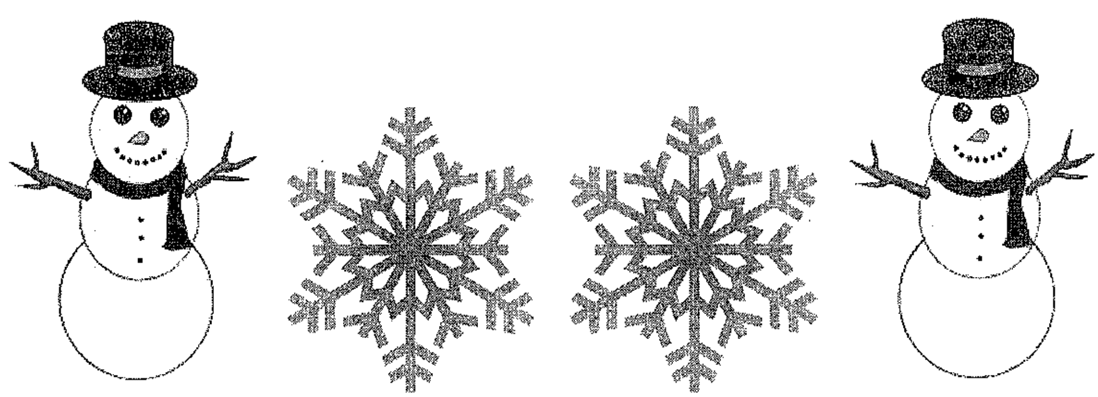
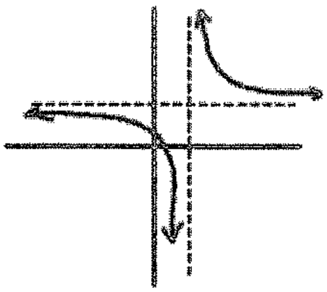
Honors Algebra
Unit 4: Graphing Rational Relationships

Wednesday January 27	Graphing Easy Rational Functions Notes & Handout Pages 1-6
Thursday January 28	Graphing Rational Functions Notes-Foldable Pages 7-8
Friday January 29	Graphing Rational Functions Pages 9-10
Monday February 1	Writing Equations of Rational Functions Notes Pages 11-13
Tuesday February 2	Graphing Rational Functions Quiz Page 14
Wednesday February 3	Graphing & Writing Rational Functions Pages 15-17
Thursday February 4	Review Matching Activity
Friday February 5	Graphing & Writing Rational Functions Test



Intro to Graphing Rational Functions

Rational Functions come in a wide variety!
All of the following graphs are graphs of rational functions.

A.


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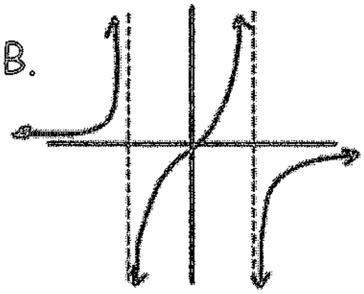

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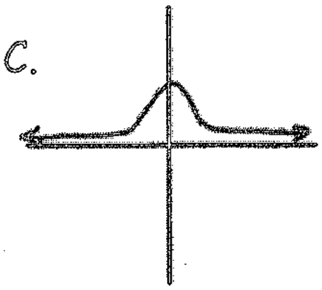

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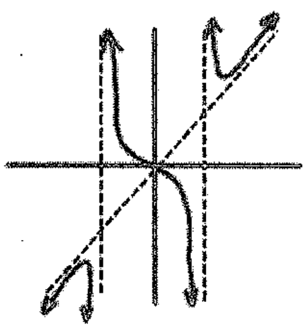

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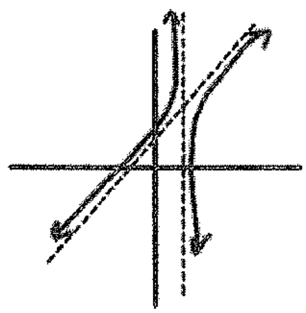
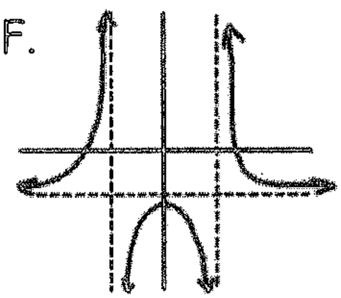


F.


Looking at the graphs above, identify ...

1) ... which have vertical asymptote(s)?

2) ... which have horizontal asymptote(s)?

3) ... which have BOTH vertical and horizontal asymptote(s)?

4) ... which have slanted asymptotes?

5) ... which have NO vertical asymptote?

Think about what you know about "excluded values" of a rational function ...

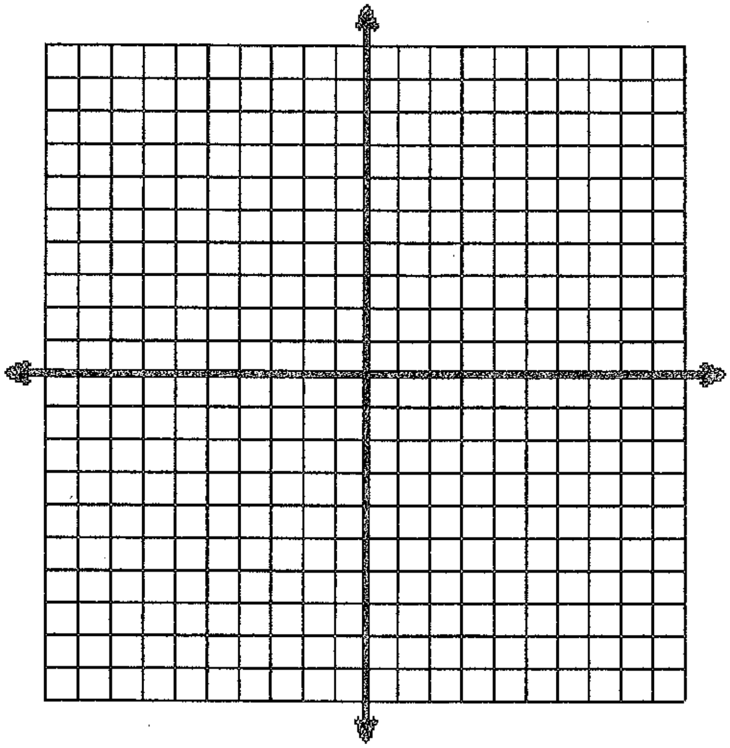
6) ... how many factors would the denominator of the function represented in graph A have?

7) ... which graphs represent a rational function that would have two different factors in the denominator?

8) ... what type of factor(s) would be in the denominator of the function represented in graph C?!

Graphic Organizer #6: RATIONAL FUNCTION
Parent Function: f(x) =1/x

$f(x)=\frac{1}{x}$	
x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	



Describe:

x – intercepts (zeros)

Domain:

Range:

y - intercept

Intervals of Increase/Decrease

End Behavior

Max or Min

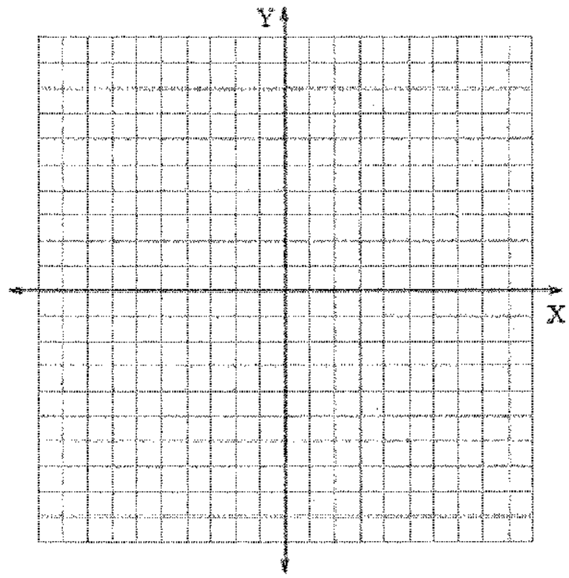
Notes: Graphing Rational Functions

Example 1:

$$f(x) = \frac{2}{x-3} + 4$$

Vertical Asymptote:____________________ Horizontal Asymptote:____________________

x	f(x)



Domain: ____________________

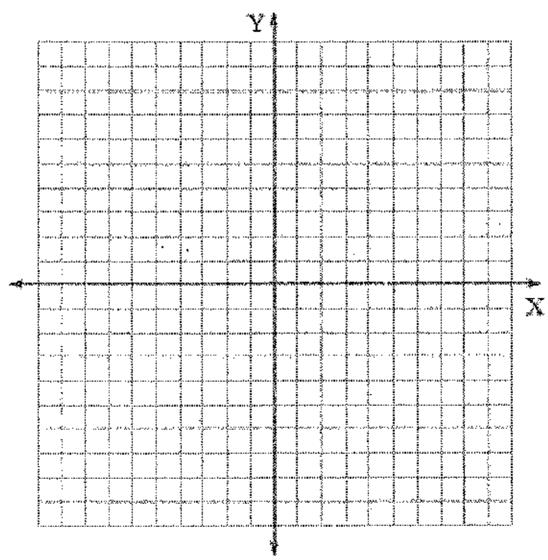
Range: ____________________

Example 2:

$$f(x) = \frac{-3}{x+2} + 4$$

Vertical Asymptote:____________________ Horizontal Asymptote:____________________

x	f(x)



Domain: ____________________

Range: ____________________

Example 3:

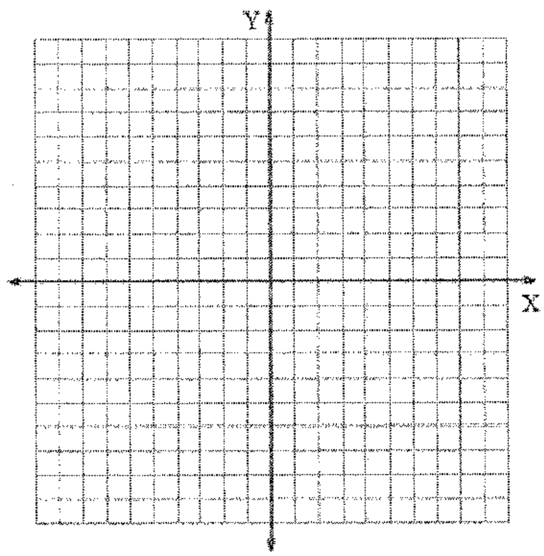
$$f(x) = -\frac{2}{2x-4} + 1$$

Vertical Asymptote:______________________ Horizontal Asymptote:____________________

x	f(x)

Domain: ____________________

Range: _____________________



Example 4:

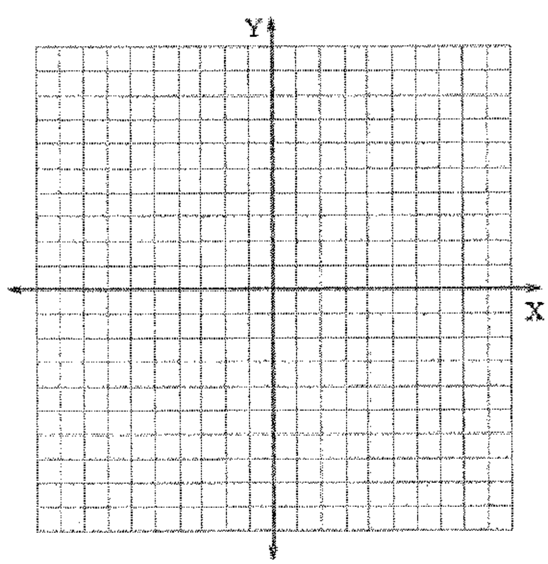
$$f(x) = \frac{1}{x} - 3$$

Vertical Asymptote:______________________ Horizontal Asymptote:____________________

x	f(x)

Domain: ____________________

Range: _____________________



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Graphing Simple Rational Functions

Date_______________ Period____

Identify the vertical asymptotes, horizontal asymptote, domain, and range of each.

1) $f(x) = -\frac{4}{x}$

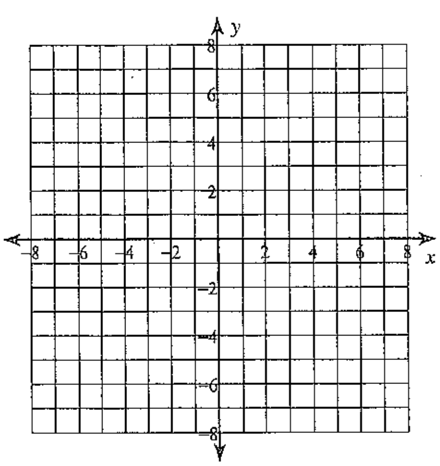
2) $f(x) = \frac{4}{x-1} + 1$

3) $f(x) = -\frac{3}{x-1} - 1$

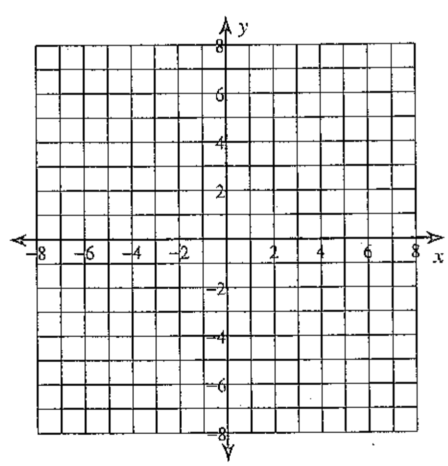
4) $f(x) = -\frac{3}{x}$

Identify the vertical asymptotes, horizontal asymptote, domain, and range of each. Then sketch the graph.

5) $f(x) = \frac{3}{x+1} - 2$

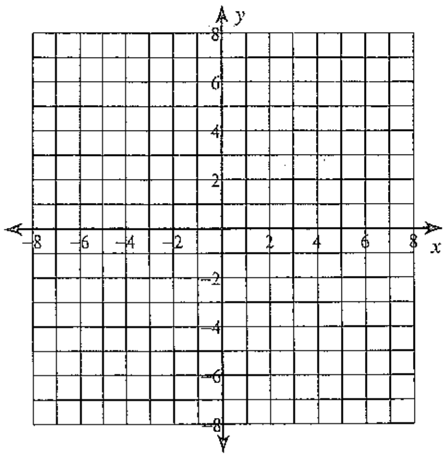


6) $f(x) = \frac{3}{x+1} + 2$

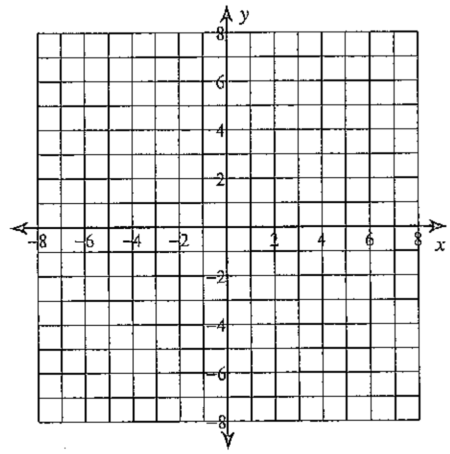


5

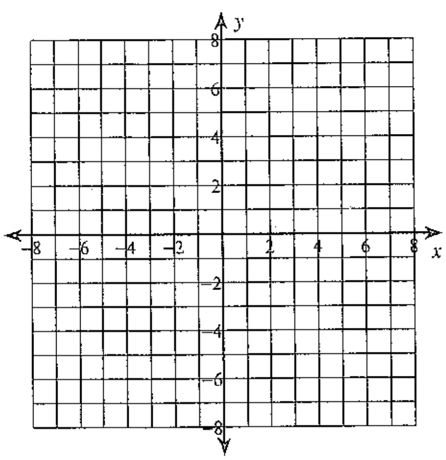
7) $f(x) = \frac{3}{x} + 1$



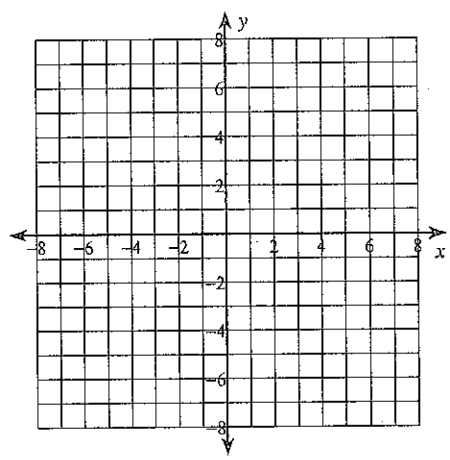
8) $f(x) = \frac{2}{x-3} + 1$



9) $f(x) = -\frac{4}{x+1} + 1$



10) $f(x) = \frac{4}{x} + 2$



Critical thinking question:

11) Write a function of the form $f(x) = \frac{a}{x-h} + k$ with a vertical asymptote at $x = 25$

Rational Functions

1] Find any holes
2] Find the vertical asymptote
3]Find any zeros
4] Find the y-int (if any)
5] Find the horizontal or oblique asymptote
6] Sketch the graph
7] Identify the domain and range

Step 1: Look for holes

1] Factor completely and cancel common factors
2] Factors that cancel form holes in the graph. To find the x-coordinate of a hole, set the canceled factor equal to zero and solve for x.
3] Substitute that result back in to the simplified form to find the y-coordinate of the hole.

A] $y = \dfrac{2x^2-8}{x^2-x-6}$

B] $y = \dfrac{x^2+x-12}{x+3}$

Steps 2 - 4: Find vertical asymptotes, zeros, and y-intercept

1] Set the denominator equal to zero to find any vertical asymptotes
2] Set the numerator equal to zero to find any zeros
3] Substitute 0 for x and solve for y to find the y-intercept, if any

A] $y = \dfrac{2x^2-8}{x^2-x-6} = \dfrac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-3)}$

B] $y = \dfrac{x^2+x-12}{x+3} = \dfrac{(x+4)(x-3)}{x+3}$

Step 5: Find horizontal OR oblique (SLANT) asymptote (can't have both)

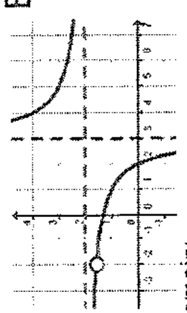
1] If the degree of the numerator is LESS than the degree of the denominator, then y=0 is the horizontal asymptote.
2] If the degree of the numerator EQUALS the degree of the denominator, then the ratio of leading coefficients is the horizontal asymptote.
3] If the degree of the numerator is MORE than the degree of the denominator, then divide to find the asymptote, ignoring the remainder. When the difference in degree is one, the asymptote will be oblique.

A] $y = \dfrac{2x^2-8}{x^2-x-6} = \dfrac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-3)}$

B] $y = \dfrac{x^2+x-12}{x+3} = \dfrac{(x+4)(x-3)}{x+3}$

Steps 6 - 7: Graph, Domain, and Range

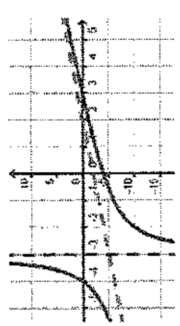
A]



Domain:

Range:

B]



Domain:

Range:

Graphing Rational Functions Worksheet

For the following function, find:

(a) the x-intercepts, y-intercepts
(b) the vertical asymptote(s)
(c) the horizontal/ slant asymptote
(d) the holes
(e) any additional points needed
(f) then, graph the function.

1. $f(x)=\dfrac{2x}{x^2-1}$

2. $y=\dfrac{8}{x^2-x-6}$

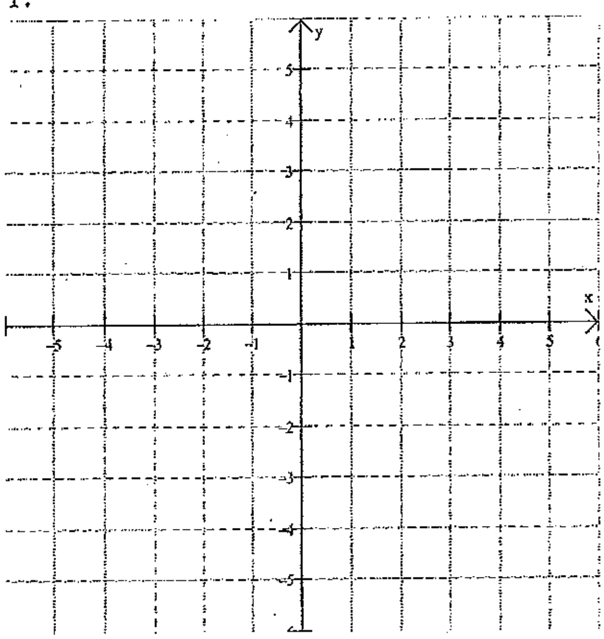
3. $f(x)=\dfrac{x^2-9}{2x^2+1}$

4. $y=\dfrac{x^2-5x+6}{x^2-4x+3}$

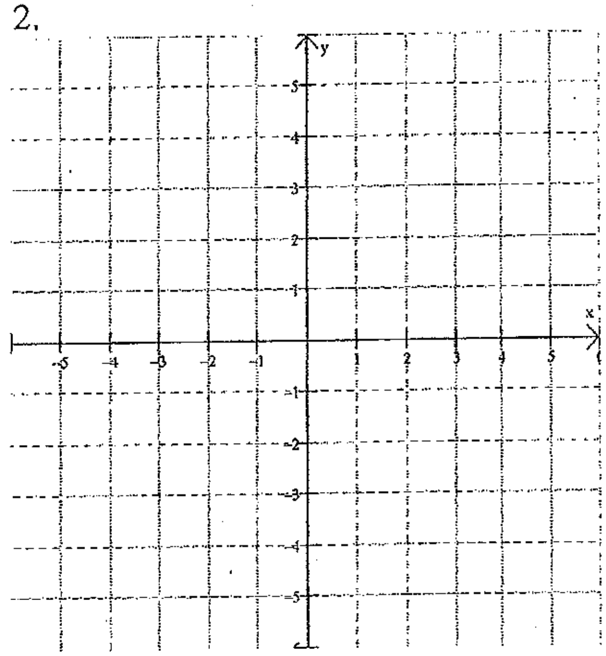
5. $y=\dfrac{x^2+11x+18}{2x+1}$

6. $g(x)=\dfrac{x-4}{x^2-3x}$

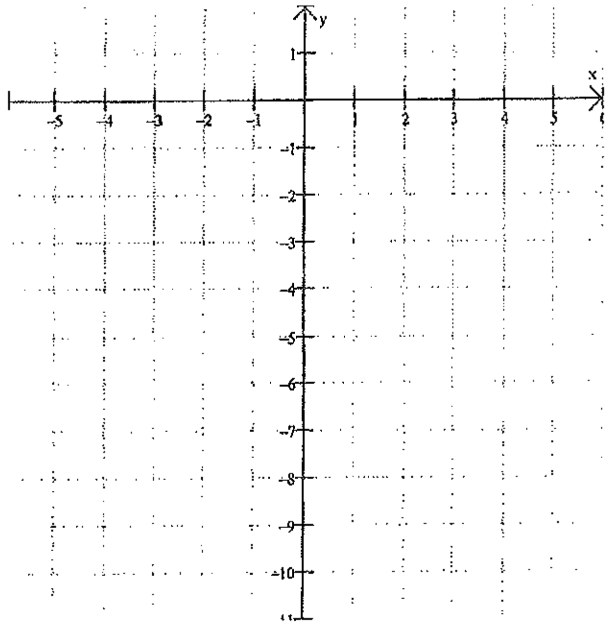
1.



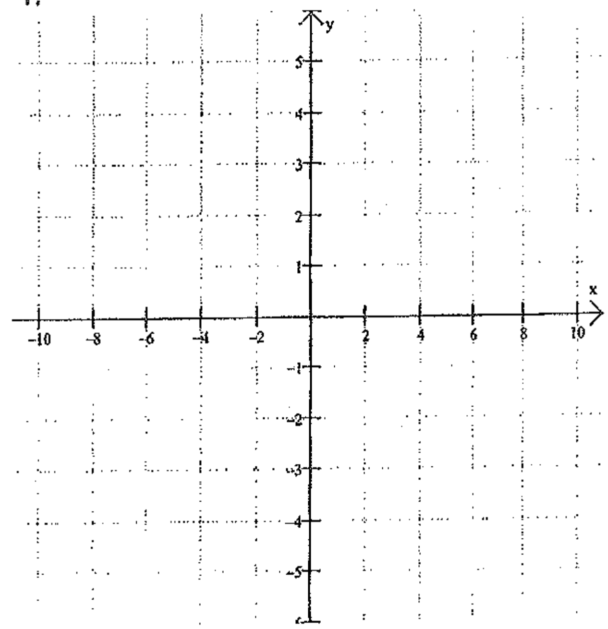
2.



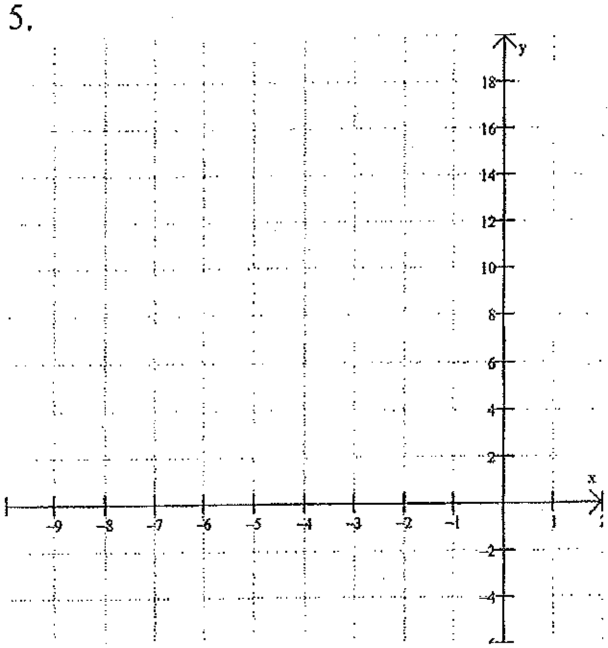
3.


y
x
1
-5 -4 -3 -2 -1 1 2 3 4 5
-1 -2 -3 -4 -5 -6 -7 -8 -9 -10

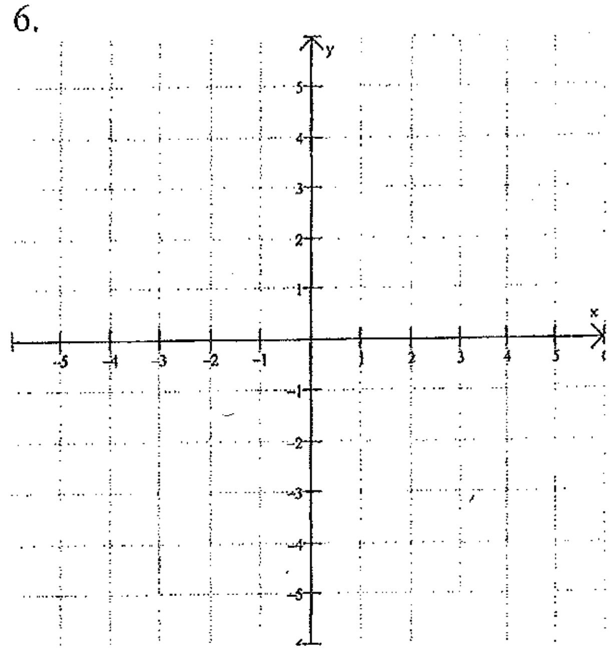
4.


y
x
5 4 3 2 1
-10 -8 -6 -4 -2 2 4 6 8 10
-1 -2 -3 -4 -5

5.


y
x
18 16 14 12 10 8 6 4 2
-9 -8 -7 -6 -5 -4 -3 -2 -1 1
-2 -4

6.


y
x
5 4 3 2 1
-5 -4 -3 -2 -1 1 2 3 4 5
-1 -2 -3 -4 -5

Name: Period: Date:

Practice Worksheet: Rational Functions

1] The graph of a rational function has a(n) ______________ ________________ at $y = 0$ if the degree of the numerator is __________ ___________ the degree of the denominator.

2] The horizontal asymptote of a rational function is the ratio of leading coefficients when the degree of the numerator is __________ ___________ the degree of the denominator.

3] If the degree of the numerator is __________ ___________ than the degree of the denominator, the rational function has a(n) ______________ ________________.

4] You must use synthetic or long ______________ to find the equation of an oblique asymptote.

5] When you cancel a common factor out of the numerator and denominator of a rational function, it forms a ____________ in the graph at that point. To find the coordinates of that point, set the canceled factor equal to ____________ and solve for x. Then ____________ to find y.

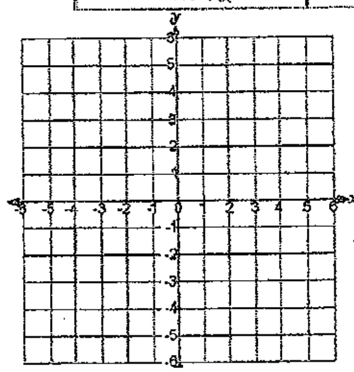
$f(x) = \frac{x^2-8x+7}{x-1}$ $\frac{(x-1)(x-7)}{x-1} = x-7$ $x-1=0$ $\quad y=1-7=-6$ $x=1$	$g(x) = \frac{x^2-15x+56}{x-3}$ $g(x) = \frac{(x-7)(x-8)}{x-3}$ $x-7=0 \quad x-8=0$ $x=7 \quad x=8$
$h(x) = \frac{x^2+2x-15}{2x^2-7x+3}$ $\frac{(x+5)(x-3)}{(2x-1)(x-3)}$ $2x-1=0$ $x=1/2$	$j(x) = \frac{2x^2-5x+5}{x-2}$ $2\,\vert\; 2 \;\; -5 \;\; 5$ $\quad\;\; 4 \;\; -2$ $2 \;\; -1 \;\; 3{:}R$ $y=2x-1$

6] Match the work shown for each process.

______a) Finding an oblique asymptote ______ b) Finding a zero

______ c) Finding a vertical asymptote ______ d) Finding a hole

7] $f(x) = \frac{x^2-25}{x^2-4x-5} = \frac{(x+5)(x-5)}{(x-5)(x+1)} = \frac{x+5}{x+1}$



Coordinates of hole:

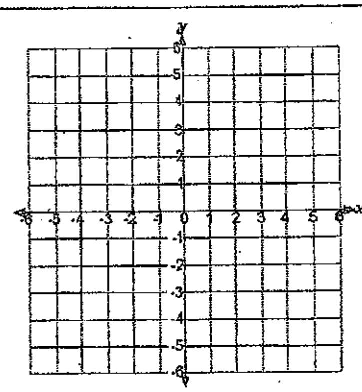
Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:

8] $f(x) = \frac{x^2+x-6}{x^2-4}$



Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:

9] $f(x) = \frac{x^2+2x-3}{x-1}$

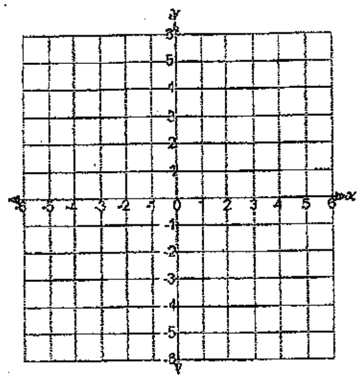
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



10] $f(x) = \frac{x-3}{x^2+x-12}$

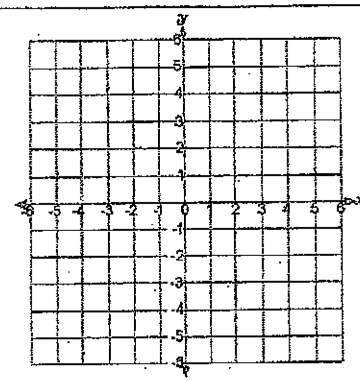
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



11] $f(x) = \frac{x^2+2x-15}{x+3}$

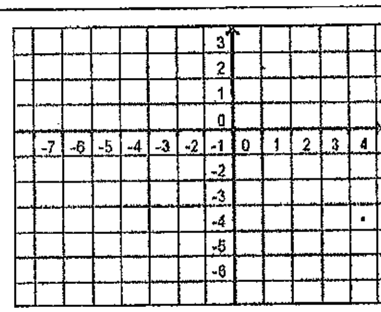
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



12] $f(x) = \frac{(x+1)(x^2+2x+1)}{x^2-1}$

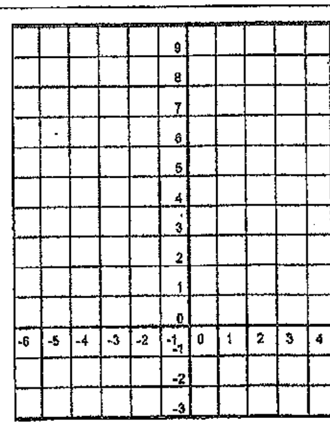
Coordinates of hole:

Equation of vertical asymptote:

Zero(s): y-intercept:

Equation of horizontal OR oblique asymptote:

Domain: Range:



Hint: The point (3, 8) is on the graph and it is a rel. minimum.

WRITING EQUATIONS OF RATIONAL FUNCTIONS

WRITE THE EQUATION OF THE RATIONAL FUNCTION GIVEN:

1. Vertical Asymptote: $x = 3$
Horizontal Asymptote: $y = -2$
Zero of the function: (-6,0)

2. Vertical Asymptote: $x = -4$
Slant Asymptote: $y = x - 2$
Zero of the function: (3,0)

Rational Functions
WS Writing Equations 1

Name: ________________

Write a rational function satisfying the following criteria.

1. VA: $x = -2$ and $x = 3$, zero $x = -5$

2. Hole at $x = -1$, Graph resembles the line $y = 2x + 1$

3. VA: $x = -7$, HA: $y = 2$, zero: $x = 3$

4. VA: $x = 4$, no zero, y intercept $(0,-2)$

5. VA: $x = 0$, $x = -5$, HA: $y = 4$, zeros: $x = \pm\frac{1}{2}$

6. VA: $x = -3$, $x = 3$, HA: $y = 0$, zero: $x = 1$

7. VA: $x = -4$, $x = 1$, HA: $y = 0$, no zeros

8. Holes at $x = -3$ and $x = -7$, resembles $y = x$

9. Hole at $x = -1$, VA: $x = -5$, and $x = 1$

10. VA: $x = 2/3$, HA: $y = 2/3$, zero: $x = 0$

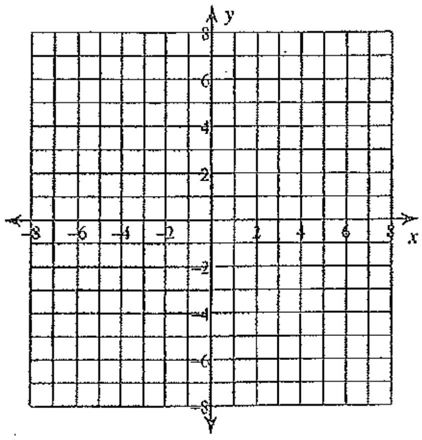
Algebra III/Trigonometry Name________________________

4.4: Graphing Rational Functions Practice

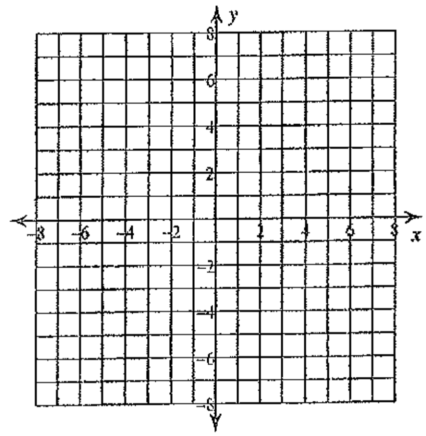
Date________________ Period____

Identify the holes, vertical asymptotes, x-intercepts, horizontal asymptote, and domain of each. Then sketch the graph.

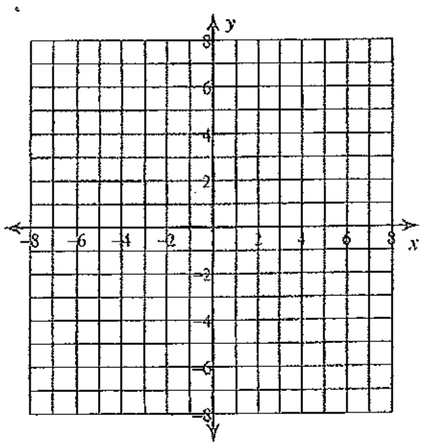
1) $f(x) = \dfrac{4}{x-3}$



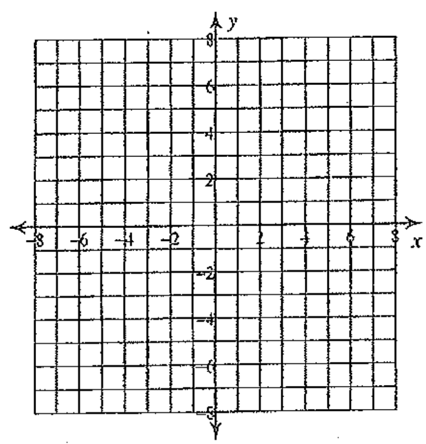
2) $f(x) = \dfrac{x^2 + 7x + 12}{-2x^2 - 2x + 12}$



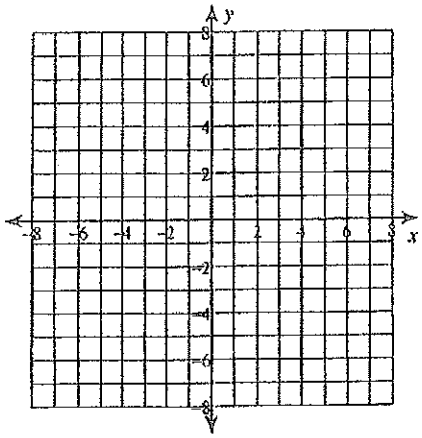
3) $f(x) = \dfrac{1}{-x+4}$



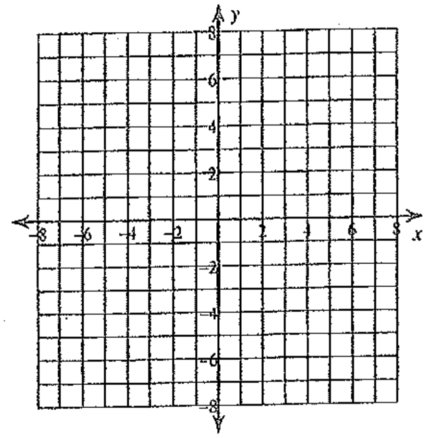
4) $f(x) = \dfrac{-3x + 12}{x^2 - 3x - 4}$



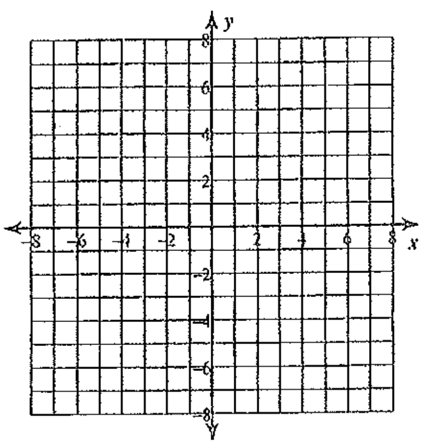
5) $f(x)=\dfrac{-2x^2+4x+16}{x^2-5x+4}$



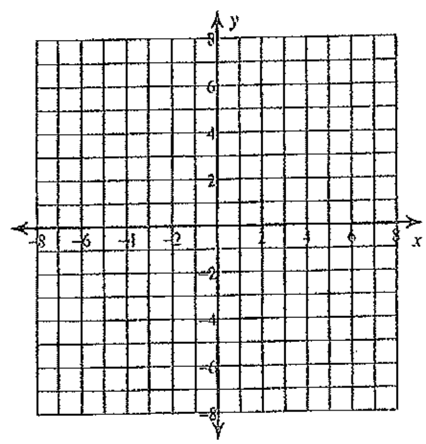
6) $f(x)=\dfrac{x^2-3x}{2x^2+2x-12}$



7) $f(x)=\dfrac{3x+6}{x+3}$



8) $f(x)=\dfrac{x^2+5x+4}{x^2-1}$



Name ______________________________

Graphing Rational Functions

Graph and give the requested information.

1. $f(x)=\dfrac{x^2-4}{x-2}$

VA: __________

HA: __________

x-int: __________

y-int: __________

SA: __________

holes: __________

2. $f(x)=\dfrac{x^2-9}{x+1}$

VA: __________

HA: __________

x-int: __________

y-int: __________

SA: __________

holes: __________

3. $f(x)=\dfrac{x}{x^2-1}$

VA: __________

HA: __________

x-int: __________

y-int: __________

SA: __________

holes: __________

4. $f(x)=\dfrac{x^2}{x-1}$

VA: __________

HA: __________

x-int: __________

y-int: __________

SA: __________

holes: __________

5. Write the equation of a rational function given:
   a) VA: $x=7$ HA: $y=-2$

   b) VA: $x=2, x=-1$ HA: $y=0$ zero at –5

Kuta Software - Infinite Algebra 2 Name____________________

Graphing Rational Functions

Date________________ Period____

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1) $f(x) = \dfrac{1}{3x^2 + 3x - 18}$

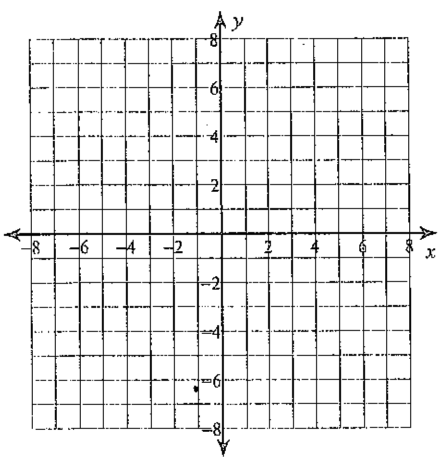
2) $f(x) = \dfrac{x - 2}{x - 4}$

3) $f(x) = \dfrac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

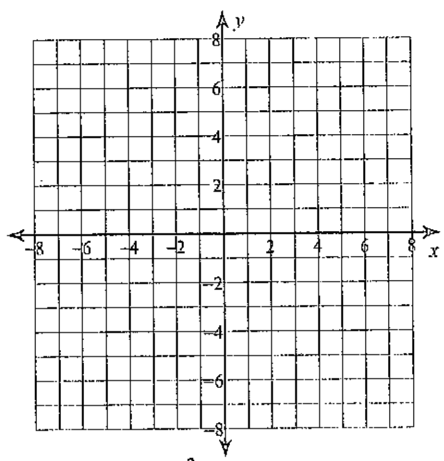
4) $f(x) = \dfrac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

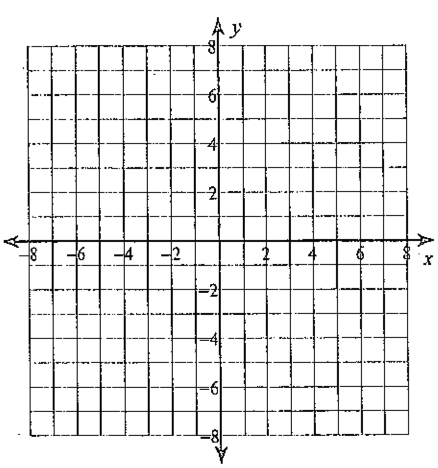
5) $f(x) = -\dfrac{4}{x^2 - 3x}$



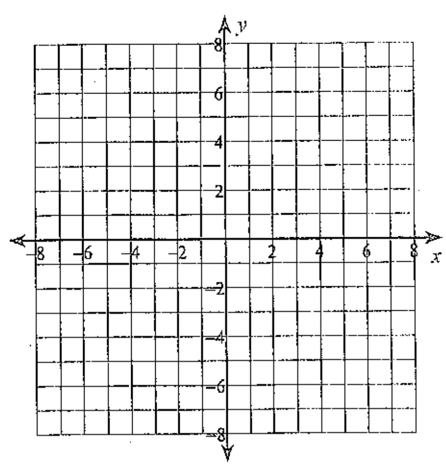
6) $f(x) = \dfrac{x - 4}{-4x - 16}$



7) $f(x) = \dfrac{x + 4}{-2x - 6}$

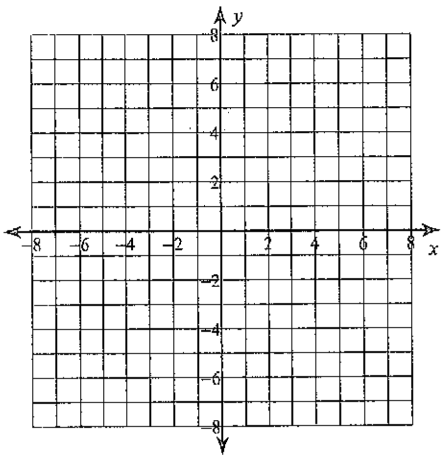


8) $f(x) = \dfrac{x^3 - 9x}{3x^2 - 6x - 9}$

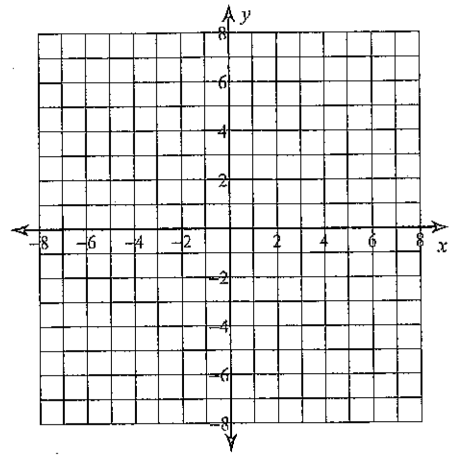


16

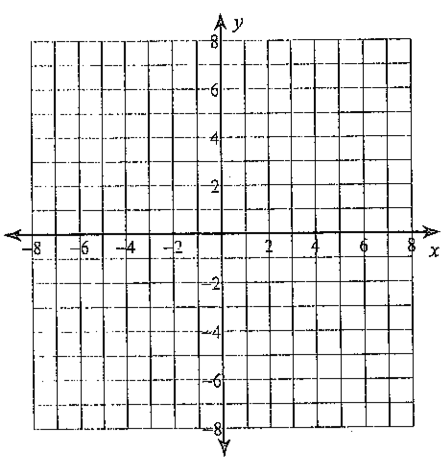
9) $f(x) = \dfrac{3x^2 - 12x}{x^2 - 2x - 3}$



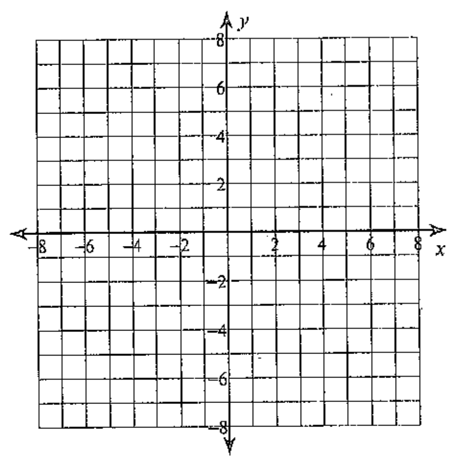
10) $f(x) = \dfrac{x^3 - 16x}{-4x^2 + 4x + 24}$



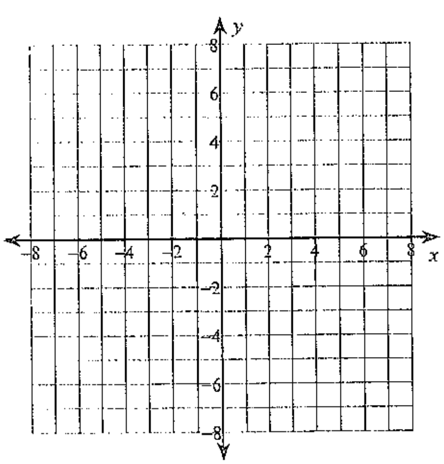
11) $f(x) = \dfrac{x^2 + 2x}{-4x + 8}$



12) $f(x) = \dfrac{x + 2}{2x + 6}$



13) $f(x) = \dfrac{2x^2 + 10x + 12}{x^2 + 3x + 2}$



14) $f(x) = \dfrac{3}{x - 2}$