

Accelerated Algebra I/Geometry A

Unit 5: Comparing and Contrasting Functions revised Agenda

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## Arithmetic Series

Evaluate the related series of each sequence.

1) 13, 15, 17, 19, 21, 23

108

2) 6, 11, 16, 21, 26, 31, 36

147

3) 22, 28, 34, 40, 46

170

4) 39, 49, 59, 69

216

Evaluate each arithmetic series described.

5)  $\sum_{k=1}^{35} (5k - 2)$

3080

6)  $\sum_{i=1}^{35} (3i - 13)$

1435

7)  $\sum_{m=1}^{15} 4m$

480

8)  $\sum_{m=1}^{10} (7m - 2)$

365

9)  $\sum_{i=1}^6 3i$

63

10)  $\sum_{n=1}^{45} (3n - 9)$

2700

11)  $a_1 = 42, a_n = 146, n = 14$

1316

12)  $a_1 = 4, a_n = 22, n = 10$

130

13)  $a_1 = 2, a_n = 122, n = 13$

806

14)  $a_1 = -18, a_n = -102, n = 13$

-780

15)  $20 + 27 + 34 + 41 \dots, n = 16$

1160

16)  $20 + 30 + 40 + 50 \dots, n = 15$

1350

17)  $7 + 9 + 11 + 13 \dots, n = 10$

160

18)  $10 + 12 + 14 + 16 \dots, n = 11$

220

**Determine the number of terms  $n$  in each arithmetic series.**

19)  $a_1 = 19, a_n = 96, S_n = 690$

12

20)  $a_1 = 16, a_n = 163, S_n = 4475$

50

21)  $a_1 = 19, a_n = 118, S_n = 822$

12

22)  $a_1 = 15, a_n = 79, S_n = 423$

9

23)  $a_1 = -3, d = 2, S_n = 21$

7

24)  $a_1 = 4, d = 7, S_n = 228$

8

25)  $(-2) + (-12) + (-22) + (-32) \dots, S_n = -224$

7

26)  $(-16) + (-26) + (-36) + (-46) \dots, S_n = -1818$

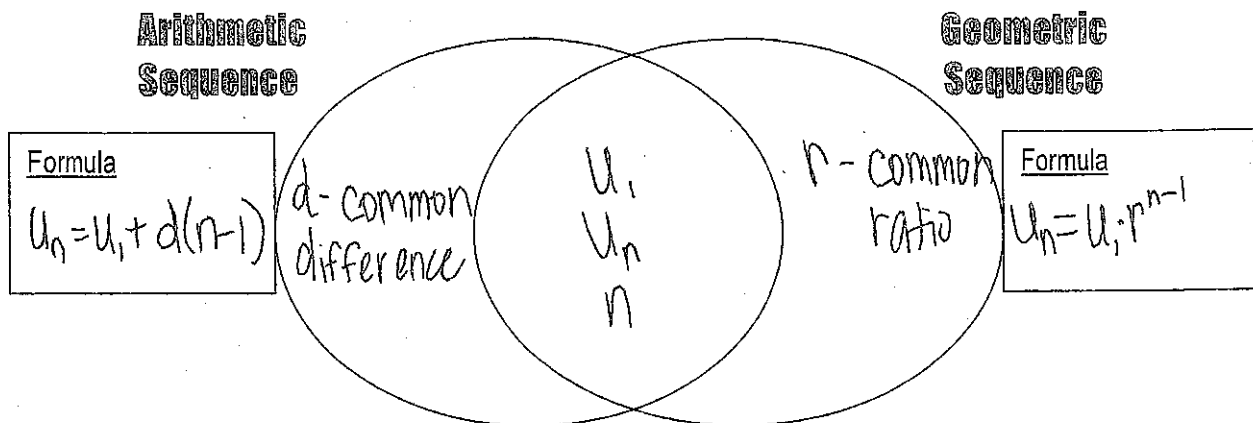
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# Geometric Sequences & Series

Notes # 6

Essential Question: What makes a sequence geometric? How do I find unknown values using the formulas for geometric sequences and geometric series?

Using the Venn Diagram below, fill in the similarities and differences of arithmetic and geometric sequences.



1. Find the 3<sup>rd</sup> and 4<sup>th</sup> terms for each of the sequences below. Then find a formula for the n<sup>th</sup> term.

a) 3, 12, 48, ...

$r = 4$

$u_4 = 192$

$u_5 = 768$

$u_n = 3(4)^{n-1}$

b) 10, 5, 2.5, ...

$r = 1/2$   $u_4 = 1.25$   $u_5 = .625$

$u_n = 10(1/2)^{n-1}$

c) 4, 12, 36, ...

$r = 3$

$u_4 = 108$

$u_5 = 324$

$u_n = 4(3)^{n-1}$

Sum of a Geometric Sequence

$$S_n = u_1 \left( \frac{r^n - 1}{r - 1} \right) = u_1 \left( \frac{1 - r^n}{1 - r} \right)$$

Formula to use for series

2. Given 2, 6, 18, 54, ... find:

a) the common ratio  $r$

$r = 3$

b) the 10<sup>th</sup> term

$u_{10} = 2(3)^{10-1} = 39,366$

c) the sum of the first 10 terms

$S_{10} = 2 \left( \frac{3^{10} - 1}{3 - 1} \right) = 3^{10} - 1 = 59,048$

3. A GS has a first term of 1 and a common ratio of  $1/4$ . Find the sum of the first four terms and show that the n<sup>th</sup> term is given by  $4^{(1-n)}$ .

$u_1 = 1$   $r = 1/4$   $S_4 = 1 \left( \frac{1 - (1/4)^4}{1 - 1/4} \right) = 85/64 \approx 1.33$

$u_n = 1(1/4)^{n-1} = (1/4)^{n-1} = (4^{-1})^{(n-1)} = 4^{-n+1} = 4^{1-n}$

## Applications of Geometric Sequences – FIXED RATE

4. Straight out of college, a young reporter goes to work for Fox 8. Her starting salary is \$16,000, but, assuming she continues to excel, she is promised an annual increase of 5% of the previous year's salary.

a) Show that the amounts of annual salary form a geometric sequence.

$$u_1 = 16,000 \quad u_2 = 16,000(1.05) = 16,800 \quad u_3 = 16,800(1.05) = 17,640$$

b) Find

$$r = 1.05$$

i) how much she earns in her 8<sup>th</sup> year with Fox 8

$$u_8 = 16,000(1.05)^7 = \$22,513.61$$

ii) the total amount earned by the employee over the first 8 years with Fox 8

$$S_8 = 16,000 \left( \frac{1.05^8 - 1}{1.05 - 1} \right) = \$152,785.74$$

## Using Systems of Equations with Geometric Sequences and Series

5. The sum of the 2<sup>nd</sup> and 3<sup>rd</sup> terms of a GS is 12. The sum of the 3<sup>rd</sup> and 4<sup>th</sup> terms is -36. Find the common ratio.

WHAT WE KNOW	WHAT WE NEED TO KNOW	FORMULA TO USE	PLUG IT IN!!
$u_2 + u_3 = 12$ $u_3 + u_4 = -36$	$r$	$u_n = u_1 r^{n-1}$	$u_1 r + u_1 r^2 = 12$ $u_1 r^2 + u_1 r^3 = -36$

Now work it out!  $u_1 r(1+r) = 12 \Rightarrow 1+r = \frac{12}{u_1 r}$

$$u_1 r^2(1+r) = -36$$

$$u_1 r^2 \left( \frac{12}{u_1 r} \right) = -36 \Rightarrow 12r = -36 \Rightarrow r = -3$$

## Comparing Arithmetic and Geometric Sequences

For each sequence, state if it is arithmetic, geometric, or neither.

1) 1, 3, 6, 10, 15, ...

Neither

2) 40, 43, 46, 49, 52, ...

Arithmetic

3)  $4, \frac{13}{3}, \frac{14}{3}, 5, \frac{16}{3}, \dots$

Arithmetic

4) -4, 12, -36, 108, -324, ...

Geometric

5) 4, 16, 36, 64, 100, ...

Neither

6) -29, -34, -39, -44, -49, ...

Arithmetic

7) 1, 5, 25, 125, 625, ...

Geometric

8) 1, 4, 9, 16, 25, ...

Neither

9) -34, -26, -18, -10, -2, ...

Arithmetic

10) 0, 3, 8, 15, 24, ...

Neither

11)  $a_n = -163 + 200n$

Arithmetic

12)  $a_n = 16 + 3n$

Arithmetic

13)  $a_n = -4 \cdot (-3)^{n-1}$

Geometric

14)  $a_n = -\frac{3}{4} + \frac{3}{2}n$

Arithmetic

15)  $a_n = -43 + 4n$

Arithmetic

16)  $a_n = (2n)^2$

Neither

17)  $a_n = -43 + 7n$

Arithmetic

18)  $a_n = \frac{n}{2^n}$

Neither

19)  $a_n = -(-3)^{n-1}$

Geometric

20)  $a_n = 2 \cdot (-3)^{n-1}$

Geometric

21)  $a_n = a_{n-1} + 6$

$a_1 = -17$

Arithmetic

22)  $a_n = na_{n-1}$

$a_1 = -1$

Neither

23)  $a_n = a_{n-1} \cdot -5$

$a_1 = 4$

Geometric

24)  $a_n = a_{n-1} + 8$

$a_1 = -17$

Arithmetic

25)  $a_n = \frac{2 + a_{n-1}}{2}$

$a_1 = -6$

Neither

26)  $a_n = a_{n-1} + 2$

$a_1 = 9$

Arithmetic

27)  $a_n = a_{n-1} + 10$

$a_1 = -1$

Arithmetic

28)  $a_n = na_{n-1}$

$a_1 = 1$

Neither

## Geometric Sequences

Determine if the sequence is geometric. If it is, find the common ratio.

1)  $-1, 6, -36, 216, \dots$

$r = -6$

2)  $-1, 1, 4, 8, \dots$

Not geometric

3)  $4, 16, 36, 64, \dots$

Not geometric

4)  $-3, -15, -75, -375, \dots$

$r = 5$

5)  $-2, -4, -8, -16, \dots$

$r = 2$

6)  $1, -5, 25, -125, \dots$

$r = -5$

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

7)  $a_n = 3^{n-1}$

First Five Terms: 1, 3, 9, 27, 81

$a_8 = 2187$

8)  $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$

First Five Terms:  $2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}$ 

$a_8 = \frac{1}{8192}$

9)  $a_n = -2.5 \cdot 4^{n-1}$

First Five Terms:  $-2.5, -10, -40, -160, -640$ 

$a_8 = -40960$

10)  $a_n = -4 \cdot 3^{n-1}$

First Five Terms:  $-4, -12, -36, -108, -324$ 

$a_8 = -8748$

Given the recursive formula for a geometric sequence find the common ratio, the first five terms, and the explicit formula.

11)  $a_n = a_{n-1} \cdot 2$

$a_1 = 2$

Common Ratio:  $r = 2$ 

First Five Terms: 2, 4, 8, 16, 32

Explicit:  $a_n = 2 \cdot 2^{n-1}$

12)  $a_n = a_{n-1} \cdot -3$

$a_1 = -3$

Common Ratio:  $r = -3$ First Five Terms:  $-3, 9, -27, 81, -243$ 

Explicit:  $a_n = -3 \cdot (-3)^{n-1}$

13)  $a_n = a_{n-1} \cdot 5$

$a_1 = 2$

Common Ratio:  $r = 5$ 

First Five Terms: 2, 10, 50, 250, 1250

Explicit:  $a_n = 2 \cdot 5^{n-1}$

14)  $a_n = a_{n-1} \cdot 3$

$a_1 = -3$

Common Ratio:  $r = 3$ First Five Terms:  $-3, -9, -27, -81, -243$ 

Explicit:  $a_n = -3 \cdot 3^{n-1}$



Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15)  $a_1 = 0.8, r = -5$

First Five Terms: 0.8, -4, 20, -100, 500

Explicit:  $a_n = 0.8 \cdot (-5)^{n-1}$

16)  $a_1 = 1, r = 2$

First Five Terms: 1, 2, 4, 8, 16

Explicit:  $a_n = 2^{n-1}$

Given the first term and the common ratio of a geometric sequence find the recursive formula and the three terms in the sequence after the last one given.

17)  $a_1 = -4, r = 6$

Next 3 terms: -24, -144, -864

Recursive:  $a_n = a_{n-1} \cdot 6$

$a_1 = -4$

18)  $a_1 = 4, r = 6$

Next 3 terms: 24, 144, 864

Recursive:  $a_n = a_{n-1} \cdot 6$

$a_1 = 4$

19)  $a_1 = 2, r = 6$

Next 3 terms: 12, 72, 432

Recursive:  $a_n = a_{n-1} \cdot 6$

$a_1 = 2$

20)  $a_1 = -4, r = 4$

Next 3 terms: -16, -64, -256

Recursive:  $a_n = a_{n-1} \cdot 4$

$a_1 = -4$

Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

21)  $a_4 = 25, r = -5$

First Five Terms: -0.2, 1, -5, 25, -125

Explicit:  $a_n = -0.2 \cdot (-5)^{n-1}$

Recursive:  $a_n = a_{n-1} \cdot -5$

$a_1 = -0.2$

22)  $a_1 = 4, r = 5$

First Five Terms: 4, 20, 100, 500, 2500

Explicit:  $a_n = 4 \cdot 5^{n-1}$

Recursive:  $a_n = a_{n-1} \cdot 5$

$a_1 = 4$

Given two terms in a geometric sequence find the 8th term and the recursive formula.

23)  $a_4 = -12$  and  $a_5 = -6$

$a_8 = -\frac{3}{4}$

Recursive:  $a_n = a_{n-1} \cdot \frac{1}{2}$

$a_1 = -96$

24)  $a_5 = 768$  and  $a_2 = 12$

$a_8 = 49152$

Recursive:  $a_n = a_{n-1} \cdot 4$

$a_1 = 3$

25)  $a_1 = -2$  and  $a_3 = -512$

$a_8 = 32768$

Recursive:  $a_n = a_{n-1} \cdot -4$

$a_1 = -2$

26)  $a_5 = 3888$  and  $a_3 = 108$

$a_8 = 839808$

Recursive:  $a_n = a_{n-1} \cdot 6$

$a_1 = 3$

## Finite Geometric Series

Evaluate the related series of each sequence.

1) 2, 12, 72, 432

518

2) -1, 5, -25, 125

104

3) -2, 6, -18, 54, -162

-122

4) -2, -12, -72, -432, -2592

-3110

Evaluate each geometric series described.

5)  $\sum_{k=1}^7 4^{k-1}$

5461

6)  $\sum_{i=1}^8 (-6)^{i-1}$

-239945

7)  $\sum_{i=1}^9 2^{i-1}$

511

8)  $\sum_{m=1}^9 -2^{m-1}$

-511

9)  $\sum_{n=1}^8 2 \cdot (-2)^{n-1}$

-170

10)  $\sum_{n=1}^9 4 \cdot 3^{n-1}$

39364

11)  $\sum_{n=1}^{10} 4 \cdot (-3)^{n-1}$

-59048

12)  $\sum_{n=1}^9 (-2)^{n-1}$

171

13)  $1 + 2 + 4 + 8 \dots, n = 6$

63

14)  $2 - 10 + 50 - 250 \dots, n = 8$

-130208

15)  $1 - 4 + 16 - 64 \dots, n = 9$

52429

16)  $-2 - 6 - 18 - 54 \dots, n = 9$

-19682

17)  $1 - 5 + 25 - 125 \dots, n = 7$

13021

18)  $-3 - 6 - 12 - 24 \dots, n = 9$

-1533

19)  $a_1 = 4, a_n = 1024, r = -2$

684

20)  $a_1 = 4, a_n = 8748, r = 3$

13120

**Determine the number of terms  $n$  in each geometric series.**

21)  $a_1 = -2, r = 5, S_n = -62$

3

22)  $a_1 = 3, r = -3, S_n = -60$

4

23)  $a_1 = -3, r = 4, S_n = -4095$

6

24)  $a_1 = -3, r = -2, S_n = 63$

6

25)  $-4 + 16 - 64 + 256 \dots, S_n = 52428$

8

26)  $\sum_{m=1}^n -2 \cdot 4^{m-1} = -42$

3

## Infinite Geometric Series

Determine if each geometric series converges or diverges.

1)  $a_1 = -3, r = 4$

Diverges

2)  $a_1 = 4, r = -\frac{3}{4}$

Converges

3)  $a_1 = 5.5, r = 0.5$

Converges

4)  $a_1 = -1, r = 3$

Diverges

5)  $81 + 27 + 9 + 3, \dots$

Converges

6)  $7.1 + 17.75 + 44.375 + 110.9375, \dots$

Diverges

7)  $-3 + \frac{12}{5} - \frac{48}{25} + \frac{192}{125}, \dots$

Converges

8)  $\frac{128}{3125} - \frac{64}{625} + \frac{32}{125} - \frac{16}{25}, \dots$

Diverges

9)  $\sum_{k=1}^{\infty} -4^{k-1}$

Diverges

10)  $\sum_{k=1}^{\infty} \frac{16 \left(\frac{3}{2}\right)^{k-1}}{9}$

Diverges

11)  $\sum_{i=1}^{\infty} 4.2 \cdot 0.2^{i-1}$

Converges

12)  $\sum_{k=1}^{\infty} \frac{7}{6} \left(-\frac{1}{4}\right)^{k-1}$

Converges

Evaluate each infinite geometric series described.

13)  $a_1 = 3, r = -\frac{1}{5}$

$$\frac{5}{2}$$

14)  $a_1 = 1, r = -4$

No sum

15)  $a_1 = 1, r = -3$

No sum

16)  $a_1 = 1, r = \frac{1}{2}$

2

$$17) 1 + 0.5 + 0.25 + 0.125 \dots,$$

$$\frac{2}{2}$$

$$18) 3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64} \dots,$$

$$\frac{12}{7}$$

$$19) 81 - 27 + 9 - 3 \dots,$$

$$\frac{243}{4}$$

$$20) 1 - 0.6 + 0.36 - 0.216 \dots,$$

$$0.625$$

$$21) \sum_{k=1}^{\infty} 5 \cdot \left(-\frac{1}{5}\right)^{k-1}$$

$$\frac{25}{6}$$

$$22) \sum_{n=1}^{\infty} -6 \cdot \left(-\frac{1}{2}\right)^{n-1}$$

$$-4$$

$$23) \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i-1}$$

$$\frac{3}{2}$$

$$24) \sum_{k=1}^{\infty} 4^{k-1}$$

No sum

Determine the common ratio of the infinite geometric series.

$$25) a_1 = 1, S = 1.25$$

$$0.2$$

$$26) a_1 = 96, S = 64$$

$$-\frac{1}{2}$$

$$27) a_1 = -4, S = -\frac{16}{5}$$

$$-\frac{1}{4}$$

$$28) a_1 = 1, S = 2.5$$

$$0.6$$

### Summation Notation

Value of  $n$  in the final term (may be  $\infty$ )

10

The Greek letter sigma means "sum."

Terms of the sum

$$a = \sum_{n=1}^{10} a_n = a_1 + a_2 + \dots + a_9 + a_{10}$$

$n$  ranges from 1 up to 10, counting by 1

The index  $n$  labels each term.  $n = 1, 2, 3, \dots$

**Last value of  $n$**

$$\sum_{n=1}^6 2n$$

**Formula for each term**

**First value of  $n$**

$$2(1) + 2(2) + 2(3) + 2(4) + 2(5) + 2(6)$$

$$\sum_{n=1}^5 n^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$= 1 + 4 + 9 + 16 + 25 = 55$$

## Introduction to Series

○ Rewrite each series as a sum.

$$1) \sum_{m=1}^5 (4m^2 + 4)$$

$$8 + 20 + 40 + 68 + 104$$

$$2) \sum_{k=1}^5 (30 - k^2)$$

$$29 + 26 + 21 + 14 + 5$$

$$3) \sum_{n=1}^5 n$$

$$1 + 2 + 3 + 4 + 5$$

$$4) \sum_{m=1}^6 (50 - m)$$

$$49 + 48 + 47 + 46 + 45 + 44$$

$$5) \sum_{a=1}^6 (3a^2 - 2)$$

$$1 + 10 + 25 + 46 + 73 + 106$$

$$6) \sum_{m=1}^5 (100 - m)$$

$$99 + 98 + 97 + 96 + 95$$

$$7) \sum_{m=1}^4 (5m^2 + 4)$$

$$9 + 24 + 49 + 84$$

$$8) \sum_{a=4}^9 (20 - a^2)$$

$$4 + (-5) + (-16) + (-29) + (-44) + (-61)$$

$$9) \sum_{m=1}^6 \frac{m^2 + 1}{m}$$

$$2 + \frac{5}{2} + \frac{10}{3} + \frac{17}{4} + \frac{26}{5} + \frac{37}{6}$$

$$10) \sum_{n=4}^9 (100 - n)$$

$$96 + 95 + 94 + 93 + 92 + 91$$

$$11) \sum_{m=0}^5 m(m+2)$$

$$0 + 3 + 8 + 15 + 24 + 35$$

$$12) \sum_{k=0}^4 (100 - k)$$

$$100 + 99 + 98 + 97 + 96$$

Evaluate each series.

$$13) \sum_{n=1}^7 (40 - n^2)$$

$$140$$

$$14) \sum_{k=1}^5 3k$$

$$45$$

$$15) \sum_{a=1}^7 (500 - a)$$

3472

$$16) \sum_{k=1}^7 (30 - k)$$

182

$$17) \sum_{a=0}^5 a$$

15

$$18) \sum_{k=0}^4 2k$$

20

$$19) \sum_{k=1}^6 k^2$$

91

$$20) \sum_{m=1}^5 3m$$

45

Rewrite each series using sigma notation.

$$21) 1 + 2 + 3 + 4$$

$$\sum_{n=1}^4 n$$

$$22) 3 + 9 + 27 + 81 + 243$$

$$\sum_{m=1}^5 3^m$$

$$23) 3 + 9 + 27 + 81$$

$$\sum_{n=1}^4 3^n$$

$$24) 1 + 4 + 9 + 16 + 25$$

$$\sum_{k=1}^5 k^2$$

$$25) 4 + 8 + 12 + 16$$

$$\sum_{k=1}^4 4k$$

$$26) \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$$

$$\sum_{a=1}^5 \frac{a}{a+1}$$

$$27) 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

$$\sum_{a=1}^6 \frac{1}{a}$$

$$28) \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$$

$$\sum_{a=1}^6 \frac{a}{a+1}$$

Critical thinking questions:

29) Are these equal? Why or why not?

$$\sum_{x=1}^{30} \frac{1}{x} \quad \text{and} \quad \sum_{x=21}^{70} \frac{1}{x-20}$$

Yes. Both are  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$  to 50 terms

30) Rewrite the following so that it starts at  $x = 0$

$$\sum_{x=7}^{10} x(x+1)$$

$$\sum_{x=0}^3 (x+7)(x+8)$$



## SOLUTIONS

What do you need to know for the test...

The point of this unit is to know the difference between linear and exponential functions. You need to be able to tell the difference between arithmetic and geometric sequences and series. Finally, you need to be able to calculate the sum of a partial arithmetic series, a finite geometric series, and an infinite geometric series.

Here are some example problems.

Find each sum.

$$\sum_{n=1}^6 3n - 2$$

This is an arithmetic sum. The common difference is 3. The first term is  $1 = (3(1) - 2)$ . There are six terms in the sum  $6 = (6 - 1 + 1)$ . The last term is  $16 = (3(6) - 2)$ . The sum is given by the partial arithmetic sum formula  $S_n = \frac{n}{2}(a_1 + a_n) = \frac{6}{2}(1 + 16) = 51$ .

$$\sum_{n=1}^6 5 \cdot (4)^{n-1}$$

This is a geometric series. It is finite because the number on top is not infinity. The common ratio is 4. The first term is 5. The last term is 5120. There are 6 terms. The sum is given by the partial geometric sum formula  $S_n = a_1 \left( \frac{r^n - 1}{r - 1} \right) = 5 \left( \frac{4^6 - 1}{4 - 1} \right) = 6825$ .

$$\sum_{n=1}^{\infty} 2 \cdot \left( \frac{1}{3} \right)^{n-1}$$

This is a geometric series. It is infinite because the number on top is infinity. It converges (has a finite sum) because  $-1 < r < 1$ . The first term is 2. The last term is 0. The common ratio is  $1/3$ . The sum is given by the infinite geometric sum formula  $S_{\infty} = \frac{a_1}{1 - r} = \frac{2}{1 - \frac{1}{3}} = 3$ .

Answer these questions for each sequence.

- Is this sequence arithmetic or geometric?
- What is the formula for the  $n$ th term?
- What is the 11 term?
- What is the sum of the first 7 terms?

Sequence 1--- 3, 6, 9, 12

Arithmetic

$$u_n = u_1 + d(n - 1) = 3 + 3(n - 1)$$

$$u_{11} = 3 + 3(11 - 1) = 33$$

$$\text{Sum of the first seven terms} = \frac{n}{2}(u_1 + u_7) = \frac{7}{2}(3 + 21) = 84.$$

Sequence 2--- 5, 15, 45,

Geometric

$$u_n = u_1 r^{n-1} = 5 \cdot 3^{n-1}$$

$$u_{11} = 5 \cdot 3^{11-1} = 295,245$$

$$S_7 = u_1 \left( \frac{r^7 - 1}{r - 1} \right) = 5 \left( \frac{3^7 - 1}{3 - 1} \right) = 5465$$

Sequence 3--- 24, 12, 6,

$$u_n = u_1 r^{n-1} = 24 \cdot \left( \frac{1}{2} \right)^{n-1}$$

$$u_{11} = 24 \cdot \left( \frac{1}{2} \right)^{11-1} = .02343$$

$$S_7 = u_1 \left( \frac{r^7 - 1}{r - 1} \right) = 24 \left( \frac{\left( \frac{1}{2} \right)^7 - 1}{\frac{1}{2} - 1} \right) = 47.625$$

Sequence 4--- 10, 3, -4, -11

Arithmetic

$$u_n = u_1 + d(n - 1) = 10 - 7(n - 1)$$

$$u_{11} = 10 + -7(11 - 1) = -60$$

$$\text{Sum of the first seven terms} = \frac{n}{2}(u_1 + u_7) = \frac{7}{2}(10 + -32) = -77.$$

Decide if each scenario is linear or exponential. Write a formula to predict the future. Answer the question.

1. Johnny is growing a bacteria for a science project. Every 3 hours, the bacteria doubles. He started with 50 bacteria. How many bacteria will he have after 11 hours?

It is exponential because it is dividing.

$$B = 50 \cdot 2^{\frac{t}{3}} = 50 \cdot 2^{\frac{11}{3}} = 634 \text{ bacteria (remember to round down)}$$

2. Karen is making cookies to sell at the bake sale. Every day she makes 45 cookies. How many days will it take her to have over 200 cookies?

It is linear because it is adding.

$$C = 45n$$

$$200 = 45n$$

$$\frac{200}{45} = n = 4.44 \text{ so } 5 \text{ days}$$