

Average Rate of Change

Notes:

Rates are used to describe how one quantity is changing in relation to another. This is called a "rate of change" or an "average rate of change." To illustrate this, consider the following statement: Reagan drove from Salt Lake to Bluffdale (a distance of about 28 miles) in 30 minutes.

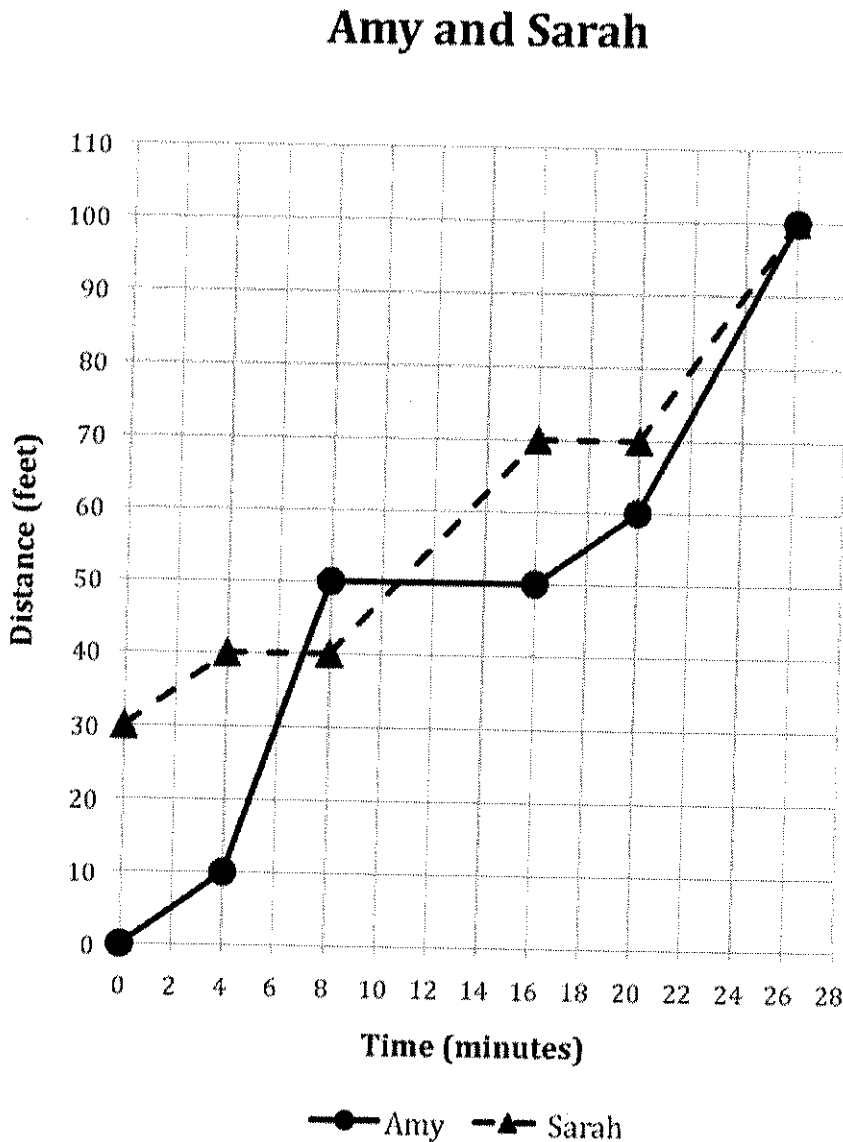
A.) What was his average speed in miles per hour?

$$\frac{28 \text{ miles}}{30 \text{ min}} \cdot \frac{2}{2} = \frac{56 \text{ miles}}{1 \text{ min}}$$

B.) Does this mean that he drove that speed the entire trip? If not, what does it mean?

C.) Did he ever drive the average speed of 56 mph?

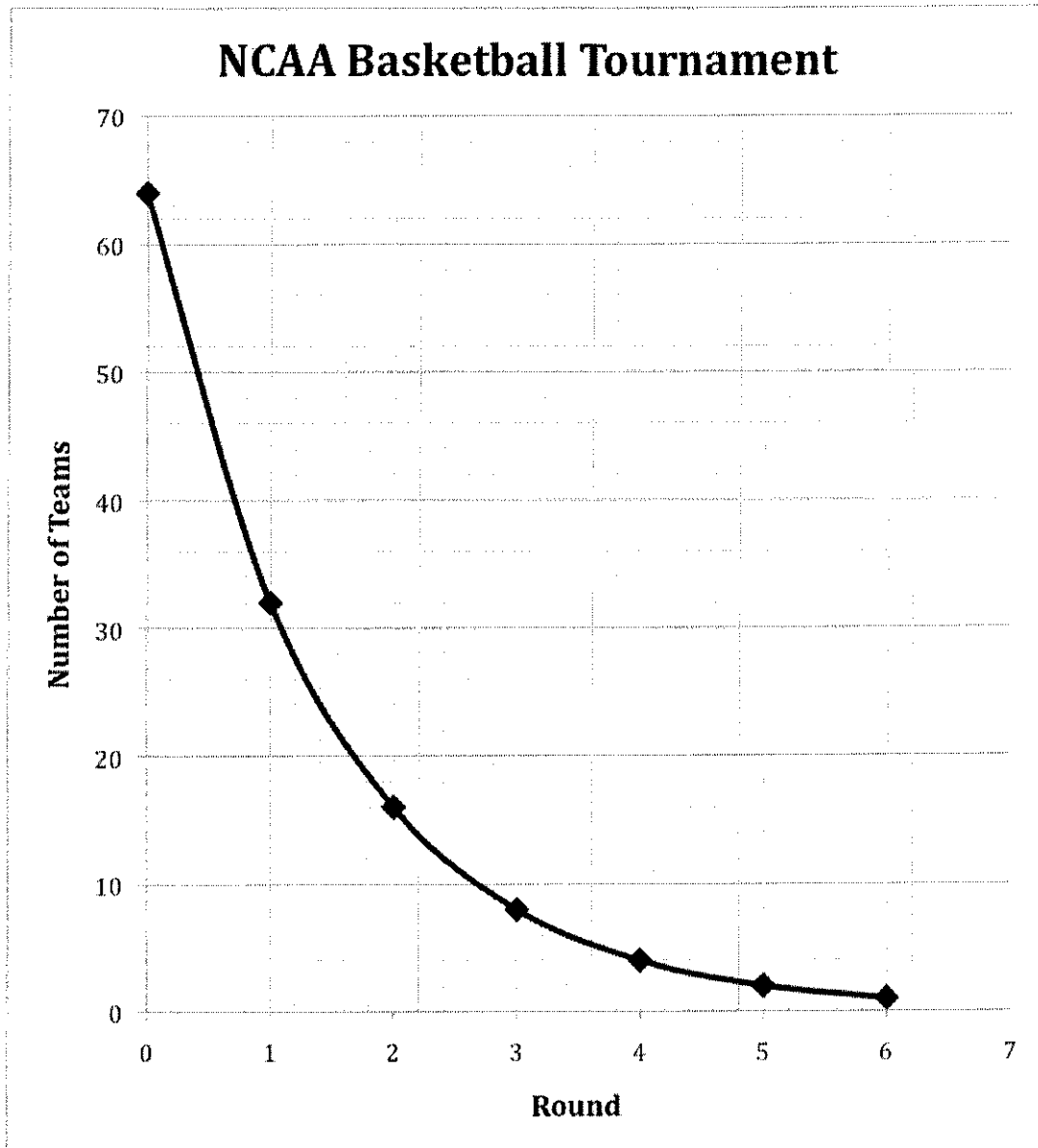
Ex: Amy and Sarah are meeting each other at the store.



Name: _____ Period: _____

Use the graph to answer the following questions.

- 1.) How far from the store is Amy at the beginning?
- 2.) How far from the store is Sarah at the beginning?
- 3.) How long does it take to get to the store?
- 4.) What happens between 6 and 7 minutes?
- 5.) Where is Amy moving faster?
- 6.) Where is Sarah moving faster?
- 7.) What is the speed of Amy between 4 and 8 minutes?
- 8.) What is the speed of Sarah 8 and 16 minutes?
- 9.) What is Amy doing during 8 and 16 minutes?
- 10.) What is Amy's average speed for the whole trip?
- 11.) What is Sarah's average speed for the whole trip?



Use the graph to answer the following questions.

- 1.) How many teams are there when the tournament starts?
- 2.) How many rounds occur before there is a winner?
- 3.) What is the rate of change between the 1st and 2nd round?
- 4.) What is the rate of change between the 2nd and 3rd round?
- 5.) What is the rate of change between the 3rd and 4th round?

Name: _____ Period: _____

6.) What is the average rate of change between the 1st and 4th round?

7.) What is the average rate of change from the beginning of the tournament to the end?

8.) The NCAA tournament chairman is considering adding another round to the tournament so more teams can participate. How many teams would start the tournament?

Notes - 3.4B Rate of Change

Ex: What is the average rate of change of the function $g(x) = 6 - 2x$

A.) Over the interval $[2, 6]$?

B.) Over the interval $[5, 7]$?

C.) Do you think it is true that $g(x)$ will have a constant average rate of change over *any* interval? Why or why not?

Ex: What is the average rate of change of the function $f(x) = 2^x$

A.) Over the interval $[1, 4]$?

B.) Over the interval $[3, 5]$?

C.) Do you think it is true that $f(x)$ will have a constant average rate of change over any interval? Why or why not?

Ex: Given a table, find the rate of change for each interval.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

A.) $[0, 3]$

B.) $[-2, 1]$

C.) $[-3, -1]$

3.4B Rate of Change

For each of the following functions, is the average rate of change over the given interval positive or negative? Justify your answer.

1.) $f(x) = 3 - 2x$ over the interval $[2, 3]$.

2.) $h(x) = 0.5^x$ over the interval $[-1, 0]$.

3.) $g(x) = 1.5^x$ over the interval $[-1, 0]$.

4.) $k(x) = 4 + 3x$ over the interval $[-2, 3]$.

5.) $m(x) = 7 - 3x$ over the interval $[2, 5]$.

Given a table, find the rate of change for each interval.

11.)

x	y
-2	-1
-1	-.5
0	0
1	.5
2	1
3	1.5
4	2

A.) $[1, 4]$

B.) $[-2, 2]$

C.) $[0, 3]$

12.)

x	y
-3	13
-2	9
-1	5
0	1
1	-3
2	-7
3	-11

A.) $[-2, 0]$

B.) $[1, 3]$

Describe how the following graphs have been transformed from their original parent graph.

Original graph: $f(x) = 5^x$

- | | |
|---------------------------------------|----------|
| 1. $f(x) = 5^{x-1}$ | 1. _____ |
| 2. $f(x) = 5^x + 2$ | 2. _____ |
| 3. $f(x) = -5^x$ | 3. _____ |
| 4. $f(x) = 3(5)^x$ | 4. _____ |
| 5. $f(x) = -\frac{1}{2}(5)^{x+2} - 3$ | 5. _____ |

Original graph $y = 2^x$

- | | |
|-------------------------------|-----------|
| 6. $y = 2^{x-6} + 1$ | 6. _____ |
| 7. $y = -2^{x+3} - 3$ | 7. _____ |
| 8. $y = \frac{1}{3}(2)^x + 8$ | 8. _____ |
| 9. $y = (-4)2^{x-1}$ | 9. _____ |
| 10. $y = 8 \cdot 2^{x+2} - 5$ | 10. _____ |

Write an equation that has transformed $y = (\frac{1}{2})^x$ in the way that is described.

- | | |
|---|-----------|
| 1. Up 6, to the left 1. | 11. _____ |
| 2. Reflected over the x axis, right 2, down 3 | 12. _____ |
| 3. Stretch of 5, up 2 | 13. _____ |
| 4. Reflected over the x axis, shrink of 1/3, left 6, up | 14. _____ |
| 5. Stretch of 2, right 3, down 4 | 15. _____ |

Topic: Transformations of graphs

What is it?

Shifting, stretching, shrinking, and reflecting of graphs

Types:

Vertical or
Horizontal shift

Add outside $y = 2^x + 3$
MOVES _____

Subtract outside $y = 2^x - 3$
MOVES _____

Add inside $y = 2^{(x+3)}$
MOVES _____

Subtract inside $y = 2^{(x-3)}$
MOVES _____

Examples

Vertical Stretch
or Shrink

Multiply by
Fraction (less
than 1) $y = \frac{1}{4}(2)^x$
Causes the graph to _____

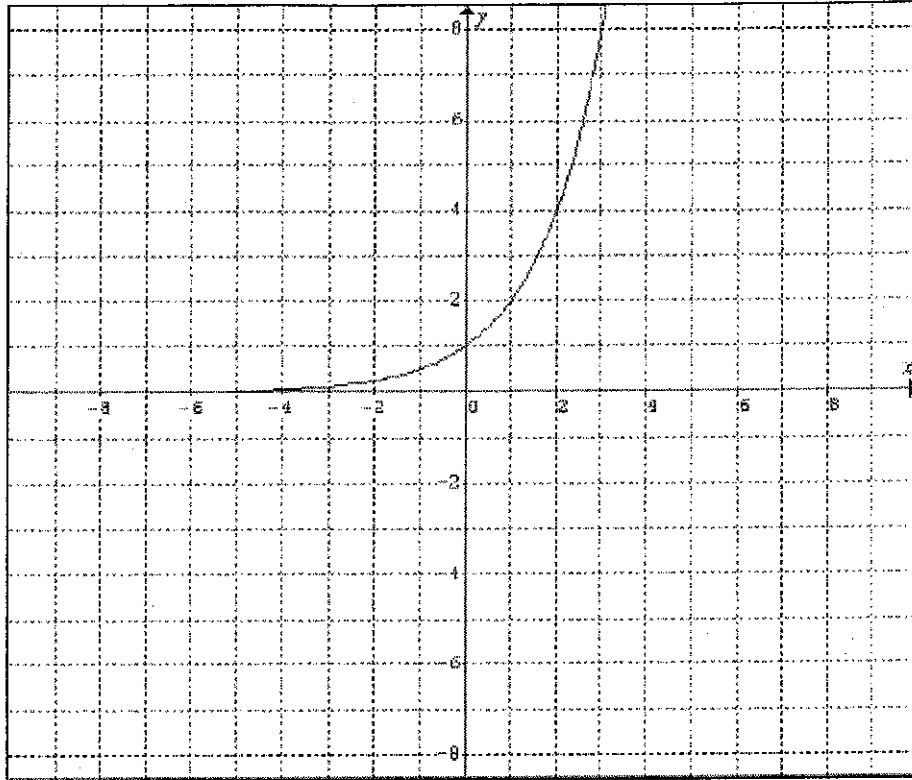
Multiply by
integer $y = 4(2)^x$
Causes the graph to _____

Reflection

Multiply by
negative (-) $y = -2^x$
Causes the graph to _____

Transformations of Exponential Functions

A. Given the graph of the exponential function $f(x) = 2^x$:



B. Graph the following functions on the specified graphs. Describe the transformations applied to the graph of $f(x)$ to obtain the graph of each of the $g(x)$, $h(x)$, $m(x)$, $n(x)$, $p(x)$, $s(x)$ and $t(x)$ functions.

$$g(x) = \frac{1}{3}(2^x)$$

Transformation:

$$h(x) = -\frac{1}{3}(2^x)$$

Transformation:

$$m(x) = 2^{3x}$$

Transformation:

$$n(x) = 2^{-3x}$$

Transformation:

$$p(x) = 2^x - 5$$

Transformation:

$$s(x) = 2^{x-5}$$

Transformation:

$$t(x) = -3(2)^{x-5} - 4$$

Transformation:

Describing and Writing Transformations of Functions

1. Identify a, b, c, and d and describe what transformation(s) the graphs of the function $f(x) = 4^x$ has undergone in each of the following cases.

$$g(x) = 4^{x-1}$$

$$h(x) = 5(4^{x+4})$$

$$m(x) = 4^x + 5$$

$$n(x) = \frac{4^x}{2}$$

$$r(x) = 3(4^{-x}) - 1$$

$$t(x) = -\frac{1}{3}(4^x) - 2$$

2. Sketch the graphs of the transformations using the given parent function, $f(x) = 3^x$. Label each one.

$$g(x) = 3^{x-2}$$

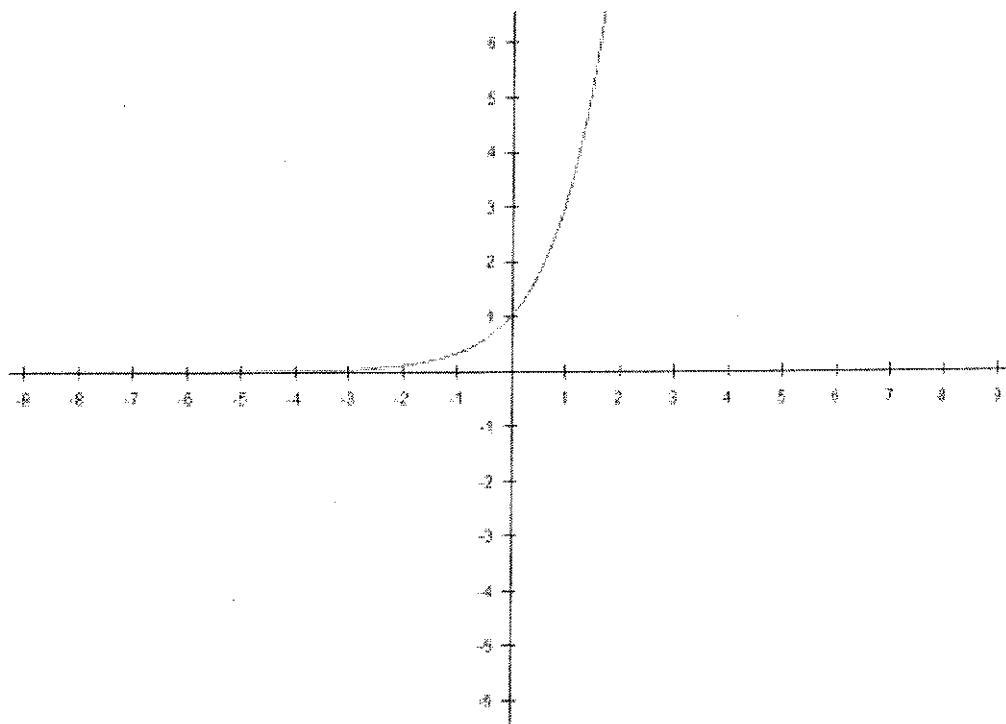
$$h(x) = -3^x$$

$$j(x) = 3^x + 4$$

$$k(x) = 2(3^x)$$

$$m(x) = 3^{x+3} - 2$$

$$n(x) = 3^{-x}$$



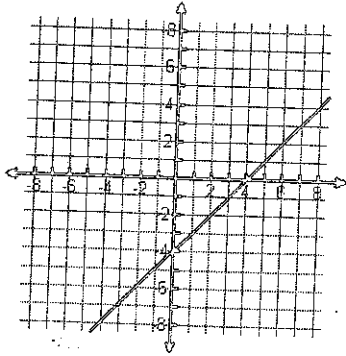
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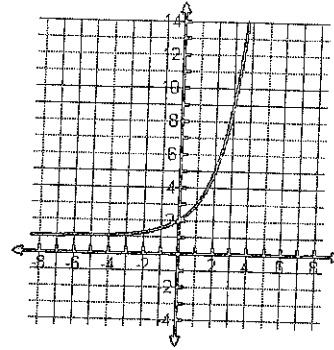
Transformations Homework

Given the following function, $f(x)$, sketch the graph of the function $g(x)$.

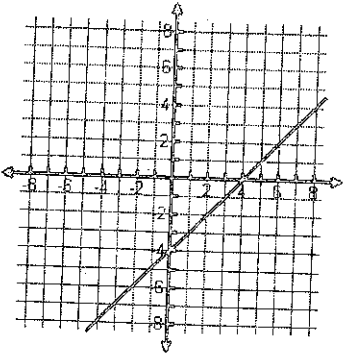
1) $g(x) = f(x) + 6$



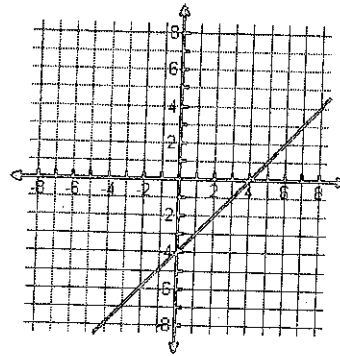
2) $g(x) = f(x) - 3$



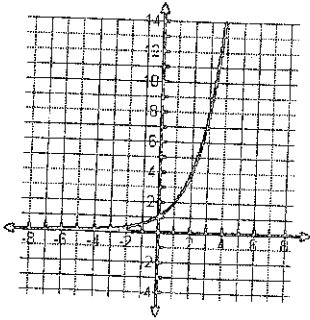
3) $g(x) = -f(x)$



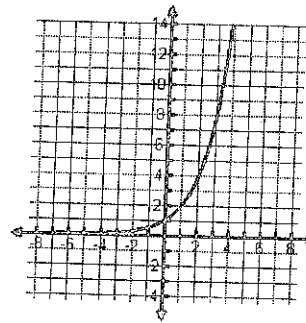
4) $g(x) = -f(x) - 2$



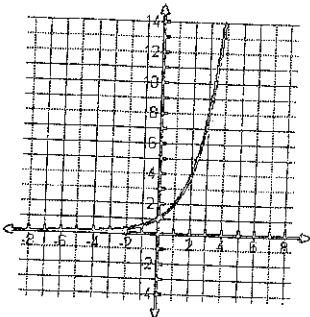
5) $g(x) = 2f(x)$



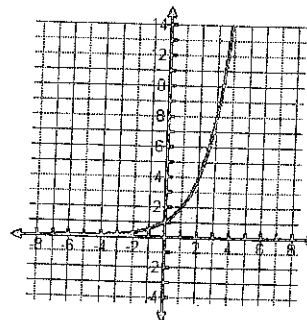
6) $g(x) = \frac{1}{2}f(x)$



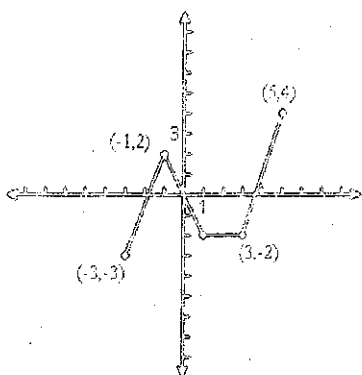
7) $g(x) = -2f(x)$



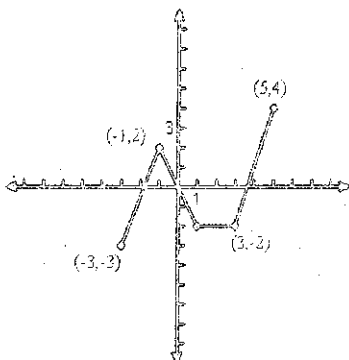
8) $g(x) = \frac{1}{2}f(x) - 6$



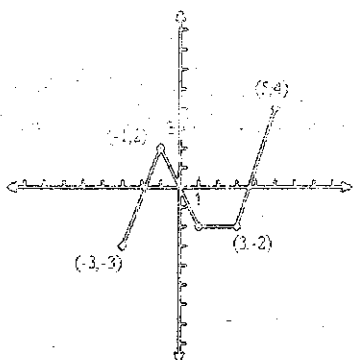
9) Graph $f(x) + 2$ on the graph, and find its domain and range.



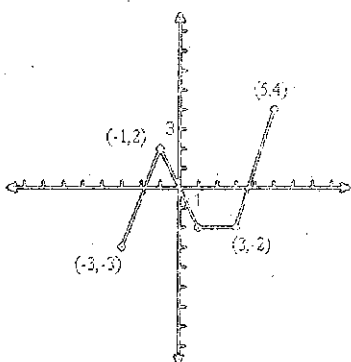
10) Graph $-f(x) - 4$ on the graph, and find its domain and range.



11) Graph the function $2f(x)$ on the graph and find its domain and range.



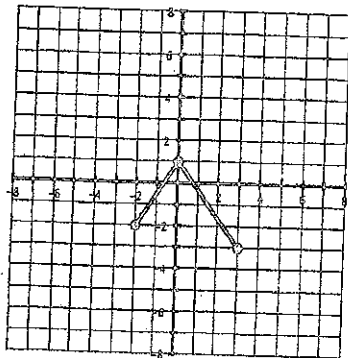
12) Graph the function $2f(x) - 2$ on the graph and find its domain and range.



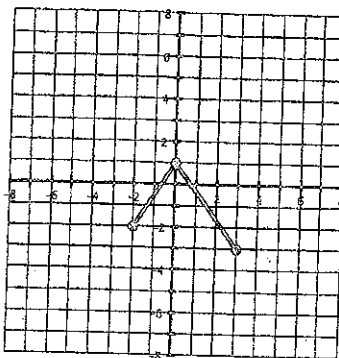
Accelerated Coordinate Algebra and Geometry
 WS 27 | Function Transformations

Each of the following drawings show the graph of function f . Function g is a transformation of f . State the transformation of f in words, then draw the new function g .

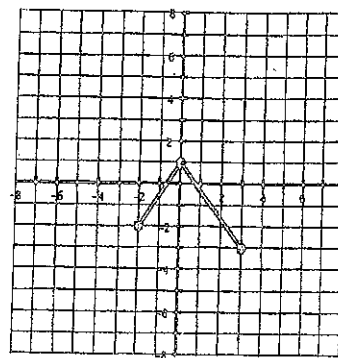
1. $g(x) = f(x+3)$



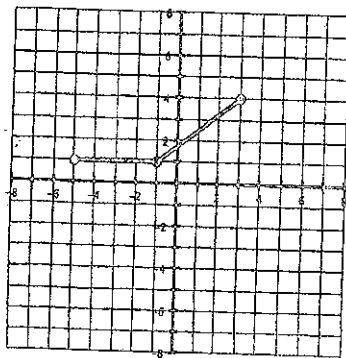
2. $g(x) = f(x)+4$



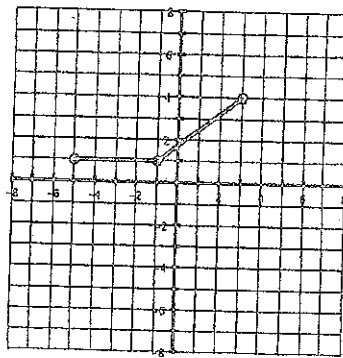
3. $g(x) = f(x-1)-2$



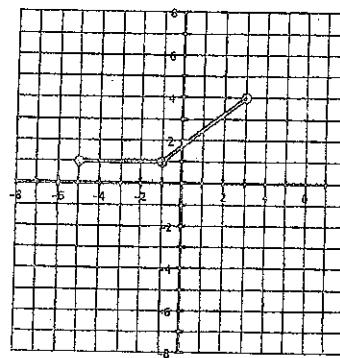
4. $g(x) = f(x)-5$



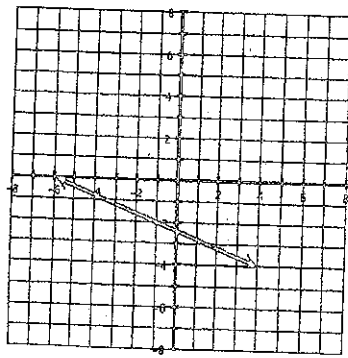
5. $g(x) = -f(x)$



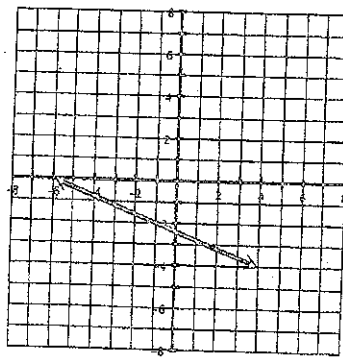
6. $g(x) = f(-x)$



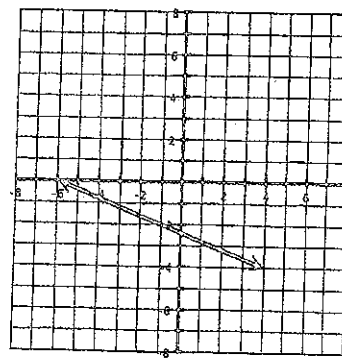
7. $g(x) = f(x-4)$



8. $g(x) = f(x)+2$



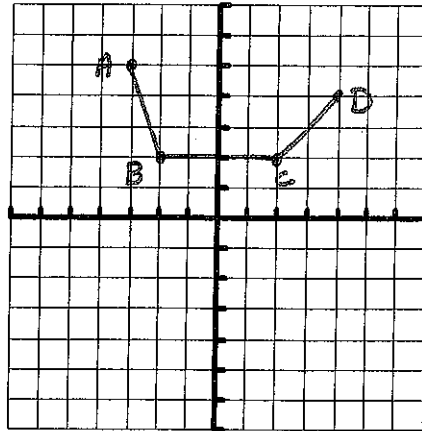
9. $g(x) = -f(x)$



Name: _____

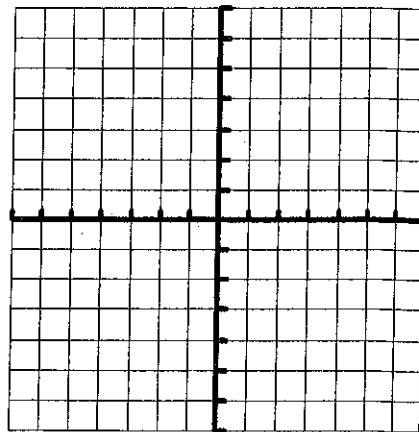
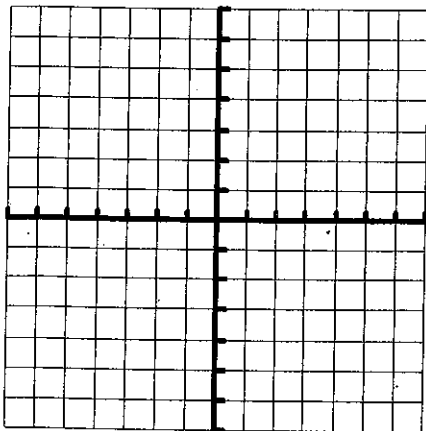
Please write out the new points and graph the new graph for each problem below.

Original Graph



1) $-2f(x-1)+3$

2) $\frac{1}{2}f(-\frac{1}{2}x)-3$



Exponential Functions

Growth and Decay Application Problems

Growth Example

$$y=(1.26)^x$$

rate:

percent:

Decay Example

$$y=(.80)^x$$

rate:

percent:

Oct 20-12:16 PM

Possible Equations and their uses:

$A=P(1+r)^t$ - used for appreciating values

$A=P(1-r)^t$ - used for the depreciating values

$A=P(1+r/n)^{nt}$ - used for values that are being compounded

monthly = 12
 weekly = 52
 daily = 365
 quarterly = 4
 semiannually = 2

$A=Pe^{rt}$ -used when a function is compounded continuously

P=principal (starting amount)

A=amount

r =rate (change to a decimal)

t = time

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Example: Ms. Benzin has a house that is worth \$200,000. It appreciates in value 5% each year. How much will it be worth in 5 years?

Which formula will we use?

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Example: Brian's car depreciates at a rate of 11% per year. If his car currently is worth \$16,500, how much will it be worth in 7 years?

Which formula will we use?

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Example: Jenni opened a savings account when she was 5 years old. She deposited \$200 and forgot about the account. The account pays 3.25% interest, compounded quarterly. Jenni is now 18 years old and just remembered she has the account. How much money is in the account now?

What formula will you use?

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Example: Sarah has \$2000 and wants to invest in an account which is continuously compounded. The rate is 5% and she plans to keep the money in her account for 10 years. How much money will she have at the end of the 10 years?

Which formula will you use?

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Working backwards:

Sammi's parents want to have \$125,000 by the time she graduates high school to pay for college (she will be 18 years old). How much money do they need to initially invest to obtain \$125,000 if they start the investment at year 1 of Sammi's life with an interest rate of 7%?

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Now you try!!!

Lets make up some problems.....one of each.....GO!!!
In your groups make an example of each!!!

Appreciate:

Depreciate:

Compounded:

Compounded Continuously:

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Exponential Application Problems

Determine whether the function represents exponential growth or decay.

1) $y = 350(0.75)^x$

2) $y = 80(1.03)^x$

3) $y = (1.87)^x$

4) $y = 500(0.9)^{-x}$

Determine the growth/decay factor.

5) $y = 10(1.35)^x$

6) $y = 742(0.60)^x$

Determine the growth/decay percent.

7) $y = (1.04)^x$

8) $y = 7500(0.42)^x$

9) A new SUV depreciates at a rate of 23% per year. If the original selling price was \$30,000, how much will the vehicle be worth after 4 years?

10) Two bacteria are discovered at the bottom of a shoe. If the bacteria multiply at a rate of 34% per hour, how many bacteria will be present after 48 hours?

11) \$3000 is deposited in an account that pays 4% annual interest compounded monthly. How much will be in the account after 20 years?

12) 10,000 molecules of radioactive material are present in the atmosphere and will dissipate at a rate of 24% per day. How many molecules will be present after one week?

13) The value of a \$100,000 house in a prime location appreciates (increases in value) at a rate of 4% per year. How much will this house be worth in 7 years?

14) You have deposited \$500 in an account that pays 6.75% interest, compounded continuously. What is the balance in the account after 15 years?

15) A retiree needs \$100,000 by the time she retires in 2035. How much should he deposit now in an account that pays 6% annual interest compounded quarterly (the current year is 2014)?

Name: _____ Date: _____

Exponential Growth and Decay Practice

$$\textit{Growth: } y = P(1 + r)^t$$

$$\textit{Decay: } y = P(1 - r)^t$$

1. You deposit \$1500 in an account that pays 5% interest compounded yearly. Find the balance after 6 years.
2. The mice population is 25,000 and is decreasing by 20% each year. Write a model for this situation. What will be the mice population after 3 years?
3. The number of mosquitoes at the beach has tripled every year since 1999. In 1999, there were 2,500 mosquitoes. Write a model for this situation. How many mosquitoes would you predict were at the beach in 2005?
4. Given the exponential model $y = 200(.80)^x$, tell whether the model represents exponential growth or decay. Then, tell what the growth/decay factor is and the growth/decay percent.
5. I bought a car for \$25,000, but its value is depreciating at a rate of 10% per year. How much will my car be worth after 8 years?

Name: _____ Date: _____

Exponential Growth and Decay Homework

Growth: $y = P(1 + r)^t$

Decay: $y = P(1 - r)^t$

- 1) Given the equation $y = 35(0.57)^x$
 - a) Does this equation represent growth or decay? _____
 - b) What is the rate of growth or decay? _____
 - c) What is the initial value? _____
 - d) Evaluate for $x = 5$ _____

- 2) Given the equation $y = 225(1.23)^x$
 - a) Does this equation represent growth or decay? _____
 - b) What is the rate of growth or decay? _____
 - c) What is the initial value? _____
 - d) Evaluate for $x = 2$ _____

- 3) Given the equation $y = 154(1.06)^x$
 - a) Does this equation represent growth or decay? _____
 - b) What is the rate of growth or decay? _____
 - c) What is the initial value? _____
 - d) Evaluate for $x = 7$ _____

- 4) Ryan is saving for his college tuition. He has \$2,550 in a savings account that pays 6.25% annual interest.
 - a) Write an exponential equation describing this situation. _____
 - b) How much money will Ryan have in his account 6 years from now?

- 5) A used car was purchased for \$12,329 this year. Each year the car's value decreases 8.5%.
 - a) Write an exponential equation describing this situation. _____
 - b) What will the car be worth in 2020?

- 6) Jeremiah owns a business. His first year he made \$11,212, each of the following years his profit increased 12%.
 - a) Write an exponential equation describing the situation. _____
 - b) What will he make in 20 years?

7) Dianna just bought a home. She paid \$240,000. She is able to pay 20% of the loan off each year.

a) Write an exponential equation describing the situation. _____

b) What will she owe in 10 years?

8) A radioactive material decays at a rate of 40% per hour.

a) If we start with 80 grams of the substance, can you find a formula that models this rate of decay? _____

b) How much will be remaining at the end of 6 hours?

c) Will we have less than a gram before the end of the day? About how many hours does it take to decay to less than a gram?

9) You just won the lotto. You have two options, you can take the full \$10,000,000 now or they will pay you 1,000 dollars this year and then double the amount they pay you every year for 10 years. Which way will get you more money?

Compound Interest and e Worksheet

The history of mathematics is marked by the discovery of special numbers such as π or i . Another special number is denoted by the letter e . It is called the *Euler number* after its discoverer and it is also called the **natural base** e . Like π , it is an irrational number.

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e = 2.71828\dots$$

It is important to remember that e is JUST A NUMBER!

One use of e is for "continuously compounded interest."

$A(t) = Pe^{rt}$	<p>where P = principal investment</p> <p>r = interest rate (as a decimal)</p> <p>t = time</p>
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There is another formula we can use to calculate interest when it is not compounded continuously:

For compounding interest a specific # of times annually:

$A(t) = P \left(1 + \frac{r}{n}\right)^{nt}$	<p>where P = principal investment</p> <p>r = interest rate (as a decimal)</p> <p>t = time</p> <p>n = # of times you compound annually</p>
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- 1) If you invest \$2500 in an account, what is the balance in the account and the amount of interest after 4 years if you earn:
 - a) 1.7% interest compounded annually?
 - b) 1.5% compounded monthly?
 - c) 1.2% compounded daily?
 - d) 0.7% compounded continuously?

- 2) Martha makes an investment of \$500 in an account that pays 6% interest compounded monthly.
- Write an equation you could use to determine the interest she earns in t years.
 - How much money will Martha have in her account one year from now if she never withdraws any money and reinvests the interest?
 - What is the effective annual rate for this account (think about what percent of her money has she earned at the end of one year)?
- 3) A credit card company charges 12.9% annual interest.
- If they compound interest monthly, how much will you owe for every dollar you do not pay off for a year?
 - If they compound interest daily, how much will you owe for every dollar you do not pay off for a year?
 - What is the effective annual rate in the situation above?
- 4) An initial investment of \$700 is worth \$725 a year later. What is the effective annual yield for this account?
- 5) A loan shark lends a gambler \$1,000.00 to cover a debt. He charges 35% annual interest compounded continuously. How much does the gambler owe the loan shark at the end of one year? Two years?
- 6) The value of a \$25,000 car depreciates at a rate of 12% per year. What will the car be worth in 5 years?

Match the function with its graph.

1. $f(x) = 3^x$

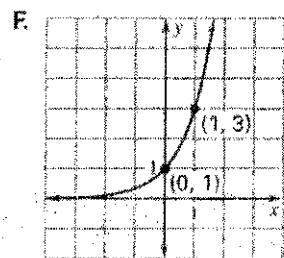
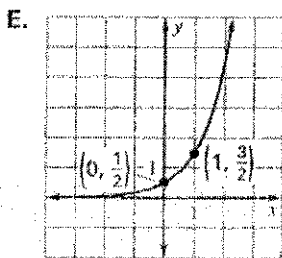
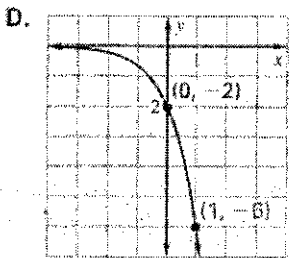
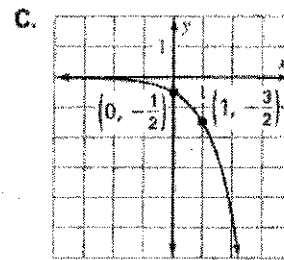
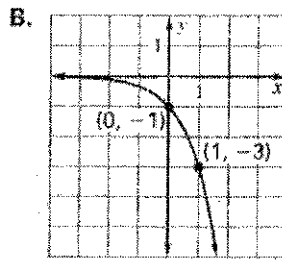
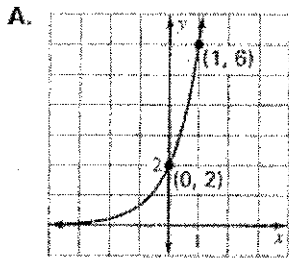
2. $f(x) = -3^x$

3. $f(x) = 2(3^x)$

4. $f(x) = \frac{1}{2}(3^x)$

5. $f(x) = -\frac{1}{2}(3^x)$

6. $f(x) = -2(3^x)$



Determine whether the function is exponential growth or decay. State the asymptote of the graph as well.

7. $y = 2(3)^{x+1} - 3$

8. $y = \frac{1}{2}(3)^x + 5$

9. $y = -3(\frac{1}{2})^x - \frac{3}{4}$

10. $y = 2(\frac{2}{3})^x$

11. $y = -(\frac{1}{3})^{-x} + 9$

12. $y = -\frac{1}{4}(2)^{-x}$

List the characteristics of the following exponential functions $y = 3(3)^x$

13.

Range: _____

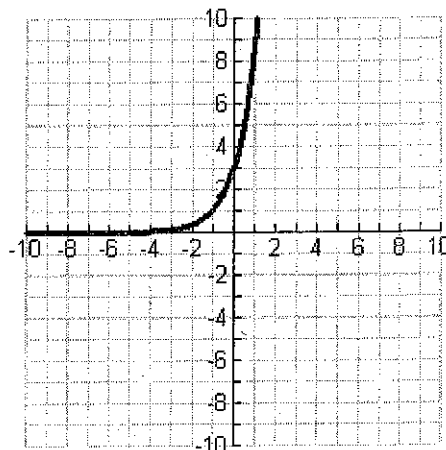
Asymptotes: _____

Intercept _____

Zeros: _____

End Behavior _____

Rate of change for $-2 \leq x \leq 2$ _____



14. $y = (\frac{1}{3})^x + 5$

Domain: _____

Range: _____

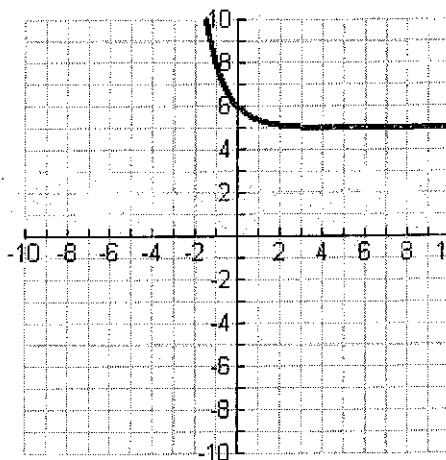
Asymptotes: _____

Intercept _____

Zeros: _____

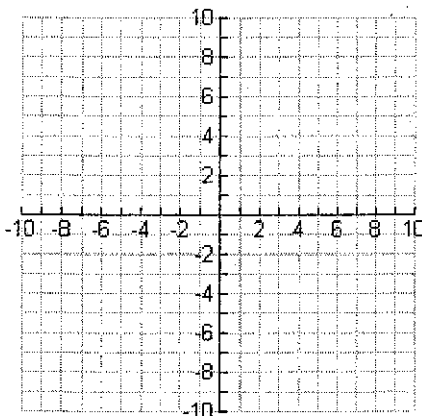
End Behavior _____

Rate of change for $-2 \leq x \leq 2$ _____

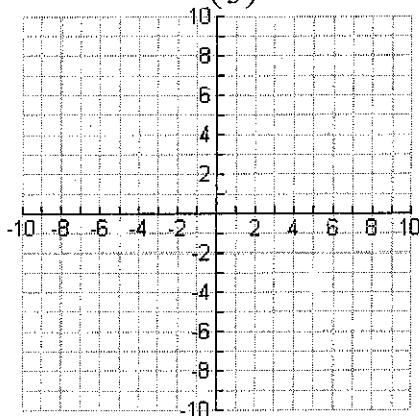


Graph the following exponential functions

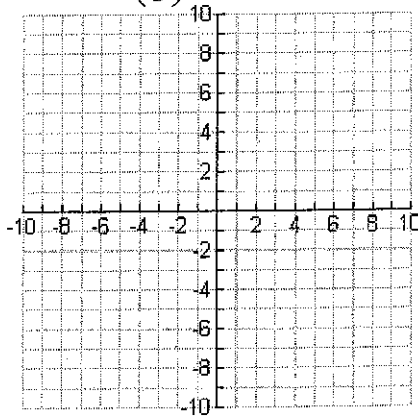
15. $f(x) = 4(2)^{x+1} - 3$



16. $f(x) = (\frac{1}{3})^{x-1} + 1$



17. $f(x) = (\frac{4}{3})^x + 1$



18. Write all transformations that have taken place in the equation $f(x) = -2(3)^{x-4} + 2$

19. Write the two money equations from memory. When do you use each equation?

20. You invest \$2,350 in an account for 6 years. How much will you have in the account if the account is...**MAKE SURE YOU WRITE YOUR EQUATION TO GET CREDIT!**

a. 6% compounded weekly	b. 2% compounded annually	c. 3.75% compounded monthly
d. 6.2% compounded semi-annually	e. 10% compounded bi-weekly	f. 7% compounded continuously
21a. In 2020 you want to have \$6,000 saved in your bank account. How much should you invest in 2011 into an account with 5% interest compounded quarterly?	21b. In 2010 you have \$8,000 saved in your bank account. How much did you invest in 1995 if your account has 8% interest compounded annually?	

<p>22a. Your cell phone is worth \$250 when you buy it in 2011. What will the phone be worth in 4 years if it depreciates by 9% each year?</p>	<p>22b. Your Barbie collection appreciates in value by 3% each year. If it is worth \$110 now, what will it be worth in 10 years?</p>
<p>22c. Your grandfather gives you an old set of baseball cards that appreciate in value by 4.5% each year. An appraiser said the set was worth \$955 this year! What was the set worth when it was given to you 6 years ago?</p>	<p>22d. Your ITOUCH just sold on ebay for the price it is currently worth- \$50. If you bought it in 2006 and it has depreciated in value by 7.5% each year, what was it worth when you bought it?</p>

23. The equation $y = 252(1.035)^t$ models the amount of cockroaches in your basement from 2010 to 2020.

- How many cockroaches were in your basement in 2010?
- How many cockroaches will be in your basement in 2018?
- How many cockroaches will be in your basement in 2020?
- What percentage do the cockroaches increase by in your basement?