Key

EXPONENTIAL EXPLORATION ACTIVITY:

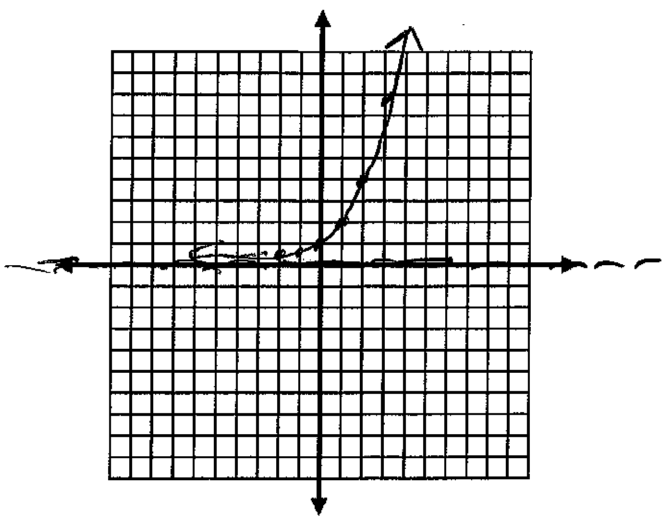
Today we will learn what an exponential function is and how it behaves graphically!!!

What is an exponential function?

How does it look graphically? Good question! Pull out your graphing calculators and let's get started!

1) In your graphing calculator, go to y= and enter in $y=2^x$, then hit "graph". Pull up the table of values and make your t-chart here:

x	y
-2	.25
-1	.5
0	1
1	2
2	4
3	8



2) Now go back to your y = and type in the following graphs, explain what happens each time compared to the graph from #1.

a) $y=2^{x-2}$ right 2

b) $y = 2^{x+2}$ left 2

c) $y = 2^x - 1$ down 1

d) $y = 2^x + 1$ up 1

e) $y = (3)2^x$ more steep - vertical stretch

f) $y = (1/2)2^x$ less steep - vertical shrink

g) $y = -2^x$ reflects over x-axis

h) $y = 2^{-x}$ reflects over y-axis

i) $y = (3)2^{x+1} - 1$ vertical stretch, left 1, down 1

$y = ab^x$

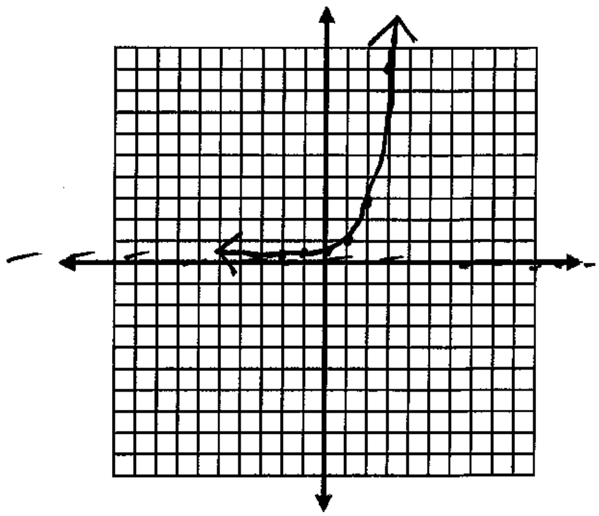
a = initial value

b = rate

Now let's graph!

$y = 3^{x-1}$ $+0$

x	y
-2	.037
-1	.11
0	.33
1	1
2	3
3	9



Characteristics:

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y=0$

y-intercept: $(0, .33)$

x-intercept: (none)

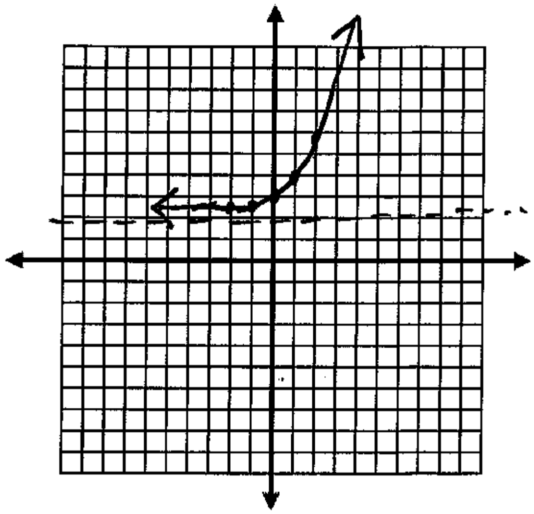
end behavior: $x \to -\infty, f(x) \to 0$
$x \to \infty, f(x) \to \infty$

interval of increase/decrease: $(-\infty, \infty)$

Growth or Decay? b/c $b > 1$

$y = 2^x + 2$

x	y
-2	2.25
-1	2.5
0	3
1	4
2	6
3	10



Characteristics:

Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

Asymptote: $y = 2$

y-intercept: ~~no~~ $(0, 3)$

x-intercept: none

end behavior: $x \to -\infty, f(x) \to 2$
$x \to \infty, f(x) \to \infty$

interval of increase/decrease: $(-\infty, \infty)$

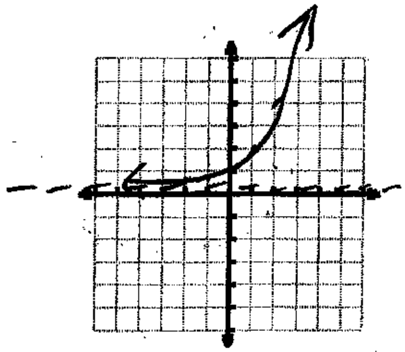
Growth/Decay: b/c $b > \cancel{2}$

CCGPS A – Linear and Exponential Functions **Name** Key
Graphing Exponential Functions

Exponential Functions: $y = b^x$, where b is a positive number other than 1

Graph $y = 2^x$ using a t-chart.

X	Y
-2	.25
-1	.5
0	1
1	2
2	4
3	8
4	16



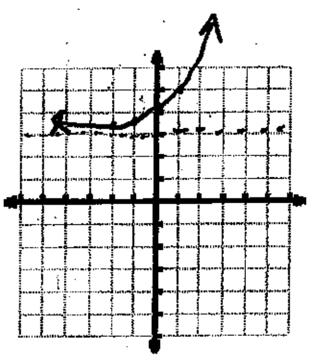
Asymptote - a line that a graph <u>approaches</u> as you move away from the origin; the graph hugs the asymptote

General Exponential Function $y = a(b^{x-h}) + k$

- Sketch the horizontal asymptote with a dashed line ($y = k$)
- Find the y-intercept of the graph by evaluating the function when x=0.
- Use a t-chart to sketch the graph of $y = ab^x$
- Transform the graph
  - Multiply y value of each coordinate in t-chart by a – move pencil to this point.
  - Shift h units horizontally
  - Shift k units vertically

1. $y = 2^x + 3$

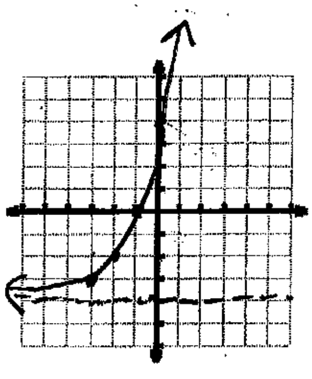
x	y
-2	3.25
-1	3.5
0	4
1	5
2	7



Y-intercept (0,5)
Asymptote $y = 3$
Domain $\mathbb{R}$
Range $y > 3$
Growth or Decay
end behavior: $x \to -\infty, f(x) \to 3$
$x \to \infty, f(x) \to \infty$

2. $y = 2^{x+3} - 4$

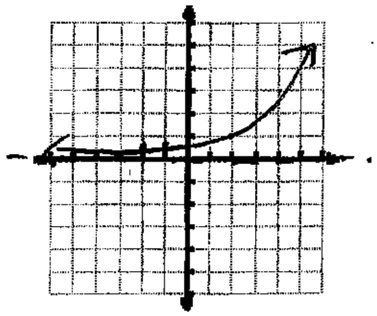
x	y
-3	-3
-2	-2
-1	0
0	4
1	12



Y-intercept (0,4)
Asymptote $y = -4$
Domain $\mathbb{R}$
Range $y > -4$
Growth or Decay
end behavior: $x \to -\infty, f(x) \to -4$
$x \to \infty, f(x) \to \infty$

3. $y = 3^{x-2} + 0$

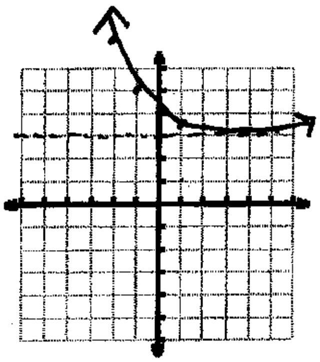
X	y
-2	.012
-1	.04
0	.11
1	.33
2	1



Y-intercept (0, .11)
Asymptote y = 0
Domain ℝ
Range y > 0
Growth or Decay
end behavior: $x \to -\infty, f(x) \to 0$
$x \to \infty, f(x) \to \infty$

4. $y = \left(\frac{1}{2}\right)^x + 3$

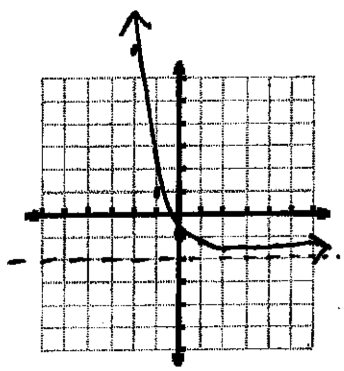
X	y
-2	7
-1	5
0	4
1	3.5
2	3.25



Y-intercept (0, 4)
Asymptote y = 3
Domain ℝ
Range y > 3
Growth or Decay
end behavior: $x \to -\infty, f(x) \to \infty$
$x \to \infty, f(x) \to 3$

5. $y = \left(\frac{1}{3}\right)^x - 2$

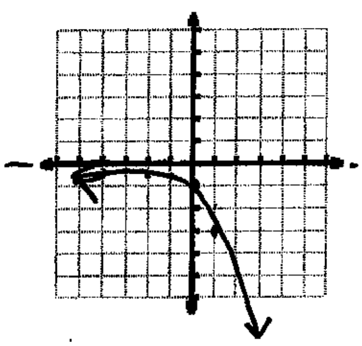
X	y
-2	7
-1	1
0	-1
1	-1.67
2	-1.89



Y-intercept (0, -1)
Asymptote y = -2
Domain ℝ
Range y > -2
Growth or Decay
end behavior: $x \to -\infty, f(x) \to \infty$
$x \to \infty, f(x) \to -2$

6. $y = -(3)^x + 0$

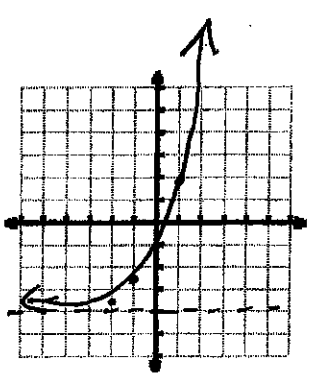
X	y
-2	-1/9
-1	-1/3
0	-1
1	-3
2	-9



Y-intercept (0, -1)
Asymptote y = 0
Domain ℝ
Range y < 0
Growth or Decay
end behavior: $x \to -\infty, f(x) \to 0$
$x \to \infty, f(x) \to -\infty$

7. $y = 3 \bullet (2)^x - 4$

X	y
-2	-3.25
-1	-2.5
0	-1
1	2
2	8



Y-intercept (0, -1)
Asymptote y = -4
Domain ℝ
Range y > -4
Growth or Decay
end behavior: $x \to -\infty, f(x) \to -4$
$x \to \infty, f(x) \to \infty$

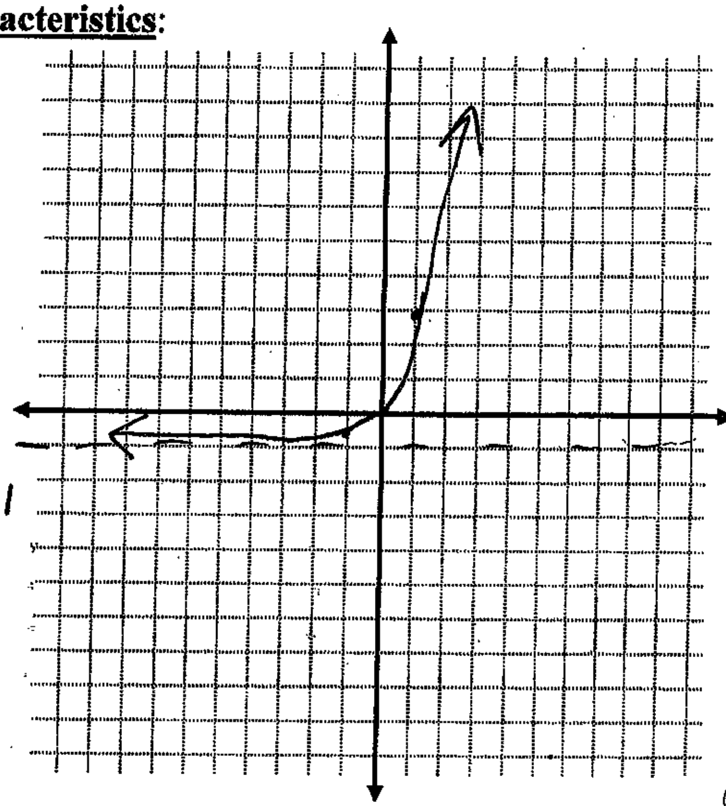
Exponential Functions Worksheet Name_______________ class_____

Graph the functions and **list all characteristics**:

1. $f(x) = 4^x - 1$

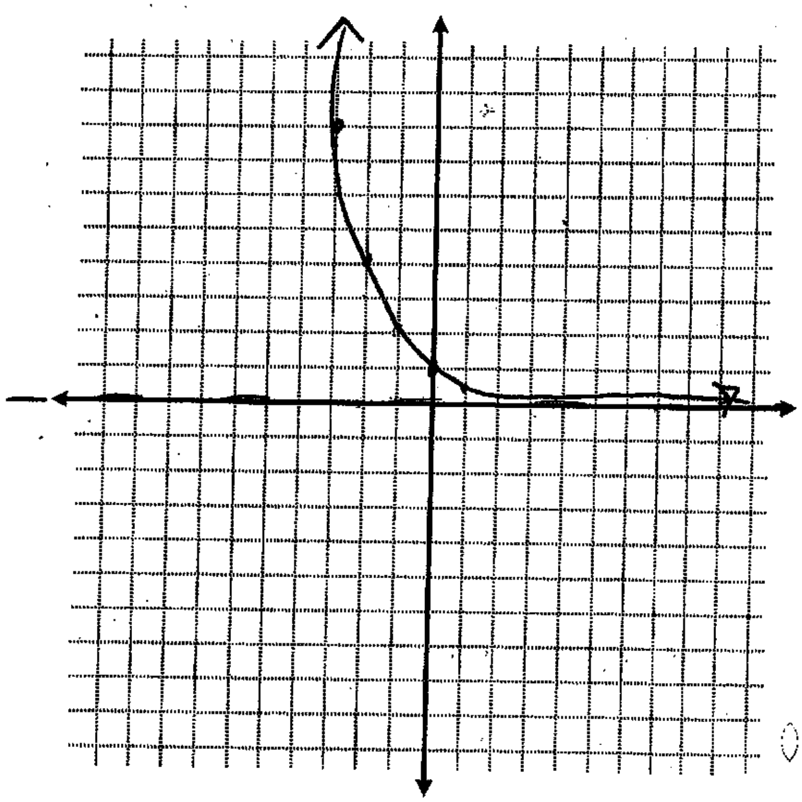
x	y
-3	-.98
-2	-.93
-1	-.75
0	0
1	3
2	15
3	63

D: $\mathbb{R}$
R: $(-1, \infty)$
zeros: (0,0)
y-inter (0,0)
end behavior:
$x \to -\infty, f(x) \to -1$
$x \to \infty, f(x) \to \infty$
increase:
$(-\infty, \infty)$



2. $f(x) = 0.5^x$

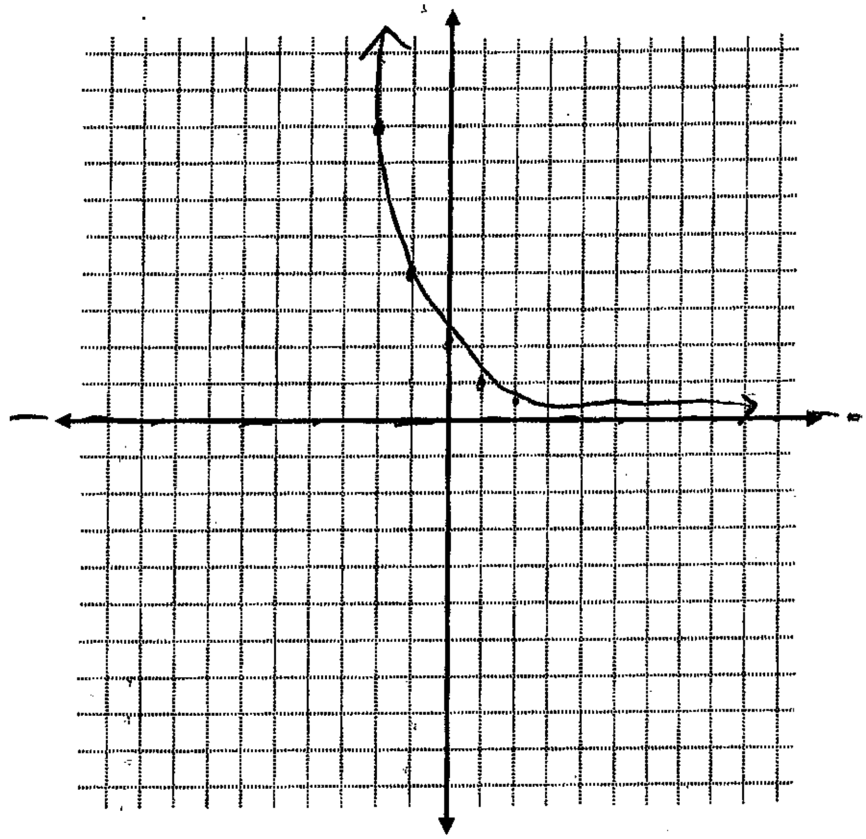
x	y
-3	8
-2	4
-1	2
0	1
1	.5
2	.25
3	.125



5b

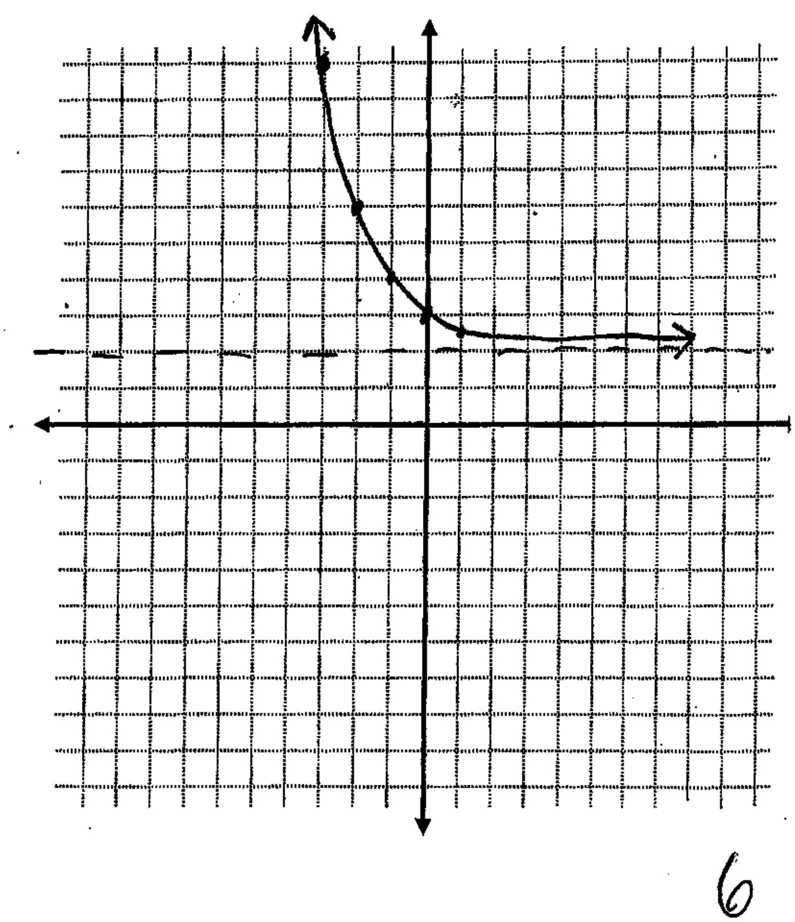
3. $f(x)=2\left(\frac{1}{2}\right)^{x}$

x	y
-3	16
-2	8
-1	4
0	2
1	1
2	1/2
3	.25



4. $f(x) = 2^{-x} + 2$

x	y
-3	10
-2	6
-1	4
0	3
1	2.5
2	2.25
3	2.125



Transformations of Exp functions Name: ____________________

Describe how the following graphs have been transformed from their original parent graph.

Original graph: $f(x)=5^x$

1. $f(x)=5^{x-1}$
2. $f(x)=5^x+2$
3. $f(x)=-5^x$
4. $f(x)=3(5)^x$
5. $f(x)=-\frac{1}{2}(5)^{x+2}-3$

1. Right 1
2. up 2
3. reflect over x-axis
4. vertical stretch
5. reflect over x-axis, vertical shrink, left 2, down 3

Original graph $y=2^x$

6. $y=2^{x-6}+1$
7. $y=-2^{x+3}-3$
8. $y=\frac{1}{3}(2)^x+8$
9. $y=(-4)2^{x-1}$
10. $y=8\cdot 2^{x+2}-5$

6. rt 2, down 1
7. reflect x-axis, left 3, down 3
8. vertical shrink, up 8
9. reflect over x-axis, rt 1
10. vertical stretch, left 2, down 5

Write an equation that has transformed $y=(\frac{1}{2})^x$ in the way that is described.

1. Up 6, to the left 1.
2. Reflected over the x axis, right 2, down 3
3. Stretch of 5, up 2
4. Reflected over the x axis, shrink of 1/3, left 6, up
5. Stretch of 2, right 3, down 4

11. $y=\frac{1}{2}^{x+1}+6$
12. $y=-(\frac{1}{2})^{x-2}-3$
13. $y=5(\frac{1}{2})^x+2$
14. $y=-\frac{1}{3}(\frac{1}{2})^{x+6}+1$
15. $y=2(\frac{1}{2})^{x-3}-4$

7

Topic: **Transformations of graphs**

What is it? **Shifting, stretching, shrinking, and reflecting of graphs**

Types:

Vertical or Horizontal shift

Examples

Add outside $y = 2^x + 3$
MOVES up

Subtract outside $y = 2^x - 3$
MOVES down

Add inside $y = 2^{(x+3)}$
MOVES left

Subtract inside $y = 2^{(x-3)}$
MOVES right

Reflection

Multiply by negative (-) $y = -2^x$
Causes the graph to reflect over x-axis

Vertical Stretch or Shrink

Multiply by Fraction (less than 1) $y = \frac{1}{4}(2)^x$
Causes the graph to vertical shrink

Multiply by integer $y = 4(2)^x$
Causes the graph to vertical stretch

Describing and Writing Transformations of Functions

1. Identify a, b, c, and d and describe what transformation(s) the graphs of the function $f(x)=4^x$ has undergone in each of the following cases.

$g(x)=4^{x-1}$
- rt 1

$h(x)=5(4^{x+4})$
- vertical stretch
- left 4

$m(x)=4^x+5$
- up 5

$n(x)=\frac{4^x}{2}$
- vertical shrink (½)

$r(x)=3(4^{-x})-1$
- vertical stretch
- reflect over y-axis
- dwn 1

$t(x)=-\frac{1}{3}(4^x)-2$
- reflect over x-axis
- vertical shrink
- dwn 2

2. Sketch the graphs of the transformations using the given parent function, $f(x)=3^x$. Label each one.

*plug in calc. to see each

$g(x)=3^{x-2}$

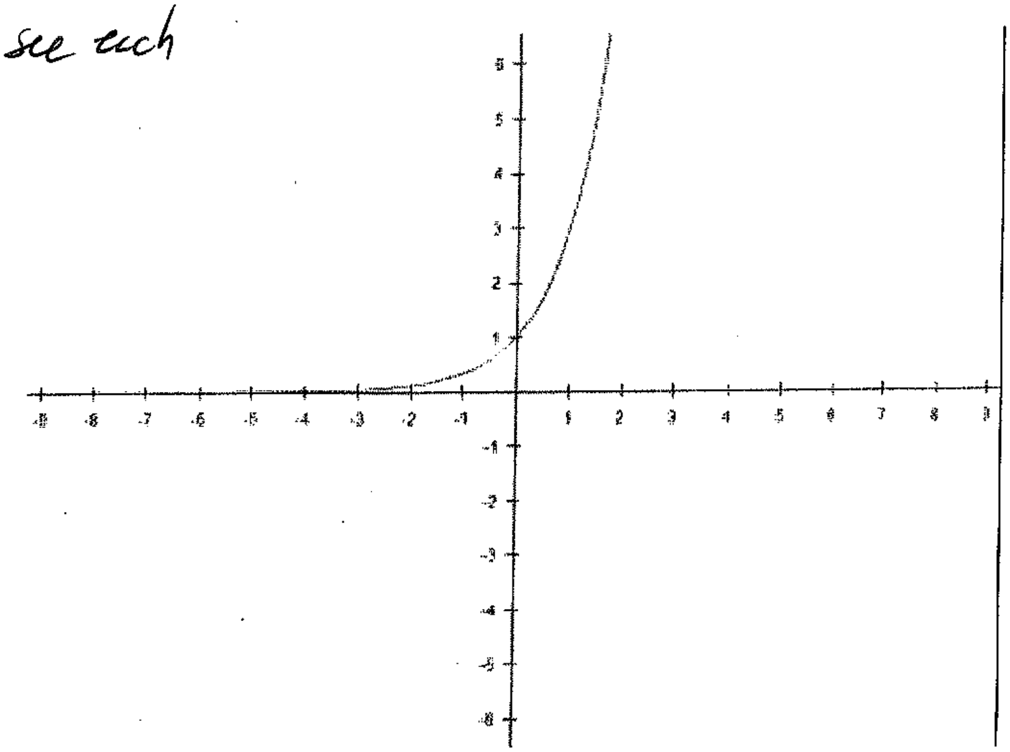
$h(x)=-3^x$

$j(x)=3^x+4$

$k(x)=2(3^x)$

$m(x)=3^{x+3}-2$

$n(x)=3^{-x}$



The properties of exponents can be used to solve exponential equations. The first step is to rewrite the equation so that the bases on both sides of the equation are the same. If the bases on both sides are the same, then the exponents must be equal. For instance,

$$3^{x+1} = 9^x$$

both bases can be made the same... $3^{x+1} = (3^2)^x$

using the exponent properties... $3^{x+1} = 3^{2x}$

if the bases are the same, then the exponents must be equal, so...

$$x+1 = 2x$$

and $x = 1$

Try these problems:

1. $2^x = 8$

   $\cancel{2}^x = \cancel{2}^3$ $\boxed{x=3}$

2. $3^{x+5} = 9^2$

   $\cancel{3}^{x+5} = \cancel{3}^{2(2)}$

   $x+5=4$ $\boxed{x=-1}$

3. $5^{2x+3} = \frac{1}{125}$

   $\cancel{5}^{2x+3} = \cancel{5}^{-3}$

   $2x=-6$ $\boxed{x=-3}$

4. $\left(\frac{1}{2}\right)^{x+4} = 8^{x-1}$

   $\cancel{2}^{-(x+4)} = \cancel{2}^{3(x-1)}$

   $-x-4 = 3x-3$ $-4x=1$ $\boxed{x=-\frac{1}{4}}$

5. $\left(\frac{1}{9}\right)^{x-2} = 81^{5-x}$

   $\cancel{9}^{-(x-2)} = \cancel{9}^{2(5-x)}$

   $-x+2 = 10-2x$ $\boxed{x=8}$

6. $8^{7x} = 16^{3x+9}$

   $\cancel{2}^{3(7x)} = \cancel{2}^{4(3x+9)}$

   $21x = 12x+36$ $9x=36$ $\boxed{x=4}$

7. $\cancel{7}^{3x+5} = \cancel{7}^{x-3}$

   $3x+5 = x-3$

   $2x=-8$ $\boxed{x=-4}$

8. $\left(\frac{1}{7}\right)^x = 7^{x+4}$

   $\cancel{7}^{-x} = \cancel{7}^{x+4}$

   $-2x=4$ $\boxed{x=-2}$

9. $\cancel{10}^{3x+5} = \cancel{10}^{x-3}$

   $3x+5 = x-3$

   $2x=-8$

   $\boxed{x=-4}$

10. $27^{7x} = 81^{3x+9}$

    $\cancel{3}^{3(7x)} = \cancel{3}^{4(3x+9)}$

    $21x = 12x+36$

    $\boxed{x=4}$

Think about these:

11. If $2^x = 8$ yields $x=3$ and $2^x = 16$ yields $x=4$, what would $2^x = 10$ yield?

12. How would you solve $5^x = 37$?

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answers: 1) 3 2) −1 3) −3 4) −1/4 5) 8 6) 4 7) −4 8) −2 9) −4 10) 4

AC Alg 1/Geo A

Name Key ID: 1

Solving Exponential Equations

Date________ Period____

Solve each equation.

1) $36^{2m} = 216$ $\left\{\frac{3}{4}\right\}$

2) $\left(\frac{1}{16}\right)^{-a-2} = 4$ $\left\{-\frac{3}{2}\right\}$

3) $3^{3-3n} = 3^{-2n}$

$\{3\}$

4) $\left(\frac{1}{243}\right)^{n} = \frac{1}{9}$ $\left\{\frac{2}{5}\right\}$

5) $6^{-2a} = 36$

$\{-1\}$

6) $6^{-3n} = 216$

$\{-1\}$

7) $2^{-3x+3} = 32$ $\left\{-\frac{2}{3}\right\}$

8) $3^{-n} = 1$

$\{0\}$

9) $64^{r} = 4^{3r}$

{ All real numbers. }

10) $81^{n+3} = 9^{2n}$

No solution.

AC Alg 1/Geo A
Exponential Equations

Name ______________________

Solve each equation.

1. $\cancel{5}^x = \cancel{5}^{-3}$ $\boxed{x=-3}$	14. $\frac{1}{27} = 3^{x-5}$ $\frac{1}{3^3} = 3^{x-5}$ $3^{-3} = 3^{x-5}$ $x-5=-3$ $\boxed{x=2}$
2. $6^x = 216$ $\cancel{6}^x = \cancel{6}^3$ $\boxed{x=3}$	15. $\left(\frac{1}{3}\right)^x = 3^{x-6}$
3. $7^y = \frac{1}{49}$ $7^y = \frac{1}{7^2}$ $7^y = 7^{-2}$ $\boxed{y=-2}$	16. $25^{2m} = 125^{m-3}$
4. $10^x = .001$ $10^x = 10^{-3}$ $\boxed{x=-3}$	17. $4^{x-1} = 8^x$
5. $2^{2x} = \frac{1}{8}$ $2^{2x} = \frac{1}{2^3}$ $2^{2x} = 2^3$ $2x=3$ $\boxed{x=3/2}$	18. $2^{x+1} = 2^{2x+3}$
6. $\left(\frac{1}{5}\right)^{x-3} = 125$ $5^{-(x-3)} = 5^3$ $-x+3=3$ $-x=0$ $\boxed{x=0}$	19. $3^{2x-1} = \frac{1}{9}$
7. $3^y = 3^{3y+1}$ $3=3y+1$ $2=3y$ $\boxed{y=2/3}$	20. $6^y = 6^{3y-1}$
8. $5^{3y+4} = 5^y$ $3y+4=y$ $4=-2y$ $\boxed{y=-2}$	21. $\left(\frac{1}{7}\right)^{6x} = 7^{2x-20}$
9. $3^x = 9^{x+1}$ $\cancel{3}^x = \cancel{3}^{2(x+1)}$ $x = 2x+2$ $-x=+2$ $\boxed{x=2}$	22. $3^{6x-5} = 9^{4x-3}$
10. $2^5 = 2^{2x-1}$ $5=2x-1$ $6=2x$ $\boxed{x=3}$	23. $5^{2x+3} = \left(\frac{1}{25}\right)^{x+4}$ $5^{2x+3} = \frac{1}{5^2}$ $5^{2x+3} = 5^{-2}$ $2x+3=-2$ $2x=-5$ $\boxed{x=-5/2}$
11. $8^{x-1} = 16^{3x}$ $2^{3(x-1)} = 2^{4(3x)}$ $3x-3=12x$ $-3=9x$ $\boxed{x=-3}$	24. $2^{3x-1} = \left(\frac{1}{8}\right)^x$
12. $2^{x+3} = \frac{1}{16}$ $2^{x+3} = \frac{1}{2^4}$ $2^{x+3} = 2^{-4}$ $x+3=-4$ $\boxed{x=-7}$	25. $\left(\frac{1}{16}\right)^{x+1} = \left(\frac{1}{8}\right)^{2x-1}$
13. $9^{3y} = 27^{y+2}$ $\cancel{3}^{2(3y)} = \cancel{3}^{3(y+2)}$ $6y=3y+6$ $3y=6$ $\boxed{y=2}$	

Name: ____ Period: _________

Key Notes:

3.4A Average Rate of Change

Rates are used to describe how one quantity is changing in relation to another. This is called a **"rate of change"** or an "**average rate of change.**" To illustrate this, consider the following statement: Reagan drove from Salt Lake to Bluffdale (a distance of about 28 miles) in 30 minutes.

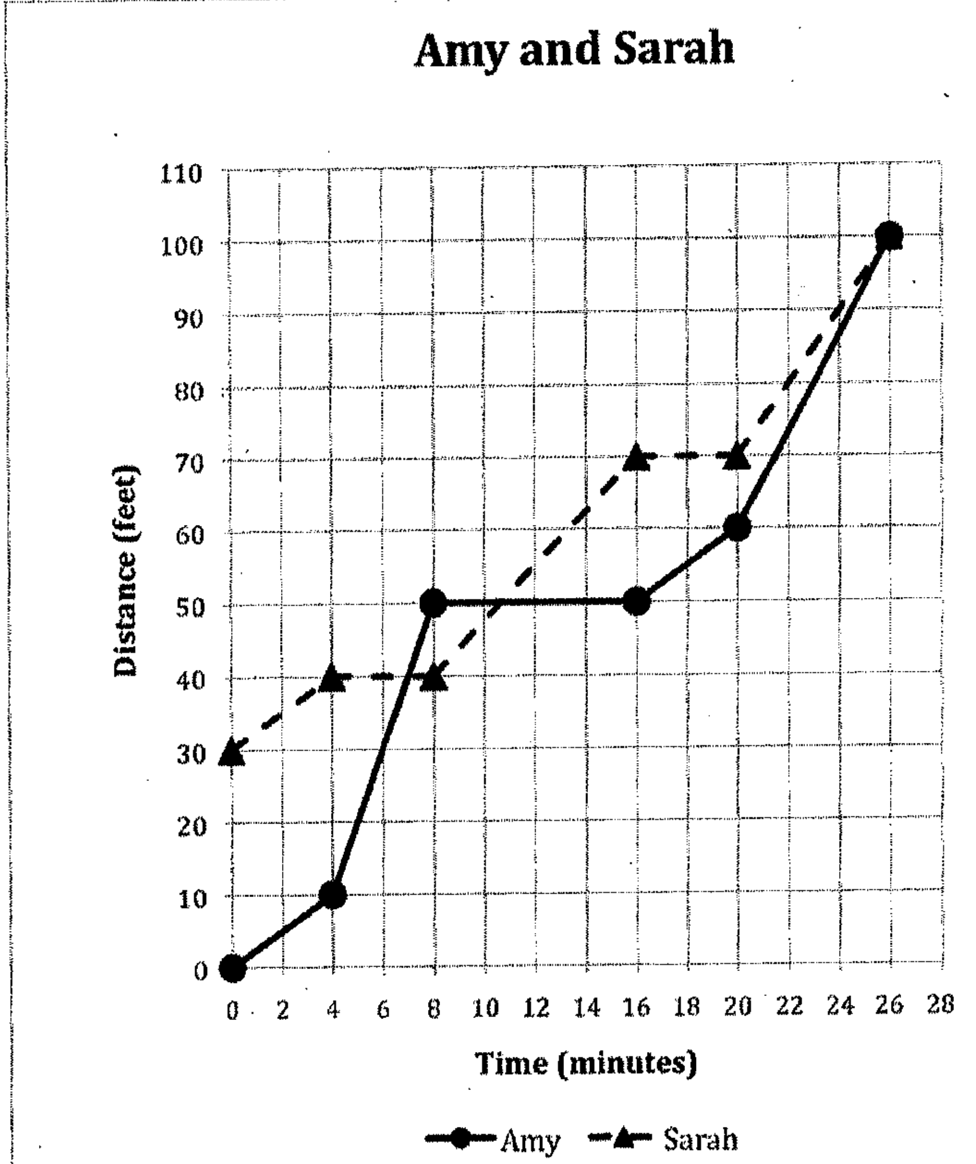
A.) What was his average speed in miles per hour?

$$\frac{28\text{ miles}}{30\text{ min}} \bullet \frac{2}{2} = \frac{56\text{ miles}}{1\text{ min}}$$

B.) Does this mean that he drove that speed the entire trip? If not, what does it mean? No,...

C.) Did he ever drive the average speed of 56 mph?
Maybe....

Ex: Amy and Sarah are meeting each other at the store.



Name: ______________________________ Period: ________

Use the graph to answer the following questions.

1.) How far from the store is Amy at the beginning?
100 ft away from the store

2.) How far from the store is Sarah at the beginning?
70 ft away from the store

3.) How long does it take to get to the store?
It took 26 min. to get to the store.

4.) What happens between 6 and 7 minutes?
They are the same distance away from the store, 60 ft away.

5.) Where is Amy moving faster?
Between 4 & 8 min. and 20 & 26 min.

6.) Where is Sarah moving faster?
Between 8 & 16 min.

7.) What is the speed of Amy between 4 and 8 minutes?

$$\frac{40 \text{ feet}}{4 \text{ min}} = \frac{10 \text{ feet}}{1 \text{ min}}$$

8.) What is the speed of Sarah 8 and 16 minutes?

$$\frac{30 \text{ feet}}{8 \text{ min}} = \frac{3.75 \text{ feet}}{1 \text{ min}}$$

9.) What is Amy doing during 8 and 16 minutes?
Answers vary: Ex: Maybe she stopped to talk on her cell phone.

10.) What is Amy's average speed for the whole trip?

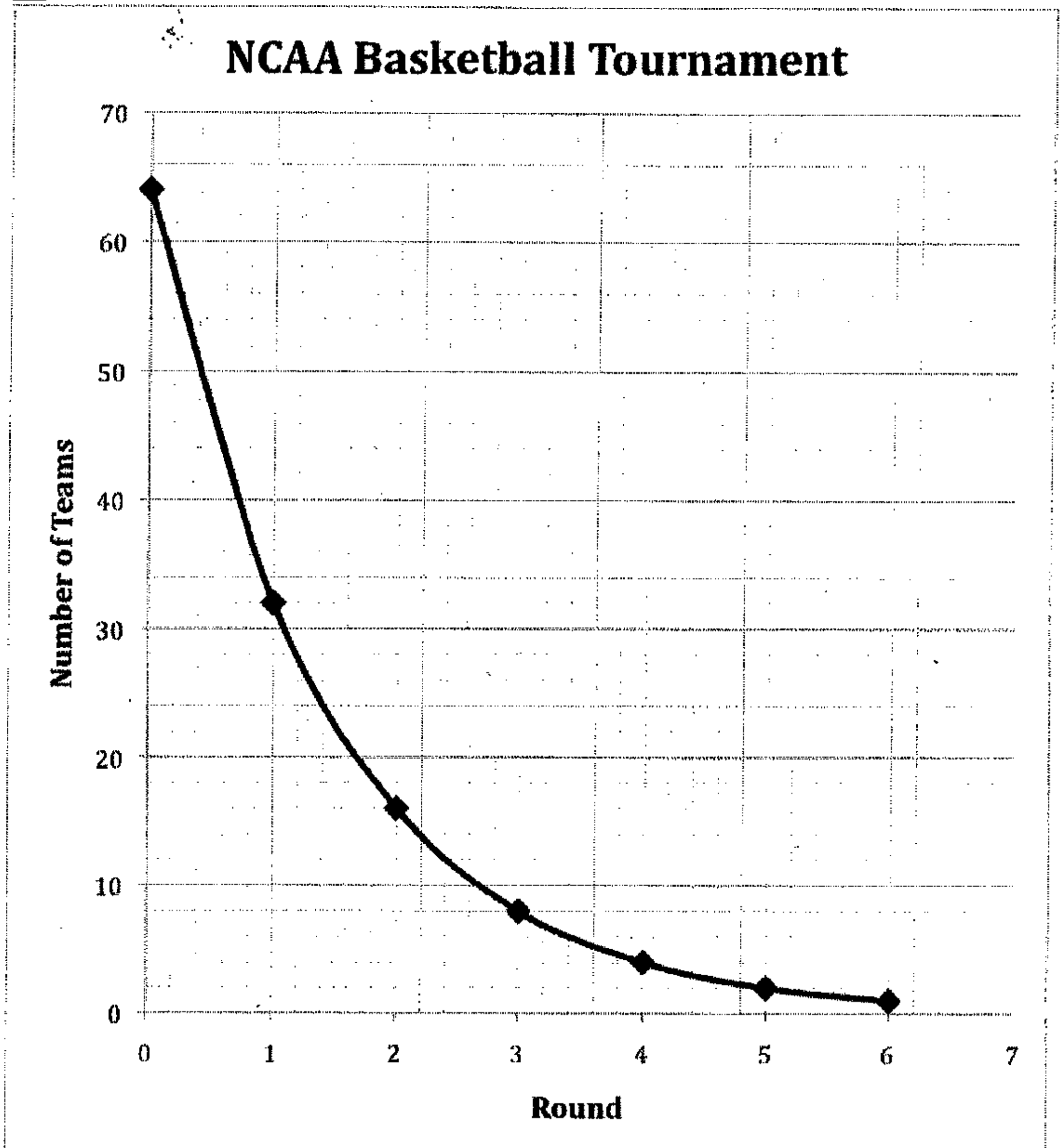
$$\frac{100 \text{ feet}}{26 \text{ min}} = \frac{3.8 \text{ feet}}{1 \text{ min}}$$

11.) What is Sarah's average speed for the whole trip?

$$\frac{70 \text{ feet}}{26 \text{ min}} = \frac{2.7 \text{ feet}}{1 \text{ min}}$$

Name: ______________________________ Period: ________

Ex: The NCAA basketball tournament.



Use the graph to answer the following questions.

1.) How many teams are there when the tournament starts?
64 teams

2.) How many rounds occur before there is a winner?
6 rounds

3.) What is the rate of change between the 1st and 2nd round?
-16/1

4.) What is the rate of change between the 2nd and 3rd round?
-8/1

5.) What is the rate of change between the 3rd and 4th round?
-4/1

Name: ____ Period: _________

6.) What is the average rate of change between the 1st and 4th round?

-28/3

7.) What is the average rate of change from the beginning of the tournament to the end?

$$\frac{63}{6} = \frac{21}{2}$$

8.) The NCAA tournament chairman is considering adding another round to the tournament so more teams can participate. How many teams would start the tournament?

128 teams

Notes - 3.4B Rate of Change

key

Ex: What is the average rate of change of the function $g(x) = 6 - 2x$

A.) Over the interval [2, 6]?

Rate of change = -2 Use table, slope formula, or graph to find slope.

B.) Over the interval [5, 7]?

Rate of change = -2 Use table, slope formula, or graph to find slope.

C.) Do you think it is true that $g(x)$ will have a constant average rate of change over *any* interval? Why or why not?

Yes,...

Ex: What is the average rate of change of the function $f(x) = 2^x$

A.) Over the interval [1, 4]?

Rate of Change = 14/3 Use table, slope formula, or graph to find slope.

B.) Over the interval [3, 5]?

Rate of Change = 12 Use table, slope formula, or graph to find slope.

C.) Do you think it is true that $f(x)$ will have a constant average rate of change over any interval? Why or why not?

No, ...

Ex: Given a table, find the rate of change for each interval.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

A.) [0, 3] rate of change = 5

B.) [-2, 1] rate of change = 1

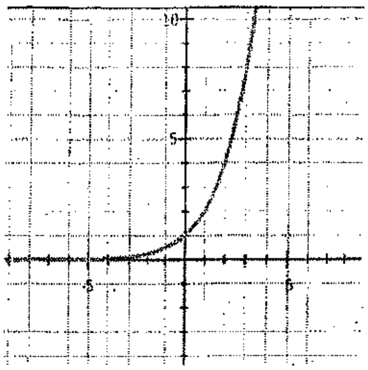
C.) [-3, -1] rate of change = -1

Name______________________________**Date**____________________**Period**_____

WS#1 Part 2: Analyzing Characteristics (Rate of Change and End Behavior)

1.

$f(x) = 2^x$



*plug -2

*plug 2

for x & find y

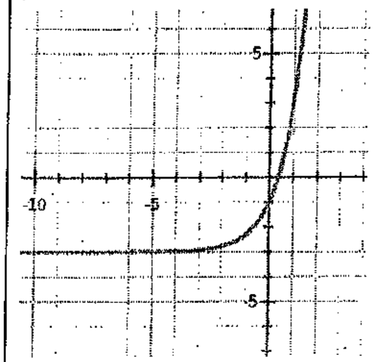
Rate of change for $-2 \le x \le 2$:

$(-2, \frac{1}{4})$ $(2, 4)$ $m = \dfrac{4 - \frac{1}{2}}{2 - -2} = \boxed{\dfrac{3.5}{4}}$

End Behavior: as $x \to -\infty$, $f(x) \to$ 0; as $x \to \infty$, $f(x) \to$ ∞

2.

$f(x) = 2(3)^x - 3$



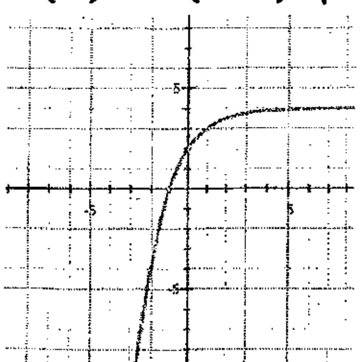
Rate of change for $-2 \le x \le 2$: $m = \dfrac{15 + 2.78}{2 - -2} = \boxed{\dfrac{17.78}{4}}$

$(-2, -2.78)$ $(2, 15)$

End Behavior: as $x \to -\infty$, $f(x) \to$ -3; as $x \to \infty$, $f(x) \to$ ∞

3.

$f(x) = -2(1/2)^x + 4$



*plug -2 & 2 in for x to find y-value

$m = \dfrac{3.5 - -4}{2 - -2}$

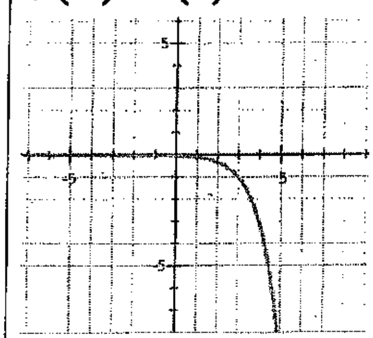
Rate of change for $-2 \le x \le 2$:

$(-2, -4)$ $(2, 3.5)$ $m = \boxed{\frac{25}{4}}$

End Behavior: as $x \to -\infty$, $f(x) \to$ $-\infty$; as $x \to \infty$, $f(x) \to$ 4

4.

$f(x) = -(3)^{x-3}$



Rate of change for $-2 \le x \le 2$:

End Behavior: as $x \to -\infty$, $f(x) \to$ ______; as $x \to \infty$, $f(x) \to$ ______

Exploring exponential functions (Decay)

You and a friend are out exploring in the woods and come across a meteorite! Since you are both geniuses, you quickly determine that the meteorite is a radioactive substance that is decaying every hour. You also determine that it weighs 30 ounces.

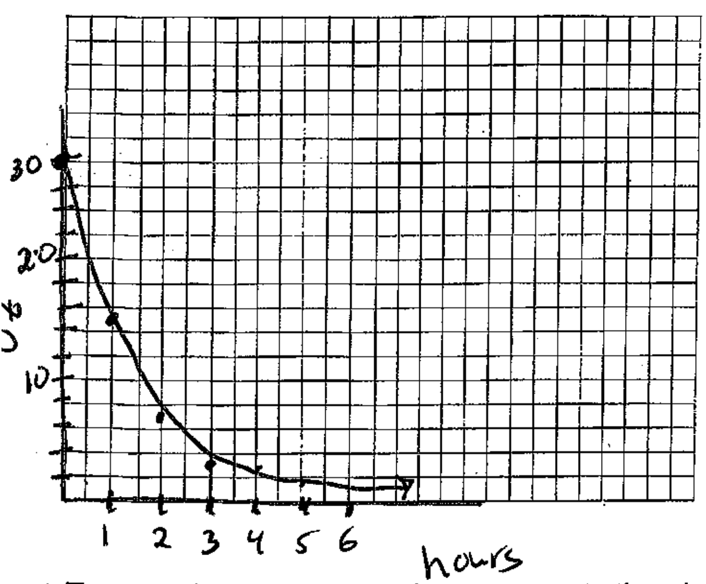
The substance has a ½ life of 1 hour (this means the substance decays at a rate of ½ every hour.) When will the substance decay and have less than 1 ounce of the original material? Use the table below to figure it out!

Hours	Ounces
0	30 oz
1	15oz
2	7.5oz
3	3.75
4	1.875
5	.9375

The substance will have less than 1 oz between 4 + 5 hours

How many hours will it take the substance to have less than 1 ounce of the original material? Between 4+5 hrs

Graph the above data



Try to write an equation that represents the above situation:

$$y = 30(1/2)^n$$

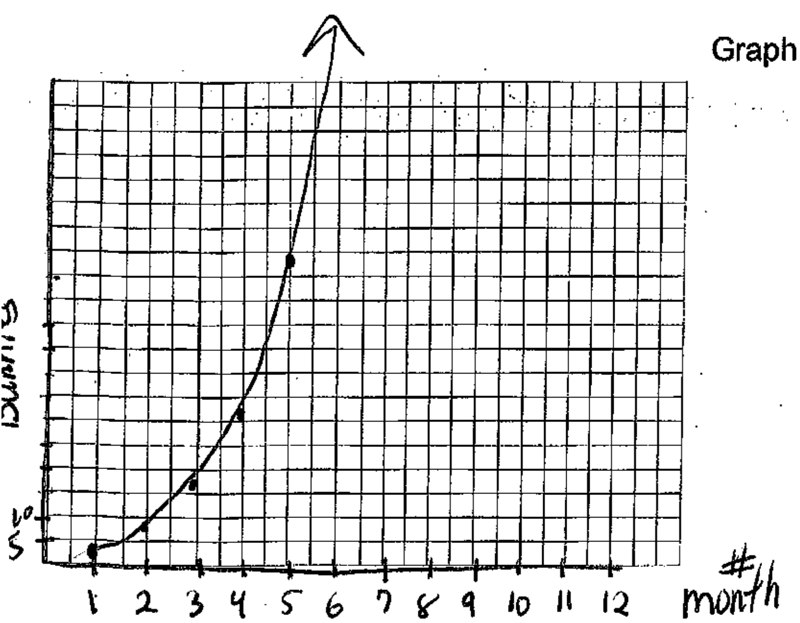
Exploring exponential functions (Growth)

You and your friend buy a bunny farm. Congratulations! You want to have as many bunnies as possible to sell for Easter. Well, you're broke and can only afford 2 bunnies. The good news is that after a month of having them home, those bunnies create 2 more bunnies. (Ask your parents if that last sentence confuses you.) Well, the new bunnies create 2 more bunnies and so on. At this rate, how many bunnies will you have after 12 months? Use the table below.

After Month:	Total # of bunnies
1	4
2	8
3	16
4	32
5	64
6	128
7	256
8	512
9	1024
10	2048
11	4096
12	8192

How many total bunnies will you have at the end of the 12 month period? 8192

Graph the above data



Try to write an equation that represents the above situation: $y = 2(2)^n$

An **exponential function** has the form

$$y = ab^x$$

b is a positive number <u>other than</u> 1

If b is *<u>greater than</u>* 1

$$y = ab^x \longleftarrow \boxed{\text{b is the “growth factor”}}$$

exponential growth function.

If b is *<u>between 0 and 1</u>*

$$y = ab^x \longleftarrow \boxed{\text{b is the “decay factor”}}$$

exponential decay function.

In the bunny problem
a = 2 (because we initially had 2 bunnies)
b = 2 (because they were having 2 babies)
So our equation was $y = 2(2)^x$

a = initial amount
b = growth/decay factor
x = time
y = ending amount

In the meteorite problem:
a = 30 (we initially had 30 ounces)
b = ½ (it was decaying by ½)
So our equation was: $y = 30(\frac{1}{2})^x$

1. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. Write an equation that models this situation. $y = 128(\frac{1}{2})^x$
How many players are left after 5 rounds? $y = 128(\frac{1}{2})^5$ $= 4$ (you can count OR plug in 5 for your x)

2. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. Write an equation that models one bacteria cell that splits into two new cells every hour. $y = 1(2)^x$
How many bacteria would you have after 24 hours? $y = 1(2)^{24}$ $= 16777216$

In the following equations, identify the initial amount and growth or decay factor. CIRCLE whether it is growth or decay.

3. $y = 100(3)^x$
Initial amount = 100
Growth/decay factor = 3

4. $y = 15(.5)^x$
Initial amount = 15
Growth/decay factor = .5

5. $y = 4^x$
Initial amount = 1
Growth/decay factor = 4

6. $y = \frac{1}{2}(\frac{3}{2})^x$
Initial amount = ½
Growth/decay factor = 3/2

Exponential Functions Growth and Decay Application Problems

Growth Example
$y=(1.26)^x$

rate: .26 or 1.26

percent: 26% → 126%

Decay Example
$y=(.80)^x$

rate: .20 (losing 20%)

percent: 20% , keeping 80%

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Possible Equations and their uses:

$A=P(1+r)^t$ - used for appreciating values

$A=P(1-r)^t$ - used for the depreciating values

$A=P(1+r/n)^{nt}$ - **used for values that are being compounded**

monthly = 12
weekly = 52
daily = 365
quarterly = 4
semiannually = 2

$A=Pe^{rt}$ -used when a function is compounded continuously

P=principal (starting amount)
A=amount
r =rate (change to a decimal)
t = time

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Example: Ms. Benzin has a house that is worth $200,000. It appreciates in value 5% each year. How much will it be worth in 5 years?

Which formula will we use?

$y = 200{,}000(1+.05)^5$

$y = \$255{,}256.31$

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Example: Brian's car depreciates at a rate of 11% per year. If his car currently is worth $16,500, how much will it be worth in 7 years?

Which formula will we use?

$y = 16{,}500(1-.11)^7$

$y = \$7298.17$

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Exponential Growth and Decay Worksheet

In the function: $y = a(b)^x$, a is the y-intercept and b is the base that determines the direction of the graph and the steepness. In real-life situations we use x as time and try to find out how things change exponentially over time. Some examples of this are money growing in a bank account by a certain percentage every year or the population of a city growing by a certain percentage every year.

Because *a* is the y-intercept it plays a very important role in word problems involving exponential growth. *a* is known as the **initial value** because it is the value of the function when $x = 0$ or at the beginning of time.

b determines how fast the function increases or decreasing. For this reason, b is known as the **growth factor.** The growth factor is determined by starting with 100% and then adding or subtracting the percentage that the function if being increased by or subtracting the percentage that the function is being decreased by. Finally you take your growth factor as a percentage and change it into a decimal before plugging it into $y = a(b)^x$.

Find the initial value and growth factor for each of the situation below then plug them in to $y = a(b)^x$ *to get the function that models the problem.* ***Answers are at the end.***

1. You deposit $200 into a bank account. Every year that account increases by 12 %. [EXAMPLE]

   Initial value: 200

   Growth factor: 1.12

   Equation: $y=200(1.12)^x$

2. The population of an apartment building is 4,000 people. Every month the population goes down by 12%.

   Initial value: 4,000

   Growth factor: 1.12

   Equation: $y = 4000(1.12)^x$

3. You start a bank account with $500 and the interest on the account is 8% every year.

   Initial value: 500

   Growth factor: 1.08

   Equation: $y = 500(1.08)^x$

Example: Jenni opened a savings account when she was 5 years old. She deposited \$200 and forgot about the account. The account pays 3.25% interest, compounded quarterly. Jenni is now 18 years old and just remembered she has the account. How much money is in the account now?

What formula will you use?

$$y = 200\left(1 + \frac{.0325}{4}\right)^{4 \cdot 18}$$

$$y = \$358.15$$

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Example: Sarah has \$2000 and wants to invest in an account which is continuously compounded. The rate is 5% and she plans to keep the money in her account for 10 years. How much money will she have at the end of the 10 years?

Which formula will you use?

$$y = 2000e^{.05(10)}$$

$$y = \$3297.44$$

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A# ________

Compound Interest and *e* Worksheet

The history of mathematics is marked by the discovery of special numbers such as π or i. Another special number is denoted by the letter e. It is called the *Euler number* after its discoverer and it is also called the **natural base** e. Like π, it is an irrational number.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = 2.71828...$$

It is important to remember that e is JUST A NUMBER!

One use of e is for "continuously compounded interest."

$$A(t) = Pe^{rt}$$

where P = principal investment

r = interest rate (as a decimal)

t = time

There is another formula we can use to calculate interest when it is not compounded continuously:

For compounding interest a specific # of times annually:

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

where P = principal investment

r = interest rate (as a decimal)

t = time

n = # of times you compound annually

1) If you invest $2500 in an account, what is the balance in the account and the amount of interest after 4 years if you earn:

   a) 1.7% interest compounded annually? $y = 2500(1+.017)^4$

   b) 1.5% compounded monthly? $y = 2500\left(1+\frac{.015}{12}\right)^{12\cdot 4}$

   c) 1.2% compounded daily? $y = 2500\left(1+\frac{.012}{365}\right)^{365\cdot 4}$

   d) 0.7% compounded continuously? $y = 2500e^{.007(4)}$

30

CCGPS Coordinate Algebra

Name:____________________
Date:______________ Block:______

Exponential Application Problems

Determine whether the function represents exponential growth or decay.

1) $y = 350(0.75)^x$ D

2) $y = 80(1.03)^x$ G

3) $y = (1.87)^x$ G

4) $y = 500(0.9)^{-x}$ G $(.9^{-1} = 1.11)$

Determine the growth/decay factor.

5) $y = 10(1.35)^x$

Growth → .35

6) $y = 742(0.60)^x$

Decay → .40

Determine the growth/decay percent.

7) $y = (1.04)^x$

Growth 4%

8) $y = 7500(0.42)^x$

Decay → 38%

9) A new SUV depreciates at a rate of 23% per year. If the original selling price was $30,000, how much will the vehicle be worth after 4 years?

$y = 30{,}000(1-.23)^4$

10) Two bacteria are discovered at the bottom of a shoe. If the bacteria multiply at a rate of 34% per hour, how many bacteria will be present after 48 hours?

$y = 2(1+.34)^t$

$y = 2(1+.34)^{48}$

11) $3000 is deposited in an account that pays 4% annual interest compounded monthly. How much will be in the account after 20 years?

$$y = 3000\left(1 + \frac{.04}{12}\right)^{12 \cdot 20}$$

12) 10,000 molecules of radioactive material are present in the atmosphere and will dissipate at a rate of 24% per day. How many molecules will be present after one week?

$$y = 10{,}000(1 - .24)^{7}$$

13) The value of a $100,000 house in a prime location appreciates (increases in value) at a rate of 4% per year. How much will this house be worth in 7 years?

$$y = 100{,}000(1 + .04)^{7}$$

14) You have deposited $500 in an account that pays 6.75% interest, compounded continuously. What is the balance in the account after 15 years?

$$y = 500e^{.0675t}$$

$$y = 500e^{.0675(15)}$$

15) A retiree needs $100,000 by the time she retires in 2035. How much should he deposit now in an account that pays 6% annual interest compounded quarterly (the current year is 2014)

$$100{,}000 = a\left(1 + \frac{.06}{4}\right)^{4(21)}$$

$$100{,}000 = a(3.49)$$

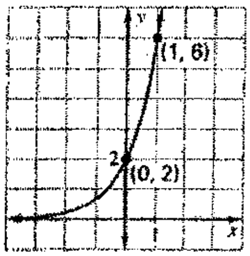
$$a \approx \$28{,}632.05$$

Math II Exponential Functions Review

Name:________________________
Date:________________ Period:____

Match the function with its graph.

1. $f(x) = 3^x$ F
2. $f(x) = -3^x$ B
3. $f(x) = 2(3^x)$ A
4. $f(x) = \frac{1}{2}(3^x)$ E
5. $f(x) = -\frac{1}{2}(3^x)$ C
6. $f(x) = -2(3^x)$ D

A.


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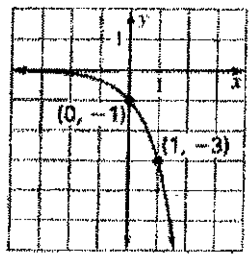

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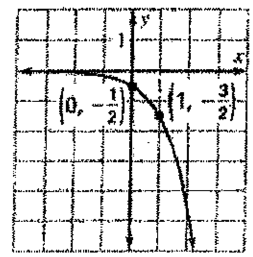

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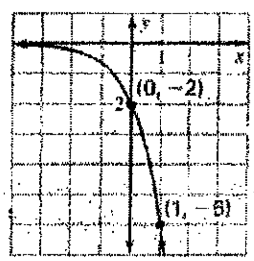
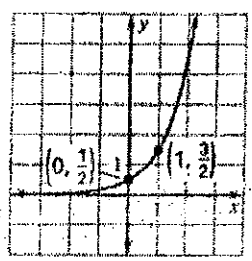
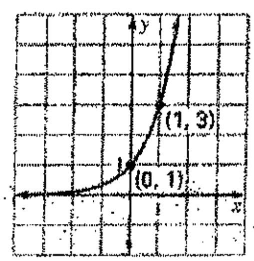


E.


F.


Determine whether the function is exponential growth or decay. State the asymptote of the graph as well.

7. $y = 2(3)^{x+1} - 3$

G
$y = -3$

8. $y = \frac{1}{2}(3)^x + 5$

G
$y = 5$

9. $y = -3(\frac{1}{2})^x - \frac{3}{4}$

D
$y = -3/4$

10. $y = 2(\frac{9}{5})^x$

G
$y = 0$

11. $y = -(\frac{1}{3})^{-x} + 9$

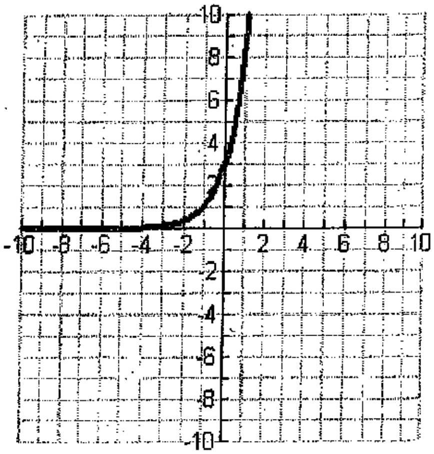
G
$y = 9$

12. $y = -\frac{1}{4}(2)^{-x}$

D
$y = 0$

$\left(\frac{1}{3}^{-x} = 3^x\right)$ $\left(2^{-x} = \frac{1}{2}^x\right)$

List the characteristics of the following exponential functions $y = 3(3)^x$



13.
Range: $(0, \infty)$

Asymptotes: $y = 0$

Intercept y-inter $(0, 3)$

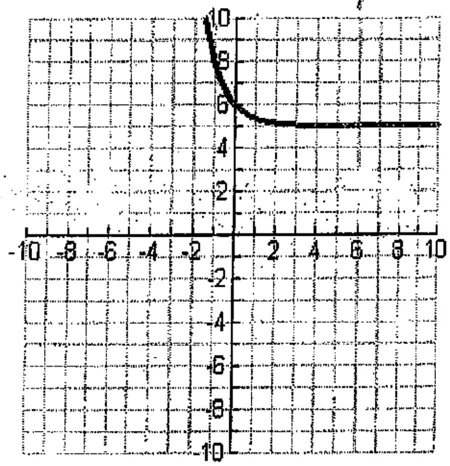
Zeros: none

End Behavior $x \to -\infty, f(x) \to 0$
$x \to \infty, f(x) \to \infty$

Rate of change for $-2 \le x \le 2$
$(-2, \frac{1}{3})$ $(2, 27)$

$m = \dfrac{27 - \frac{1}{3}}{2 - -2} = \dfrac{26.67}{4}$

14. $y = (\frac{1}{3})^x + 5$



Domain: $\mathbb{R}$

Range: $(5, \infty)$

Asymptotes: $y = 5$

Intercept $(0, 6)$

Zeros: none

End Behavior $x \to -\infty, f(x) \to \infty$
$x \to \infty, f(x) \to 5$

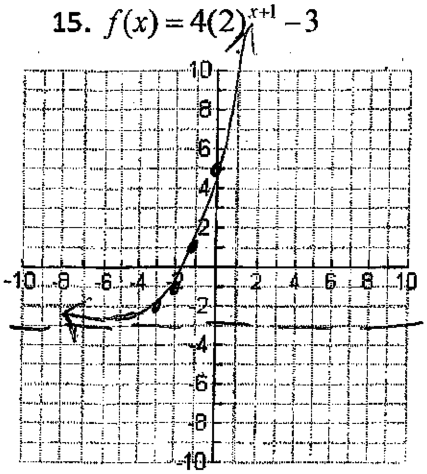
Rate of change for $-2 \le x \le 2$
$(-2, 14)$ $(2, 5.11)$

$m = \dfrac{5.11 - 14}{2 - -2} = \dfrac{-8.9}{4}$

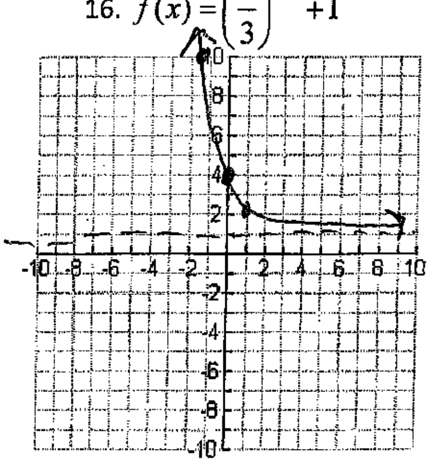
Graph the following exponential functions

15. $f(x) = 4(2)^{x+1} - 3$

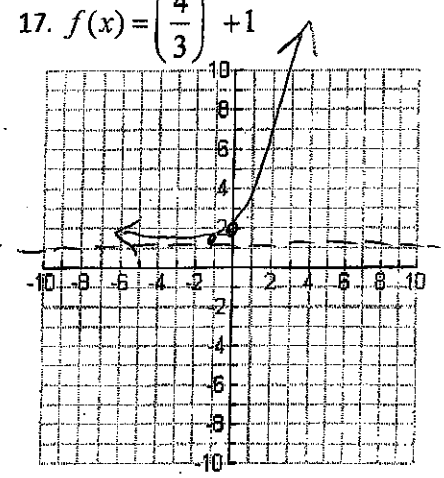
x	y
-3	-2
-2	-1
-1	1
0	5
1	13



16. $f(x) = \left(\frac{1}{3}\right)^{x-1} + 1$



17. $f(x) = \left(\frac{4}{3}\right)^x + 1$



18. Write all transformations that have taken place in the equation $f(x) = -2(3)^{x-4} + 2$

$-$ → reflects over x-axis

2 → vertical stretch

-4 → rt 4

$+2$ → up 2

19. Write the two money equations from memory. When do you use each equation?

Appreciate $y = a(1+r)^t$ $\quad$ $y = Pe^{rt}$

Depreciate $y = a(1-r)^t$ $\quad$ $y = a\left(1+\frac{r}{n}\right)^{nt}$

20. You invest $2,350 in an account for 6 years. How much will you have in the account if the account is...**MAKE SURE YOU WRITE YOUR EQUATION TO GET CREDIT!**

a. 6% compounded weekly $y = 2350\left(1+\frac{.06}{52}\right)^{52(6)}$ $\boxed{y \approx \$3367.63}$	b. 2% compounded annually $y = 2350(1+.02)^6$	c. 3.75% compounded monthly $y = 2350\left(1+\frac{.0375}{12}\right)^{12(6)}$ $\boxed{y = \$2941.93}$
d. 6.2% compounded semi-annually $y = 2350\left(1+\frac{.062}{2}\right)^{2(6)}$	e. 10% compounded bi-weekly $y = 2350\left(1+\frac{.10}{26}\right)^{26(6)}$	f. 7% compounded continuously $y = 2350e^{.07(6)}$ $\boxed{y = \$3576.61}$

21a. In 2020 you want to have $6,000 saved in your bank account. How much should you invest in 2011 into an account with 5% interest compounded quarterly? $6000 = a\left(1+\frac{.05}{4}\right)^{4(9)}$	21b. In 2010 you have $8,000 saved in your bank account. How much did you invest in 1995 if your account has 8% interest compounded annually? $8,000 = a(1+.08)^{2010-1995}$ $\frac{8,000}{3.17} = \frac{a(3.17)}{3.17}$ $\boxed{a \approx \$2521.93}$

$6000 = a(1.56)$

$\boxed{a \approx \$3836.45}$

22a. Your cell phone is worth \$250 when you buy it in 2011. What will the phone be worth in 4 years if it depreciates by 9% each year? $y=250(1-.09)^4$	22b. Your Barbie collection appreciates in value by 3% each year. If it is worth \$110 now, what will it be worth in 10 years? $y=110(1+.03)^{10}$
22c. Your grandfather gives you an old set of baseball cards that appreciate in value by 4.5% each year. An appraiser said the set was worth \$955 this year! What was the set worth when it was given to you 6 years ago? $955=a(1+.045)^6$ $955=a(1.30)$ $\boxed{a \approx \$733.34}$	22d. Your ITOUCH just sold on ebay for the price it is currently worth- \$50. If you bought it in 2006 and it has depreciated in value by 7.5% each year, what was it worth when you bought it? $50=a(1-.075)^{2021-2006}$ $50=a(.31)$ $a \approx \$161.00$

23. The equation $y=252(1.035)^t$ **models the amount of cockroaches in your basement from 2010 to 2020.**

a. How many cockroaches were in your basement in 2010? $y=252(1.035)^0=252$

b. How many cockroaches will be in your basement in 2018? $y=252(1.035)^8=331$

c. How many cockroaches will be in your basement in 2020? $y=252(1.035)^{10}=355$

d. What percentage do the cockroaches increase by in your basement?

3.5% Increase