1 = -2$ and $a_5 = -512$

$a_8 = 32768$
Recursive: $a_n = a_{n-1} \cdot -4$
$a_1 = -2$

26) $a_5 = 3888$ and $a_3 = 108$

$a_8 = 839808$
Recursive: $a_n = a_{n-1} \cdot 6$
$a_1 = 3$

3

Name: _____ Date: _____ Key

Linear vs. Exponential Functions Tasks

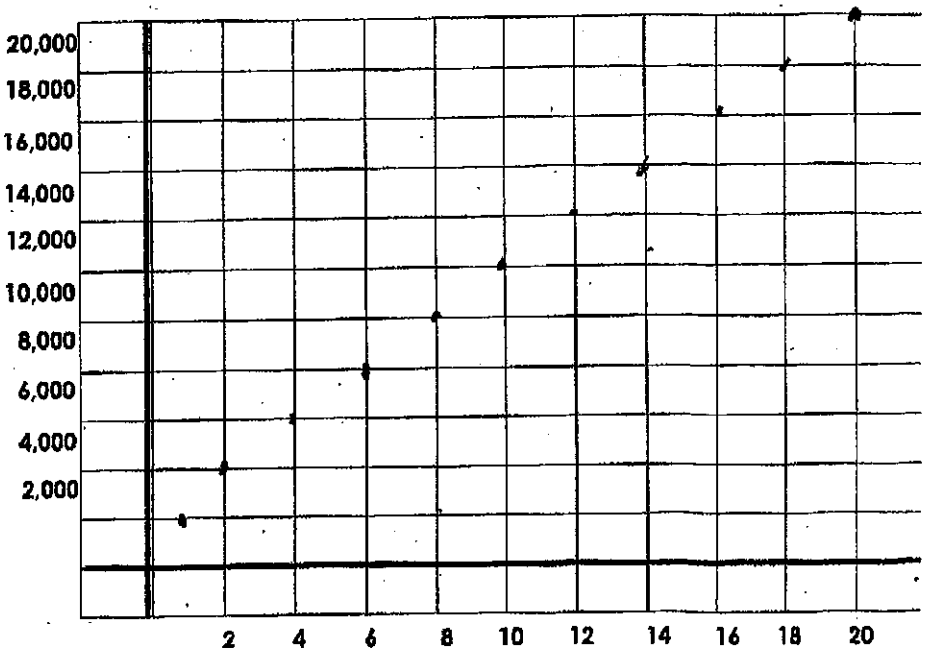
Which option would you choose and why? _____

- **Option 1:** You can have $1000 a year for twenty years
- **Option 2:** You can get $1 the first year, $2 the second year, $4 the 3rd, doubling the amount each year for twenty years.

Use the chart below to figure out how much money you would gain at the end of 20 years for option 1 and Option 2.

Graph both options to see which will give you more money.

1	$1,000	$1
2	$2,000	$1+$2=$3
3	$3,000	$3+$4=$7
4	4,000	7+8
5	5,000	8+16
6	6,000	16+32
7	7,000	32+64
8	8,000	64+128
9	9,000	128+256
10	10,000	512
11	11,000	1024
12	12,000	2048
13	13,000	4096
14		8192
15		16,384
16		32768
17		65536
18		131072
19	19,000	262144
20	20,000	524288

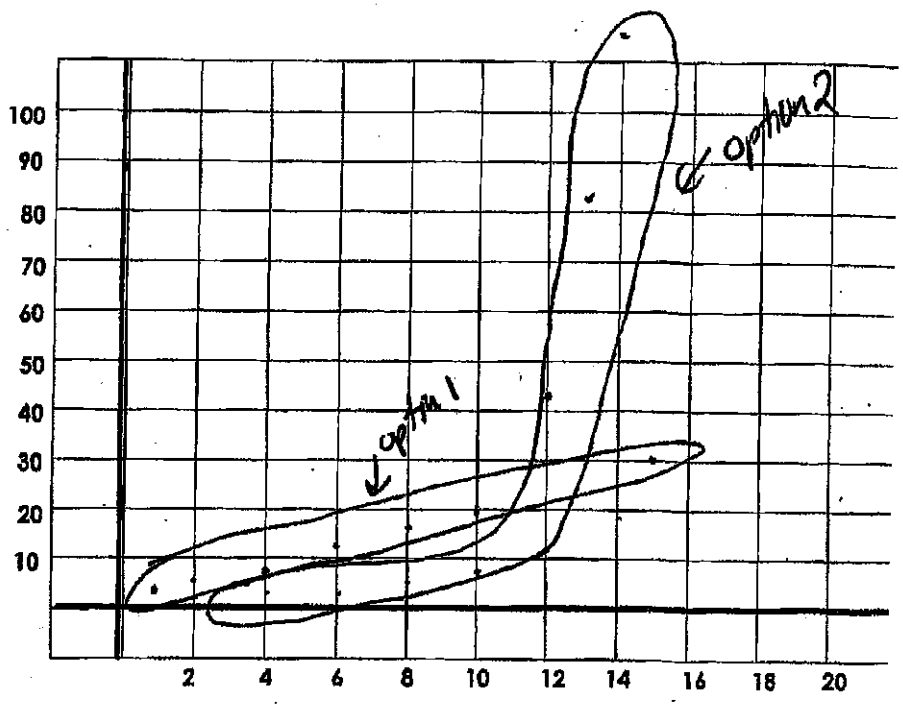


Task: Raking Leaves

Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves:

- ***Choice 1***: Two dollars for each bag of leaves,
- ***Choice 2***: She will be paid for the number of bags she rakes as follows: two cents for one bag, four cents for two bags, eight cents for three bags, and so on with the amount doubling for each additional bag.

1	2	.02
2	4	.04
3	6	.08
4	8	.16
5	10	.32
6	12	.64
7	14	1.28
8	16	2.56
9	18	5.12
10	20	10.24
11	22	20.48
12	24	40.96
13	26	81.92
14	28	163.84
15	30	329.68



1. If Celia rakes five bags of leaves, should she opt for payment method 1 or 2? What if she rakes ten bags of leaves?
   option 1 → if only 5 bags
   option 1 → if only 10 bags
2. How many bags of leaves does Celia have to raking before method 2 pays more than method 1?
   12 bags or more would more $, otherwise option 1 is best
3. Describe the differences in payment plans.
   if only bagging 1–11 bags use option 1
   if bagging 12–15 bags use option 2
4. Describe the difference in the way the payment grows in the table and on the graph.
   option 1 – linear
   option 2 – exponential
5. Is this growth situation continuous or discrete? How do you know?
   Discrete.... can not just do ½ a bag

5

HOMEWORK Task: Talk is Cheap!

To encourage communication between parents and their children and to prevent children from having extremely large monthly bills due to additional minute charges, two cell phone companies are offering special service plans for students.

Talk Fast cellular phone service charges $0.10 for each minute the phone is used.
Talk Easy cellular phone service charges a basic monthly fee of $18 plus $0.04 for each minute the phone is used.

Your parents are willing to purchase for you one of the cellular phone service plans listed above. However, to help you become fiscally responsible they ask you to use the following questions to analyze the plans before choosing one.

1. How much would each company charge per month if you talked on the phone for 100 minutes in a month? How much if you talked for 200 minutes in a month?

$f(100) = .10(100)$ $= \$10$ $f(200) = .10(200)$ $= \$20$ $g(100) = .04(100) + 18$ $= \$22$ $g(200) = .04(200) + 18$ $= \$26$

2. Build a table, make a graph, and write a function rule, f(x) or g(x), to represent the cost of each cellular service in terms of the number of minutes, x.

Tables:

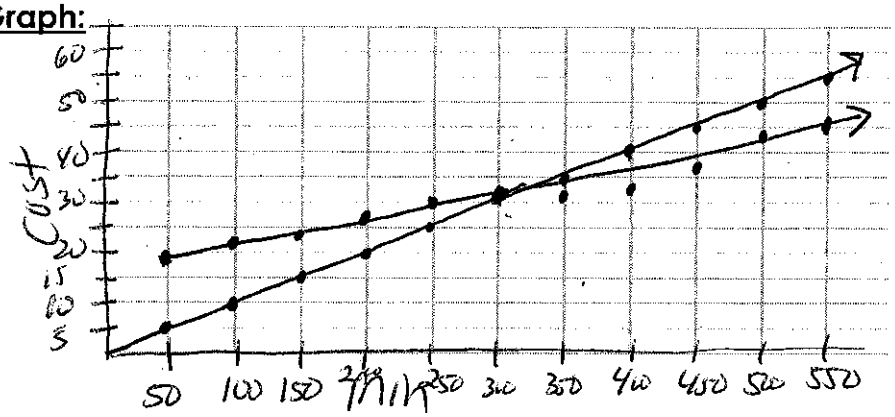
Talk Fast: f(x) = $.10x$

(numbers of minutes) x	1	2	3	4	5	6	7	8
(cost in dollars) f(x)	.10	.20	.30	.40	.50	.60	.70	.80

Talk Easy: g(x) = $.04x + 18$

(numbers of minutes) x	1	2	3	4	5	6	7	8
(cost in dollars) g(x)	18.04	18.08	18.12	18.16	18.20	18.24	18.28	18.32

Graph:



Rule:

Use the table, graph, and/or rule to help answer the following questions:

3. Which company would be a better financial deal if you plan to use the phone for 200 minutes a month? Explain your reasoning.

Fast talk would only be $20 where Talk Easy would be $26.

4. Which company would be a better financial deal if you plan to use the phone for 500 minutes a month? Explain your reasoning.

$f(500) = .10(500)$
$= \$50$

$g(500) = .04(500) + 18$
$= \$38$

Talk Easy

5. Depending on the number of minutes you talk on the phone each month, explain to your parents which cellular phone plan is more economical. Include in your explanation the point at which both cellular phone plans cost the same amount of money.

*If you talk for less than ~~or equal to~~ 300 min use talk fast. If you talk for 300 or more use Talk easy for a lower payment

6. If you know the cost of each plan for 300 minutes, can you double this cost to find the cost for 600 minutes? Explain your answer.

NO, you can't double it for the Talk Easy plan b/c it would then include the $18 twice.

Name: _____ Date: Key

Combining Functions Homework

Combining Functions

Given the functions $f(x) = 4x + 8$ and $g(x) = 2x - 12$

1. Find f(x) + g(x). = $6x - 4$

2. Find f(x) – g(x). $(4x+8) - (2x-12)$ $4x+8-2x+12 = 2x+20$

Given the functions $f(x) = 3x^2 + 5x - 8$ and $g(x) = 2x^2 - 9$

3. Find f(x) + g(x). $5x^2+5x-17$

4. Find f(x) – g(x). $(3x^2+5x-8) - (2x^2-9) = x^2+5x+1$

5. Find f(2) and g(2). $f(2) = 3(2)^2+5(2)-8 = 14$ $g(2) = 2(2)^2-9 = -1$

6. If e(x) = f(x) – g(x), find e(2). $e(2) = (2)^2+5(2)+1 = 15$

7. What do you notice about your answers to questions 5 and 6?

Given the functions $f(x) = 2x^2 + 3x$ and $g(x) = 5x + 1$

8. Find 2f(x) + 3g(x). $4x^2+6x+15x+3 = 4x^2+21x+3$

9. Find 5f(x) – 2g(x). $10x^2+15x-(10x+2) = 10x^2+5x+2$

10. Jill has a regular savings account that has $350 in it. She saves $55 each month in this account. She is also going on tour with her school choir next year. She opens up a new savings account just for tour. She deposits $25 to start the account and then, decides to save $40 each month from her paycheck into her tour savings account.

a. Write a function to represent the prices r(x) for Jill's regular savings account. $r(x) = 350 + 55x$

b. Write a function t(x) to represent Jill's tour savings account. $t(x) = 25 + 40x$

c. Combine the two functions into one function s(x) = r(x) + t(x). $= 95x + 375$

d. Calculate her totals savings after 3 months, 6 months, and 10 months.
3 months $= 95(3)+375 = 660$ 6 months $= 95(6)+375 = 945$ 10 m $= 95(10)+375 = 1325$

11. Joseph's Plumbing Company employs 3 workers. They employ out at the following rates.

- Joseph (owner): $75 (flat fee) + $65 per hour
- Sam (an apprentice): $10 (flat fee) + $25 per hour
- Sally: $50 (flat fee) + $45 per hour

a. Write 3 functions, one for each employee.
$y = 75 + 65x$ $z = 10 + 25x$ $k = 50 + 45x$

b. Write a new function to show the total amount of money coming in for the company in terms of hours worked? $f(x) = 135x + 135$

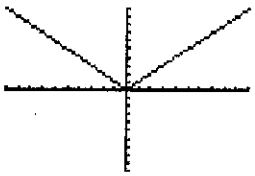
c. How much money will the company make if all the employees work an 8 hour day?
$f(8) = 135(8) + 135$
$f(8) = 1215$

Name: _____ **Date:** _____ **Period:** _____

Now, let's look at the graphs of the parent functions for symmetry.

A function is an ***even function*** when:
- → $f(-x) = f(x)$
- → Graph is symmetric about the y-axis.

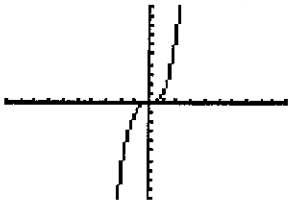
Example: $y = |x|$



Trick: Can you fold the graph in half along the y-axis and it aligns perfectly? Then it is **even.**

A function is an ***odd function*** when:
- → $f(-x) = -f(x)$
- → Graph is symmetric about the origin. (or rotated 180° about the origin)

Example: $y = x^3$

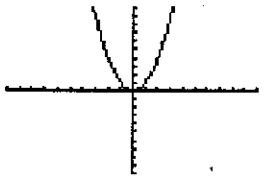


Trick: Can you rotate the graph upside down (OR, flip the graph over the y-axis and then the x-axis) and it still looks the same? Then it is **odd.**

A function is ***neither*** even ***nor*** odd if it doesn't meet the requirements to be an even function or an odd function.

Example 2: Decide if the following graphs are *even*, *odd*, or *neither*. Explain.

a) $y = x^2$



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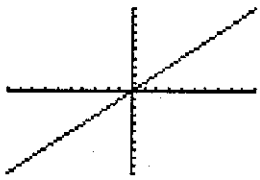
b) $y = x$



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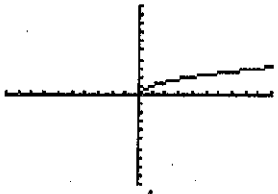
c) $y = \sqrt{x}$



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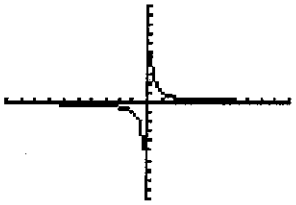
d) $y = \frac{1}{x}$



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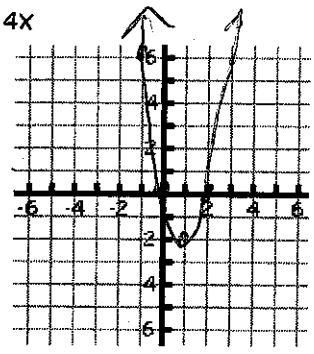
Example 3: Graph the following functions. Then, determine if the function is *even*, *odd*, or *neither*. Justify your answer by stating how the graph is or isn't symmetrical.

a) $f(x) = 2x^2 - 4x$

x	f(x)
-2	16
-1	6
0	0
1	-2
2	0
3	6
4	16



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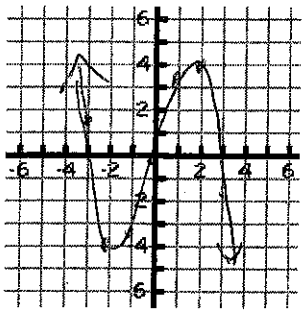
b) $g(x) = -\frac{1}{2}x^3 + 4x$

x	g(x)
-3	1.5
-2	-4
-1	-3.5
0	0
1	3.5
2	4
3	-1.5



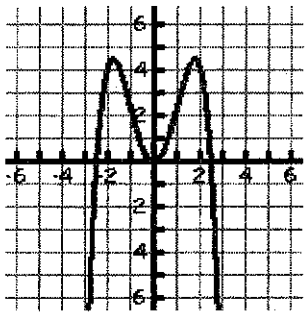
odd

Name: _____ Date: _____ Period: _____

Integrated Advanced Algebra
Worksheet: Graphs of Even and Odd Function

Determine whether each of the following functions is *even*, *odd*, or *neither*. Justify your answer by stating how the graph is or isn't symmetrical.

1. $f(x) = -\frac{1}{2}x^4 + 3x^2$



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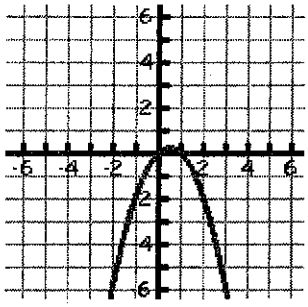
2. $g(x) = -x^2 + x$



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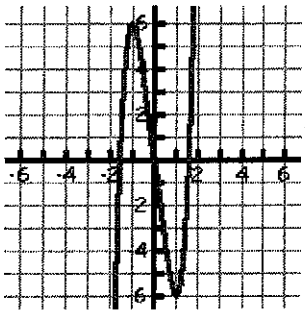
3. $h(x) = x^5 + x^3 - 8x$



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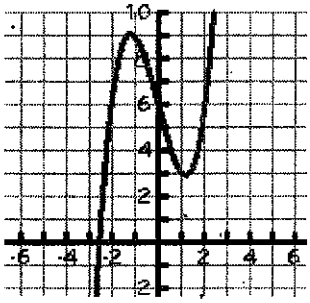
4. $f(x) = x^3 - 4x + 6$



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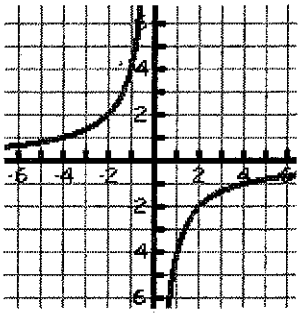
5. $g(x) = \frac{-4}{x}$



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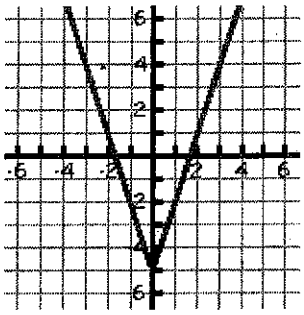
6. $h(x) = 3|x| - 5$



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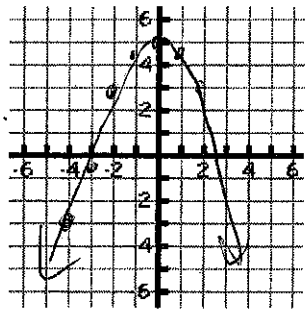
Graph the each of the following functions. Then, determine if the function is *even*, *odd*, or *neither*. Justify your answer by stating how the graph is or isn't symmetrical.

7. $f(x) = -\frac{1}{2}x^2 + 5$

x	f(x)
-4	-3
-3	.5
-2	3
-1	4.5
0	5
1	4.5
2	3



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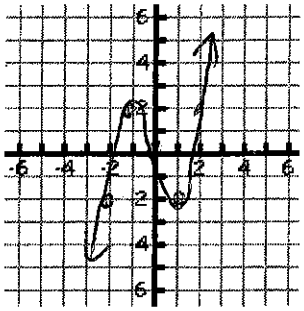
8. $g(x) = x^3 - 3x$

x	g(x)
-2	-2
-1	2
0	0
1	-2
2	2



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Name: Key Date: _____ Period: _____

Arithmetic and Geometric Sequences Practice

Directions: For each of the following tables:

a) Describe how to find the next term in the sequence.

b) Find the next term in the table.

c) Write a recursive rule for the function.

d) Write an explicit rule for the function.

e) Tell whether the function is linear, exponential, or neither.

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1)

x	y
1	10
2	20
3	40
4	?
...	...
n	?

a) To find the next term, common ratio of 2

b) Next term in table: 80, 160...

c) Recursive Rule: $a_1 = 10 \quad a_n = a_{n-1}(2)$

d) Explicit Rule: $a_1 = 10 \quad a_n = 10(2)^{n-1}$

e) Type of function: exponential

2)

x	y
1	40
2	200
3	1000
4	?
...	...
n	?

a) To find the next term, common ratio of 5

b) Next term in table: 5,000, 25,000...

c) Recursive Rule: $a_1 = 40 \quad a_n = a_{n-1}(5)$

d) Explicit Rule: $a_1 = 40 \quad a_n = 40(5)^{n-1}$

e) Type of function: exponential

3)

x	y
1	9
2	10
3	11
4	?
...	...
n	?

a) To find the next term, add 1 common difference

b) Next term in table: 12, 13, 14...

c) Recursive Rule: $a_1 = 1 \quad a_n = a_{n-1} + 1$

d) Explicit Rule: $a_n = 1 \quad a_n = 9 + (n-1)(1)$

e) Type of function: arithmetic

4)

x	y
1	16
2	64
3	256
4	?
...	...
n	?

a) To find the next term, Common ratio of 4

b) Next term in table: 1024, 4096

c) Recursive Rule: $a_1 = 16 \quad a_n = a_{n-1}(4)$

d) Explicit Rule: $a_1 = 16 \quad a_n = 16(4)^{n-1}$

e) Type of function: exponential

5)

x	y
1	-2
2	-5
3	-8
4	?
...	...
n	?

a) To find the next term, Common difference -3

b) Next term in table: -11, -14...

c) Recursive Rule: $a_1 = -2 \quad a_n = a_{n-1} - 3$

d) Explicit Rule: $a_n = -2 \quad a_n = -2 + -3(n-1)$

e) Type of function: arithmetic

6)

x	y
1	32
2	64
3	128
4	?
...	...
n	?

a) To find the next term, Common ratio of 2

b) Next term in table: 256

c) Recursive Rule: $a_n = 32 \quad a_n = a_{n-1}(2)$

d) Explicit Rule: $a_1 = 32 \quad a_n = 32(2)^{n-1}$

e) Type of function: geometric

7)

x	y
1	3,125
2	625
3	125
4	?
...	...
n	?

a) To find the next term, Common ratio of 1/5

b) Next term in table: 25, 5

c) Recursive Rule: $a_1 = 3,125 \quad a_n = a_{n-1}(1/5)$

d) Explicit Rule: $a_1 = 3125 \quad a_n = 3125(1/5)^{n-1}$

e) Type of function: geometric

Name: _____ Date: _____ Per: ____

ARITHMETIC AND GEOMETRIC SEQUENCE WORD PROBLEMS PRACTICE
All final solutions MUST use the formula.

1. Edgar is getting better at math. On his first quiz he scored 57 points, then he scores 61 and 65 on his next two quizzes. If his scores continued to increase at the same rate, what will be his score on his 9th quiz? Show all work.

   a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula. arithmetic
   $a_n = 57 + 4(n-1)$ $a_1 = 57$ $d = 4$

   b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.
   $n = 9$
   $a_9 = 57 + 4(9-1) \rightarrow a_9 = 57 + 32$ $a_9 = 89\%$

   c. Write your final answer as a sentence.
   Edgar would score an 89% on his 9th quiz.

2. Suppose you drop a tennis ball from a height of 15 feet. After the ball hits the floor, it rebounds to 85% of its previous height. How high will the ball rebound after its third bounce? Round to the nearest tenth.

   a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula. $a_n = 15(.85)^{n-1}$ $a_1 = 15$ $r = .85$

   b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.
   n = term value $a_3 = 15(.85)^{3-1}$
   $a_3 \approx 10.84$ ft

   c. Write your final answer as a sentence.
   On the 3rd bounce the ball would get up to 10.84 ft.

3. Viola makes gift baskets for Valentine's Day. She has 13 baskets left over from last year, and she plans to make 12 more each day. If there are 15 work days until the day she begins to sell the baskets, how many baskets will she have to sell?

   a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula. $a_n = 13 + 12(n-1)$ $a_1 = 13$ $r = 12$

   b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.
   $n = 15$ (days) $a_{15} = 13 + 12(15-1)$
   $a_{15} = 181$

   c. Write your final answer as a sentence.
   On the 15th day Viola will have 181 baskets

4. In a certain region, the number of highway accidents increased by 20% over a four year period. How many accidents were there in 2006 if there were 5120 in 2002? Hint: When the percent increases, you want the original 100% plus the additional 20%.

a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula.

$y = 5120(1+.20)^{t/4}$ t = # of years from 2002

$n = t$

b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

n = # of years from 2002

$y = 5120(1+.20)^{4/4} = 6144$ accidents

c. Write your final answer as a sentence.

There were 6144 accidents over the span of 2002-2006

5. A house worth $350,000 when purchased was worth $335,000 after the first year and $320,000 after the second year. If the economy does not pick up and this trend continues, what will be the value of the house after 6 years.

a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula.

$y = 350,000(1- \quad)^{t}$ $t = n$ = time in years

↑ initial amount R

b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

$y = 350,000(1-.086)^{6}$

$y = \$204,054.05$

c. Write your final answer as a sentence.

In 6 years the house will be worth $204,054.05. eeekkk!!!!

6. Write about anything that you need help with on the word problems we have done so far. Be specific so we can help you. Do you need to come for tutoring?

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