Notes 10-2

Adding, Subtracting, and Multiplying Radical Expressions

I. Adding and Subtracting Radical Expressions

A. Like Radicals

Square-root expressions with the same radicand are examples of like radicals.

Like Radicals	$2\sqrt{5}$ and $4\sqrt{5}$	$6\sqrt{x}$ and $-2\sqrt{x}$	$3\sqrt{4t}$ and $\sqrt{4t}$
Unlike Radicals	2 and $\sqrt{15}$	$6\sqrt{x}$ and $\sqrt{6x}$	$3\sqrt{2}$ and $2\sqrt{3}$

Helpful Hint

Combining like radicals is similar to combining like terms.

COMPARE:

$$2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5}$$

$$2x + 4x = 6x$$

You can combine like radicals by adding or subtracting the numbers multiplied by the radical and keeping the radical the same.

B. Basic Examples

Example 1: Adding and Subtracting Square-Root Expressions

Add or subtract.

A. $\mathbf{9\sqrt{3} - 4\sqrt{3}}$

$9\sqrt{3} - 4\sqrt{3}$ *The terms are like radicals.*

$5\sqrt{3}$

B. $\mathbf{6\sqrt{x} + 8\sqrt{y}}$

$6\sqrt{x} + 8\sqrt{y}$ *The terms are unlike radicals. Do not combine.*

Add or subtract.

C. $\sqrt{m} - 7\sqrt{m}$

$1\sqrt{m} - 7\sqrt{m}$

$\sqrt{m} = 1\sqrt{m}$, *the terms are like radicals.*

$-6\sqrt{m}$

Combine like radicals.

D. $2\sqrt{xy} + 2\sqrt{y} + 9\sqrt{xy}$

$2\sqrt{xy} + 2\sqrt{y} + 9\sqrt{xy}$

Identify like radicals.

$11\sqrt{xy} + 2\sqrt{y}$

Combine like radicals.

More Examples

Add or subtract.

a. $5\sqrt{7} - 6\sqrt{7}$

$5\sqrt{7} - 6\sqrt{7}$ *The terms are like radicals.*

$-\sqrt{7}$ *Combine like radicals.*

b. $8\sqrt{3} - 5\sqrt{3}$

$8\sqrt{3} - 5\sqrt{3}$ *The terms are like radicals.*

$3\sqrt{3}$ *Combine like radicals.*

Add or subtract.

c. $4\sqrt{n} + 4\sqrt{n}$

$4\sqrt{n} + 4\sqrt{n}$ *The terms are like radicals.*

$8\sqrt{n}$ *Combine like radicals.*

d. $\sqrt{2s} - \sqrt{5s} + 9\sqrt{5s}$

$\sqrt{2s} - 1\sqrt{5s} + 9\sqrt{5s}$ *Identify like radicals.*

$\sqrt{2s} + 8\sqrt{5s}$ *Combine like radicals.*

C. Simplifying before combining

Sometimes radicals do not appear to be like until they are simplified. Simplify all radicals in an expression before trying to identify like radicals.

Example 1:

Simplify each expression. All variables represent nonnegative numbers.

$$\sqrt{45} - \sqrt{20}$$

$$\sqrt{9(5)} - \sqrt{4(5)}$$ *Factor the radicands using perfect squares.*

$$\sqrt{9}\sqrt{5} - \sqrt{4}\sqrt{5}$$ *Product Property of Square Roots*

$$3\sqrt{5} - 2\sqrt{5}$$ *Simplify.*

$$\sqrt{5}$$ *Combine like radicals.*

Example 2

Simplify each expression. All variables represent nonnegative numbers.

$$9\sqrt{75} + 2\sqrt{50}$$

$$9\sqrt{3(25)} + 2\sqrt{2(25)}$$

Factor the radicands using perfect squares.

$$9\sqrt{3}\sqrt{25} + 2\sqrt{2}\sqrt{25}$$

Product Property of Square Roots

$$9(5)\sqrt{3} + 2(5)\sqrt{2}$$

Simplify.

$$45\sqrt{3} + 10\sqrt{2}$$

The terms are unlike radicals. Do not combine.

Example 3:

Simplify each expression. All variables represent nonnegative numbers.

$$\sqrt{75y} - 2\sqrt{27y} + \sqrt{48y}$$

$$\sqrt{25(3y)} - 2\sqrt{9(3y)} + \sqrt{16(3y)}$$

Factor the radicands using perfect squares.

$$\sqrt{25}\sqrt{3y} - 2\sqrt{9}\sqrt{3y} + \sqrt{16}\sqrt{3y}$$

Product Property of Square Roots

$$5\sqrt{3y} - 2(3)\sqrt{3y} + 4\sqrt{3y}$$

Simplify.

$$5\sqrt{3y} - 6\sqrt{3y} + 4\sqrt{3y}$$

$$3\sqrt{3y}$$

Combine like radicals.

Example 4

Simplify each expression. All variables represent nonnegative numbers.

$$\sqrt{54} + \sqrt{24}$$

$$\sqrt{9(6)} + \sqrt{4(6)}$$ *Factor the radicands using perfect squares.*

$$\sqrt{9}\sqrt{6} + \sqrt{4}\sqrt{6}$$ *Product Property of Square Roots*

$$3\sqrt{6} + 2\sqrt{6}$$ *Simplify.*

$$5\sqrt{6}$$ *Combine like radicals.*

Example 5

Simplify each expression. All variables represent nonnegative numbers.

$$4\sqrt{27} - \sqrt{18}$$

$$4\sqrt{9(3)} - \sqrt{9(2)}$$

Factor the radicands using perfect squares.

$$4\sqrt{9}\sqrt{3} - \sqrt{9}\sqrt{2}$$

Product Property of Square Roots

$$4(3)\sqrt{3} - 3\sqrt{2}$$

Simplify.

$$12\sqrt{3} - 3\sqrt{2}$$

The terms are unlike radicals. Do not combine.

Example 6

Simplify each expression. All variables represent nonnegative numbers.

$$\sqrt{12y} + \sqrt{27y}$$

$$\sqrt{4(3y)} + \sqrt{9(3y)}$$ *Factor the radicands using perfect squares.*

$$\sqrt{4}\sqrt{3y} + \sqrt{9}\sqrt{3y}$$ *Product Property of Square Roots*

$$2\sqrt{3y} + 3\sqrt{3y}$$ *Simplify.*

$$5\sqrt{3y}$$ *Combine like radicals.*

More examples

Ex 7: $\sqrt{27}+\sqrt{75}=\ \sqrt{9\cdot 3}+\sqrt{25\cdot 3}=\ 3\sqrt{3}+5\sqrt{3}=\ 8\sqrt{3}$

Ex 8: $3\sqrt{20}-7\sqrt{45}=\ 3\sqrt{4\cdot 5}-7\sqrt{9\cdot 5}=$

$3\cdot 2\sqrt{5}-7\cdot 3\sqrt{5}=\ 6\sqrt{5}-21\sqrt{5}=\ -15\sqrt{5}$

Ex 9:

$\sqrt{36}-\sqrt{48}-4\sqrt{3}-\sqrt{9}=\ 6-\sqrt{16\cdot 3}-4\sqrt{3}-3=$

$6-4\sqrt{3}-4\sqrt{3}-3=\qquad 3-8\sqrt{3}$

More Examples

Ex 10:

$$\sqrt{9x^4} - \sqrt{36x^3} + \sqrt{x^3} = \quad 3x^2 - 6\sqrt{x^2 x} + \sqrt{x^2 x} =$$

$$3x^2 - 6x\sqrt{x} + x\sqrt{x} = \quad 3x^2 - 5x\sqrt{x}$$

Ex 11:

$$10\sqrt[3]{81p^6} - \sqrt[3]{24p^6} = \quad 10\sqrt[3]{27 \cdot 3p^6} - \sqrt[3]{8 \cdot 3p^6} =$$

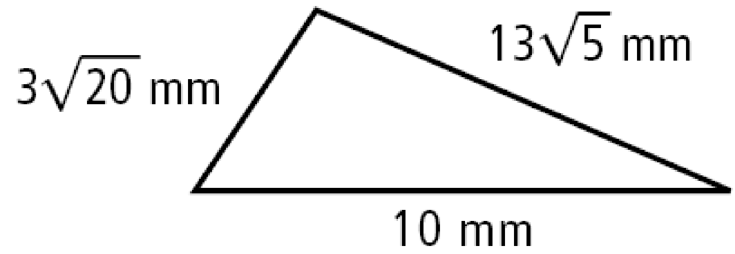
$$10 \cdot 3p^2\sqrt[3]{3} - 2p^2\sqrt[3]{3} = \quad 30p^2\sqrt[3]{3} - 2p^2\sqrt[3]{3} =$$

$$28p^2\sqrt[3]{3}$$

D. Applications

Example 1: ***Geometry Application***

Find the perimeter of the triangle. Give the answer as a radical expression in simplest form.



$$10 + 13\sqrt{5} + 3\sqrt{20}$$ *Write an expression for perimeter.*

$$10 + 13\sqrt{5} + 3\sqrt{4(5)}$$ *Factor 20 using a perfect square.*

$$10 + 13\sqrt{5} + 3\sqrt{4}\sqrt{5}$$ *Product Property of Square Roots*

$$10 + 13\sqrt{5} + 3(2)\sqrt{5}$$ *Simplify.*

$$10 + 13\sqrt{5} + 6\sqrt{5}$$ *Combine like radicals.*

$$10 + 19\sqrt{5}$$

The perimeter is $\left(10 + 19\sqrt{5}\right)$ mm.

II. Multiplying Radical Expressions

A. Using the Distributive Property

Ex 1: Multiply. Write the product in simplest form. All variables represent nonnegative numbers.

$$\sqrt{3}\left(7-\sqrt{8}\right)$$

$\sqrt{3}(7)-\sqrt{3}\sqrt{8}$ — *Distribute* $\sqrt{3}$.

$7\sqrt{3}-\sqrt{3(8)}$ — *Product Property of Square Roots.*

$7\sqrt{3}-\sqrt{24}$ — *Multiply the factors in the second radicand.*

$7\sqrt{3}-\sqrt{4(6)}$ — *Factor 24 using a perfect-square factor.*

$7\sqrt{3}-\sqrt{4}\sqrt{6}$ — *Product Property of Square Roots*

$7\sqrt{3}-2\sqrt{6}$ — *Simplify.*

Ex 2: Multiply. Write the product in simplest form. All variables represent nonnegative numbers.

$$\sqrt{2}\left(\sqrt{8}+\sqrt{18}\right)$$

$$\sqrt{2}\sqrt{8}+\sqrt{2}\sqrt{18}$$ *Distribute* $\sqrt{2}$.

$$\sqrt{2(8)}+\sqrt{2(18)}$$ *Product Property of Square Roots*

$$\sqrt{16}+\sqrt{36}$$ *Simplify the radicands.*

$$4+6$$ *Simplify.*

$$10$$

Ex 3: Multiply. Write the product in simplest form. All variables represent nonnegative numbers.

$$\sqrt{6}\left(\sqrt{8}-3\right)$$

$$\sqrt{6}\sqrt{8}-3\sqrt{6}$$

Distribute $\sqrt{6}$.

$$\sqrt{8(6)}-3\sqrt{6}$$

Product Property of Square Roots

$$\sqrt{48}-3\sqrt{6}$$

Multiply the factors in the first radicand.

$$\sqrt{16(3)}-3\sqrt{6}$$

Factor 48 using a perfect-square factor.

$$\sqrt{16}\sqrt{3}-3\sqrt{6}$$

Product Property of Square Roots

$$4\sqrt{3}-3\sqrt{6}$$

Simplify.

Ex 4: Multiply. Write the product in simplest form. All variables represent nonnegative numbers.

$$\sqrt{5}\left(\sqrt{10}+4\sqrt{3}\right)$$

$$\sqrt{5}\sqrt{10}+\sqrt{5}\left(4\sqrt{3}\right)$$ *Distribute* $\sqrt{5}$.

$$\sqrt{5(10)}+4\sqrt{15}$$ *Product Property of Square Roots*

$$\sqrt{50}+4\sqrt{15}$$

$$\sqrt{2(25)}+4\sqrt{15}$$ *Factor 50 using a perfect-square factor.*

$$5\sqrt{2}+4\sqrt{15}$$ *Simplify.*

More Examples

Ex 5:

$$\sqrt{7}\left(\sqrt{7}-\sqrt{3}\right)=\sqrt{7}\cdot\sqrt{7}-\sqrt{7}\cdot\sqrt{3}=\sqrt{49}-\sqrt{21}=$$

$$7-\sqrt{21}$$

Ex 6:

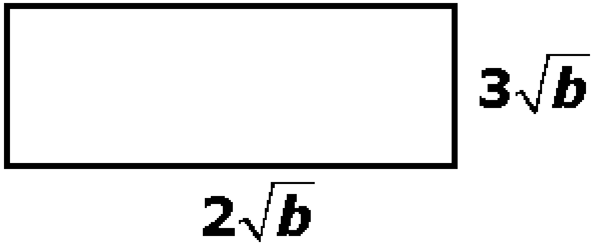
$$\sqrt{5x}\left(\sqrt{x}-3\sqrt{5}\right)=\sqrt{5x^2}-3\sqrt{25x}=\quad x\sqrt{5}-3\cdot5\sqrt{x}=$$

$$x\sqrt{5}-15\sqrt{x}$$

B. Applications

Example 1

Find the perimeter of a rectangle whose length is $3\sqrt{b}$ inches and whose width is $2\sqrt{b}$ inches. Give your answer as a radical expression in simplest form.



$2\left(3\sqrt{b} + 2\sqrt{b}\right)$ *Write an expression for perimeter 2 (l + w).*

$(2)3\sqrt{b} + (2)2\sqrt{b}$ *Multiply each term by 2 .*

$6\sqrt{b} + 4\sqrt{b}$ *Simplify.*

$10\sqrt{b}$ *Combine like radicals.*

The perimeter is in $10\sqrt{b}$

Lesson Quiz

Multiply. Write each product in simplest form. All variables represent nonnegative numbers.

1. $\sqrt{5}\sqrt{10}$ $5\sqrt{2}$

2. $3\sqrt{6x}\sqrt{8x}$ $12x\sqrt{3}$

3. $\sqrt{2}\left(\sqrt{7}+\sqrt{2}\right)$ $\sqrt{14}+2$

4. $\left(2+\sqrt{5}\right)^2$ $9+4\sqrt{5}$

5. $\left(3\sqrt{6}\right)^2$ 54

6. $\sqrt{3}\left(5-\sqrt{18}\right)$ $5\sqrt{3}-3\sqrt{6}$

7. $\left(6+\sqrt{3}\right)\left(2-\sqrt{3}\right)$ $9-4\sqrt{3}$

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