

Key

3.4A Average Rate of Change

Notes:

Rates are used to describe how one quantity is changing in relation to another. This is called a **“rate of change”** or an **“average rate of change.”** To illustrate this, consider the following statement: Reagan drove from Salt Lake to Bluffdale (a distance of about 28 miles) in 30 minutes.

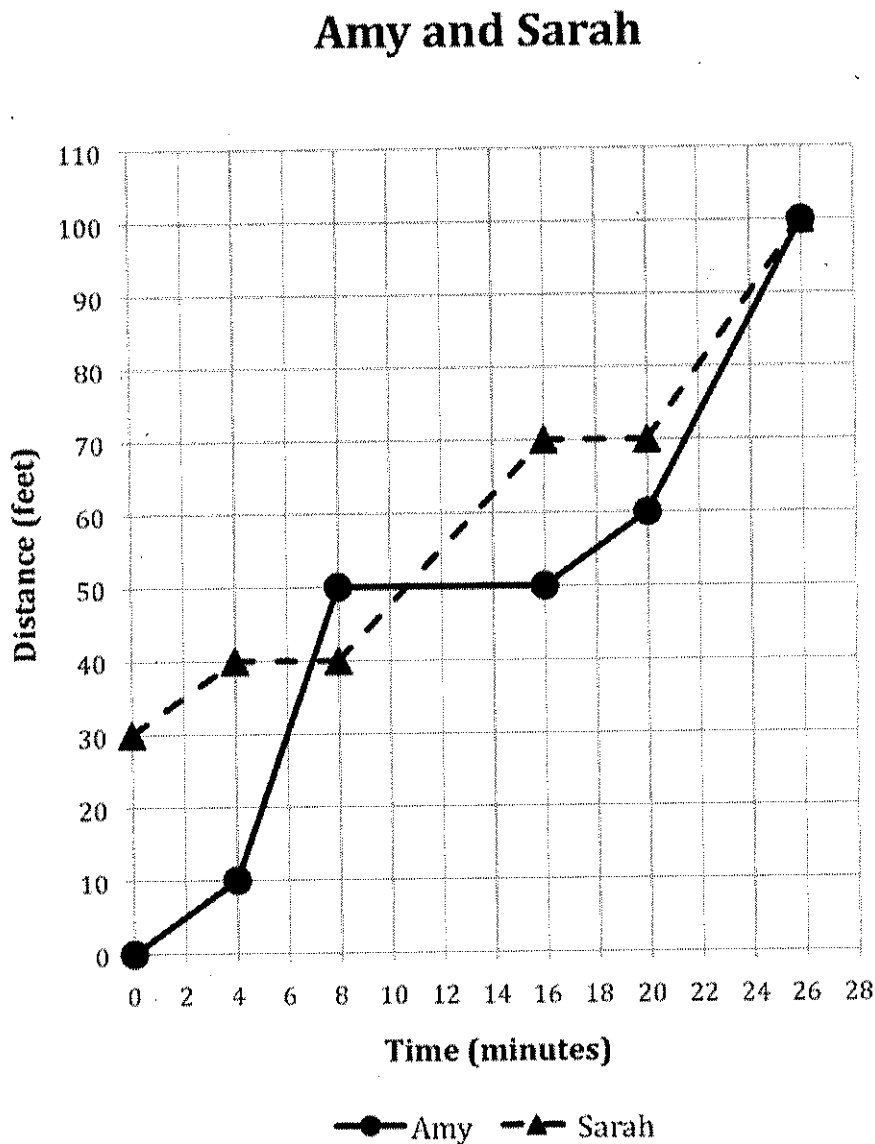
A.) What was his average speed in miles per hour?

$$\frac{28 \text{ miles}}{30 \text{ min}} \cdot \frac{2}{2} = \frac{56 \text{ miles}}{1 \text{ min}}$$

B.) Does this mean that he drove that speed the entire trip? If not, what does it mean? No,...

C.) Did he ever drive the average speed of 56 mph? Maybe....

Ex: Amy and Sarah are meeting each other at the store.

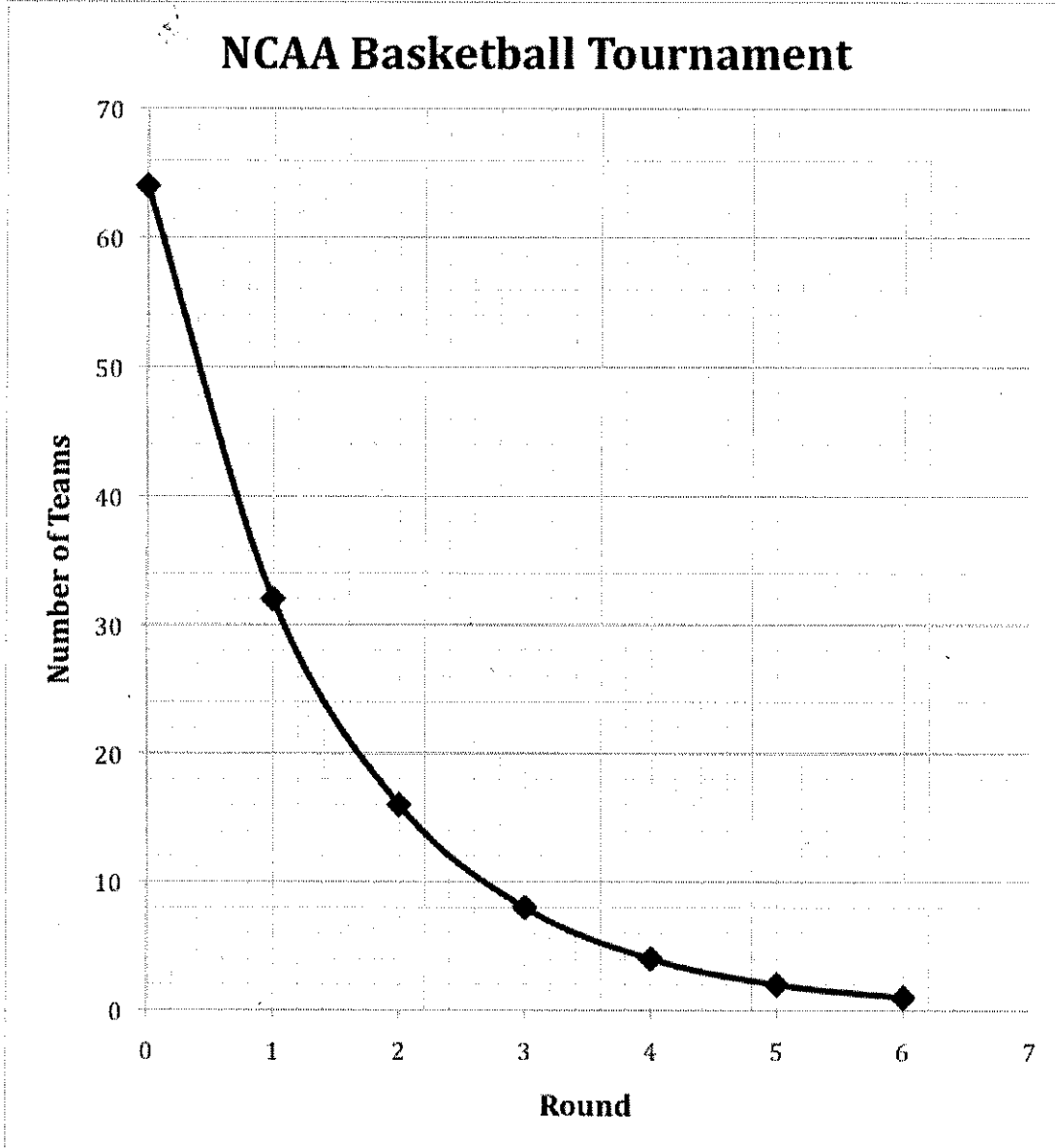


Name: _____ Period: _____

Use the graph to answer the following questions.

- 1.) How far from the store is Amy at the beginning?
100 ft away from the store
- 2.) How far from the store is Sarah at the beginning?
70 ft away from the store
- 3.) How long does it take to get to the store?
It took 26 min. to get to the store.
- 4.) What happens between 6 and 7 minutes?
They are the same distance away from the store, 60 ft away.
- 5.) Where is Amy moving faster?
Between 4 & 8 min. and 20 & 26 min.
- 6.) Where is Sarah moving faster?
Between 8 & 16 min.
- 7.) What is the speed of Amy between 4 and 8 minutes?
$$\frac{40 \text{ feet}}{4 \text{ min}} = \frac{10 \text{ feet}}{1 \text{ min}}$$
- 8.) What is the speed of Sarah 8 and 16 minutes?
$$\frac{30 \text{ feet}}{8 \text{ min}} = \frac{3.75 \text{ feet}}{1 \text{ min}}$$
- 9.) What is Amy doing during 8 and 16 minutes?
Answers vary: Ex: Maybe she stopped to talk on her cell phone.
- 10.) What is Amy's average speed for the whole trip?
$$\frac{100 \text{ feet}}{26 \text{ min}} = \frac{3.8 \text{ feet}}{1 \text{ min}}$$
- 11.) What is Sarah's average speed for the whole trip?
$$\frac{70 \text{ feet}}{26 \text{ min}} = \frac{2.7 \text{ feet}}{1 \text{ min}}$$

Ex: The NCAA basketball tournament.



Use the graph to answer the following questions.

- 1.) How many teams are there when the tournament starts?
64 teams
- 2.) How many rounds occur before there is a winner?
6 rounds
- 3.) What is the rate of change between the 1st and 2nd round?
 $-16/1$
- 4.) What is the rate of change between the 2nd and 3rd round?
 $-8/1$
- 5.) What is the rate of change between the 3rd and 4th round?
 $-4/1$

Name: _____ Period: _____

6.) What is the average rate of change between the 1st and 4th round?

$$-28/3$$

7.) What is the average rate of change from the beginning of the tournament to the end?

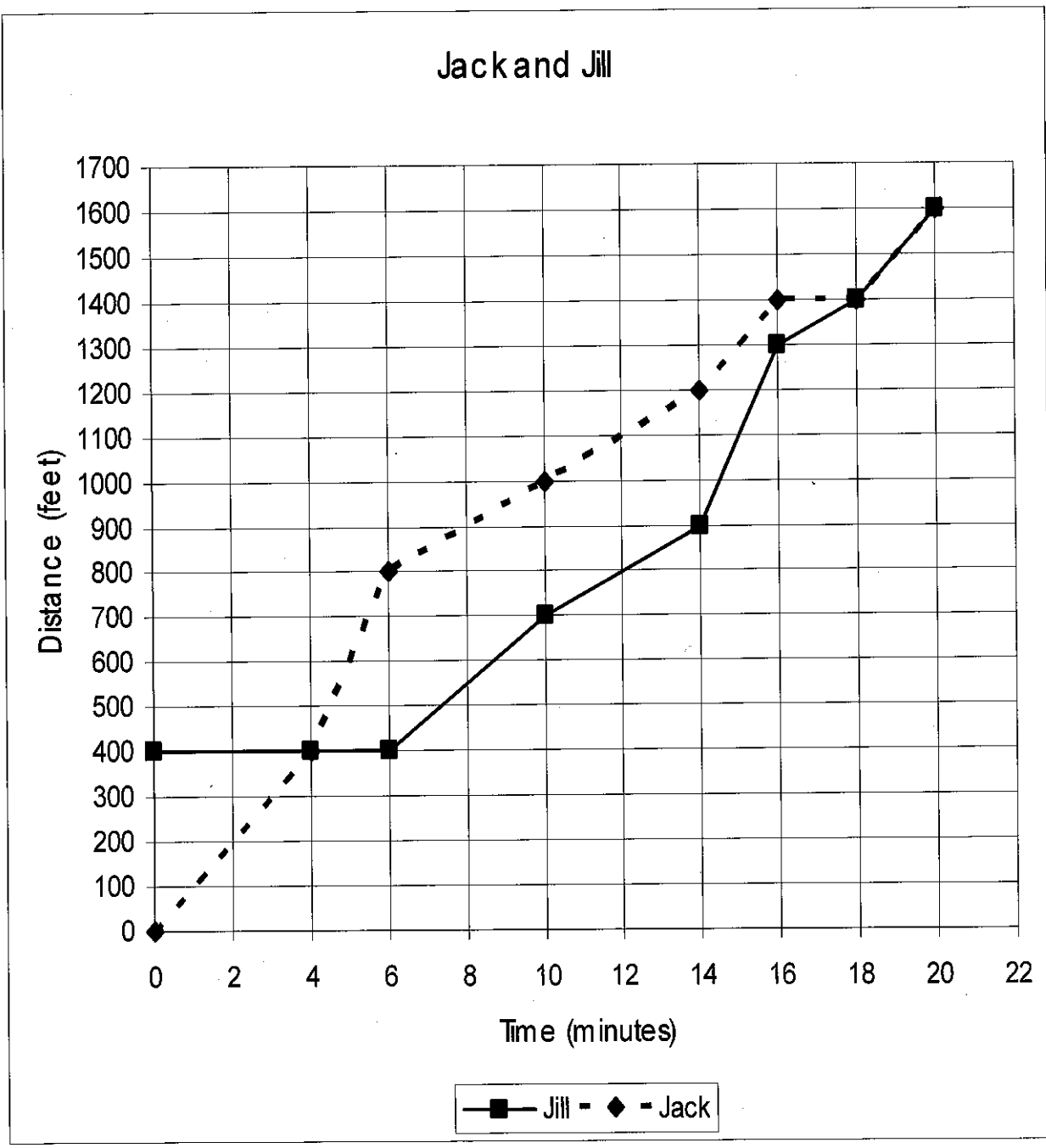
$$\frac{63}{6} = \frac{21}{2}$$

8.) The NCAA tournament chairman is considering adding another round to the tournament so more teams can participate. How many teams would start the tournament?

128 teams

3.4 Average Rate of Change

The graph below shows Jack and Jill's trip up the hill.



Use the graph to answer the following questions:

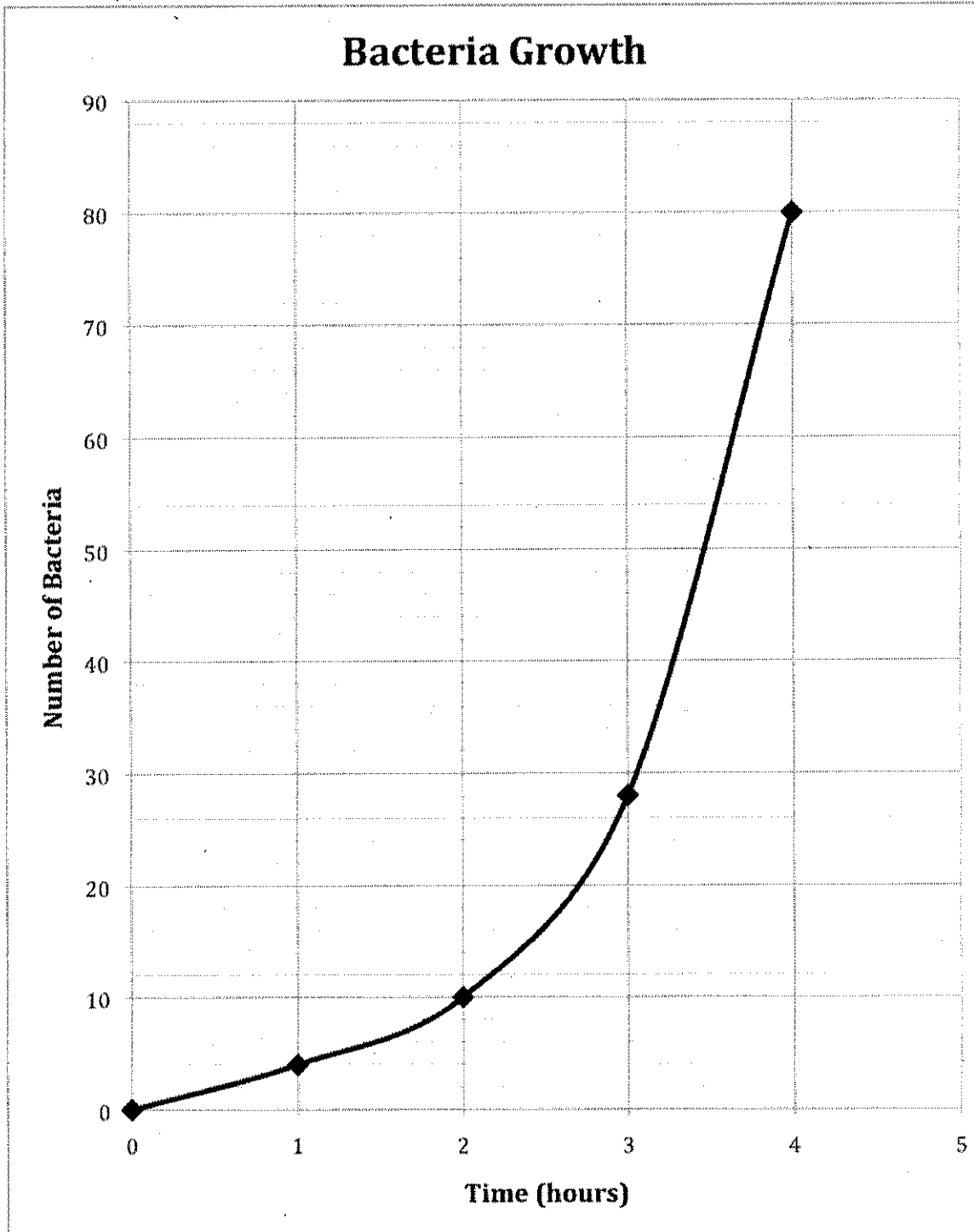
1. How far from the hill is Jack at the beginning? 1600 ft away
2. How far from the hill is Jill at the beginning? 1200 ft away

Name: _____ Period: _____

3. How long does the trip take? 20 min.
4. What is Jill doing for the first 6 minutes? Answers will vary: She stopped to talk.
5. What happens at minute 4? Jack & Jill are both 1200 ft away from the hill.
6. What is happening in the last 2 minutes? They are together, walking the same rate.
7. Where is Jack moving fastest? Between 4 and 16 min.
8. Where is Jill moving fastest? Between 14 and 16 min.
9. What is the speed of Jack between 10 and 14 minutes? $200/4 = 50$ ft per min
10. What is the speed of Jill between 6 and 10 minutes? $300/4 = 75$ ft per min
11. What is Jack doing during 16 and 18 minutes? Answers vary: He is stopped tying his shoes.
12. What is Jill's average speed for the whole trip? $1200/20 = 60$ ft per min
13. What is Jack's average speed for the whole trip? $1600/20 = 80$ ft per min

Name: _____ Period: _____

The graph below shows the number of bacteria in a colony is growing.



Use the graph to answer the following questions.

14.) How many bacteria were there to start? 0 bacteria

15.) What is the rate of change between 1 and 2 hours? $\frac{2 \text{ bacteria}}{1 \text{ hr}}$

Name: _____ Period: _____

- 16.) What is the rate of change between 2 and 3 hours? $70/2 = 35$ bacteria per hr
- 17.) Which interval of time has the biggest rate of change? Between 3 & 4 hours
- 18.) What is the average rate of change between 1 and 3 hours? $24/2 = 12$ bacteria per hr
- 19.) What is the average rate of change from 0 to 4 hours? $80/4 = 20$ bacteria per hr

Notes - 3.4B Rate of Change

key

Ex: What is the average rate of change of the function $g(x) = 6 - 2x$

A.) Over the interval $[2, 6]$?

Rate of change = -2 Use table, slope formula, or graph to find slope.

B.) Over the interval $[5, 7]$?

Rate of change = -2 Use table, slope formula, or graph to find slope.

C.) Do you think it is true that $g(x)$ will have a constant average rate of change over any interval? Why or why not?

Yes,...

Ex: What is the average rate of change of the function $f(x) = 2^x$

A.) Over the interval $[1, 4]$?

Rate of Change = $14/3$ Use table, slope formula, or graph to find slope.

B.) Over the interval $[3, 5]$?

Rate of Change = 12 Use table, slope formula, or graph to find slope.

C.) Do you think it is true that $f(x)$ will have a constant average rate of change over any interval? Why or why not?

No, ...

Ex: Given a table, find the rate of change for each interval.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

A.) $[0, 3]$ rate of change = 5

B.) $[-2, 1]$ rate of change = 1

C.) $[-3, -1]$ rate of change = -1

Name _____ Date _____ Period _____

3.4B Rate of Change

For each of the following functions, is the average rate of change over the given interval positive or negative? Justify your answer.

1.) $f(x) = 3 - 2x$ over the interval $[2, 3]$.

Negative rate of change. Use table or slope formula to justify rate of change = -2.

2.) $h(x) = 0.5^x$ over the interval $[-1, 0]$.

Negative rate of change. Use table or slope formula to justify rate of change = -2.

3.) $g(x) = 1.5^x$ over the interval $[-1, 0]$.

Positive rate of change. Use table or slope formula to justify rate of change = $1/3$.

4.) $k(x) = 4 + 3x$ over the interval $[-2, 3]$.

Positive rate of change. Use table or slope formula to justify rate of change = 3.

5.) $m(x) = 7 - 3x$ over the interval $[2, 5]$.

Negative rate of change. Use table or slope formula to justify rate of change = -3.

Given a table, find the rate of change for each interval.

11.)

x	y
-2	-1
-1	-5
0	0
1	.5
2	1
3	1.5
4	2

A.) $[1, 4]$ rate of change = $1/2$

$$\frac{.5 - 2}{1 - 4} = \frac{1}{2}$$

B.) $[-2, 2]$ rate of change = $1/2$

$$\frac{-1 - 1}{-2 - 2} = \frac{1}{2}$$

C.) $[0, 3]$ rate of change = $1/2$

$$\frac{0 - 1.5}{0 - 3} = \frac{1}{2}$$

12.)

x	y
-3	13
-2	9
-1	5
0	1
1	-3
2	-7
3	-11

A.) $[-2, 0]$ rate of change = -4B.) $[1, 3]$ rate of change = -4C.) $[-3, 1]$ rate of change = -4

13.)

x	y
-3	10
-2	5
-1	2
0	1
1	2
2	5
3	10

A.) $[1, 3]$ rate of change = 4B.) $[-1, 1]$ rate of change = 0C.) $[-3, 0]$ rate of change = -3

14.)

x	y
-2	-8
-1	-1
0	0
1	1
2	8
3	27
4	64

A.) $[-2, 4]$ rate of change = 12B.) $[-1, 3]$ rate of change = 7C.) $[0, 2]$ rate of change = 4

15.)

x	y
-4	.125
-3	.25
-2	.5
-1	1
0	2
1	4
2	8

A.) $[-2, 2]$ rate of change = $15/8$ B.) $[-4, 1]$ rate of change = $31/40$ C.) $[-3, 0]$ rate of change = $7/12$