KEY

Step 1: Look for holes

1] Factor completely and cancel common factors
2] Factors that cancel form holes in the graph. To find the x-coordinate of a hole, set the canceled factor equal to zero and solve for x.
3] Substitute that result back in to the simplified form to find the y-coordinate of the hole.

A] $y=\frac{2x^2-8}{x^2-x-6} = \frac{2(x^2-4)}{(x-3)(x+2)} = \frac{2\cancel{(x+2)}(x-2)}{(x-3)\cancel{(x+2)}}$

$x+2=0$
$x=-2$

$y \Rightarrow \frac{2(-2-2)}{(-2-3)} = \frac{2(-4)}{-5} = \frac{-8}{-5} = \frac{8}{5}$

hole at $(-2, 8/5)$

B] $y=\frac{x^2+x-12}{x+3} = \frac{(x+4)(x-3)}{x+3}$

no holes

no common factor to cancel

On your whiteboard...

$y=\frac{x-5}{x^2-2x-15} = \frac{\cancel{x-5}}{\cancel{(x-5)}(x+3)} = \frac{1}{x+3}$

Find any holes: (5, 1/8)

$x-5=0$
$x=5$

$y \Rightarrow \frac{1}{5+3} = \frac{1}{8}$

KEY

On your whiteboard...

$$y=\frac{(x-1)(x^2+4x+4)}{x^2+2x-3} = \frac{\cancel{(x-1)}(x+2)(x+2)}{(x+3)\cancel{(x-1)}}$$

Find any holes: (1, 9/4)

$x-1=0$
$x=1$

$$y \Rightarrow \frac{(1+2)(1+2)}{(1+3)} = \frac{(3)(3)}{4} = \frac{9}{4}$$

Steps 2 - 4: Find vertical asymptotes, zeros, and y-intercept

1] Set the denominator equal to zero to find any vertical asymptotes
2] Set the numerator equal to zero to find any zeros
3] Substitute 0 for x and solve for y to find the y-intercept, if any

A] $y=\dfrac{2x^2-8}{x^2-x-6} = \dfrac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-3)}$

V.A. $x-3=0$
$x=3$

zeros: $2(x-2)=0$
$x-2=0$
$x=2$
$(2,0)$

y-int: $(0,\frac{4}{3})$
$\dfrac{2(0-2)}{(0-3)} = \dfrac{4}{3}$

B] $y=\dfrac{x^2+x-12}{x+3} = \dfrac{(x+4)(x-3)}{x+3}$

V.A. $x+3=0$
$x=-3$

zeros: $x+4=0$
$x=-4$
$x-3=0$
$x=3$
$(-4,0)$
$(3,0)$

y-int: $(0,-4)$
$\dfrac{(0+4)(0-3)}{(0+3)} = -4$

KEY

On your whiteboard...

$$y=\frac{x-5}{x^2-2x-15}=\frac{\cancel{x-5}}{\cancel{(x-5)}(x+3)}=\frac{1}{x+3}$$

Find any holes: (5, 1/8)

Find the vertical asymptote: x= -3

Find any zeros: none

Find the y-intercept: (0,1/3)

V.A. $x+3=0$

$x=-3$

zeros:

$1\neq 0$

none

y-int: $(0,\frac{1}{3})$

$\frac{1}{0+3}=\frac{1}{3}$

On your whiteboard...

$$y=\frac{(x-1)(x^2+4x+4)}{x^2+2x-3}=\frac{\cancel{(x-1)}(x+2)(x+2)}{\cancel{(x-1)}(x+3)}$$

Find any holes: (1, 9/4)

Find the vertical asymptote: x= -3

Find any zeros: x= -2 with multiplicity 2

Find the y-intercept: (0, 4/3)

V.A. $x+3=0$

$x=-3$

zeros:

$x+2=0$

$x+2=0$

$x=-2$ w/mult. 2

y-int: $(0,\frac{4}{3})$

$\frac{(0+2)(0+2)}{(0+3)}=\frac{4}{3}$

KEY

Step 5: Find horizontal OR oblique (SLANT) asymptote (can't have both)

1] If the degree of the numerator is LESS than the degree of the denominator, then y=0 is the horizontal asymptote.
2] If the degree of the numerator EQUALS the degree of the denominator, then the ratio of leading coefficients is the horizontal asymptote.
3] If the degree of the numerator is MORE than the degree of the denominator, then divide to find the asymptote, ignoring the remainder. When the difference in degree is one, the asymptote will be oblique.

A] $y=\dfrac{2x^2-8}{x^2-x-6}=\dfrac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-3)}$

degrees are equal so $\frac{2}{1}=2$

horizontal asymptote at $y=2$

B] $y=\dfrac{x^2+x-12}{x+3}=\dfrac{(x+4)(x-3)}{x+3}$

degree of top is 1 more than bottom

$$\begin{array}{r|rrr} -3 & 1 & 1 & -12 \\ & \downarrow & -3 & 6 \\ \hline & 1 & -2 & -6:R \\ & X & C & \end{array}$$

oblique asymptote at $y=X-2$

On your whiteboard...

$y=\dfrac{x-5}{x^2-2x-15}=\dfrac{\cancel{x-5}}{\cancel{(x-5)}(x+3)}=\dfrac{1}{x+3}$ $\dfrac{\text{degree: }0}{\text{degree: }1}$

so H.A. is at $y=0$

Find any holes: (5, 1/8)

Find the vertical asymptote: x= -3

Find any zeros: none

Find the y-intercept: (0,1/3)

Find the horizontal or ~~oblique~~ asymptote: y= 0

KEY

On your whiteboard...

$$y=\frac{(x-1)(x^2+4x+4)}{x^2+2x-3}=\frac{\cancel{(x-1)}(x+2)(x+2)}{\cancel{(x-1)}(x+3)}$$

$\dfrac{\text{degree}:2}{\text{degree}:1}$

Find any holes: (1, 9/4)

Find the vertical asymptote: x= -3

$\dfrac{x^2+4x+4}{x+3}$

Find any zeros:
x= -2 with multiplicity 2

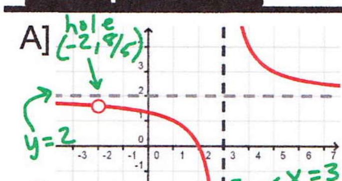
Find the y-intercept: (0, 4/3)

$$\begin{array}{r|rrr} -3 & 1 & 4 & 4 \\ & \downarrow & -3 & -3 \\ \hline & 1 & 1 & 1:R \\ & x & c & \end{array}$$

Find the ~~horizontal~~
or oblique asymptote: y=x+1

Steps 6 - 7: Graph, Domain, and Range

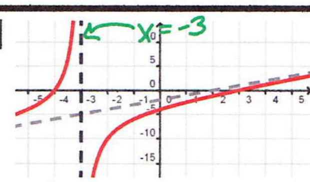
A]



Domain:
$(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

Range:
$(-\infty,8/5)\cup(8/5,2)\cup(2,\infty)$

B]



Domain:
$(-\infty,-3)\cup(-3,\infty)$

Range:
$(-\infty,\infty)$

KEY

On your whiteboard...

$$y=\frac{x-5}{x^2-2x-15}=\frac{\cancel{x-5}}{\cancel{(x-5)}(x+3)}=\frac{1}{x+3}$$

Find any holes: (5, 1/8)

Find the vertical asymptote: x= -3

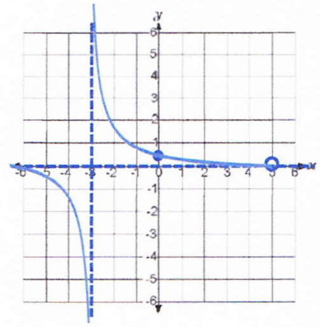
Find any zeros: none

Find the horizontal or
oblique asymptote: y= 0

Find the y-intercept: (0,1/3)

Find the domain and range:
domain is (-∞,-3)U(-3,5)U(5,∞)
range is (-∞,0)U(0,1/8)U(1/8,∞)

Sketch the graph:



On your whiteboard...

$$y=\frac{(x-1)(x^2+4x+4)}{x^2+2x-3}=\frac{\cancel{(x-1)}(x+2)(x+2)}{\cancel{(x-1)}(x+3)}$$

Find any holes: (1, 9/4)

Find the vertical asymptote: x= -3

Find any zeros:
x= -2 with multiplicity 2

Find the horizontal
or oblique asymptote: y=x+1

Find the y-intercept: (0, 4/3)

Find the domain and range:
domain (-∞,-3)U(-3,1)U(1,∞)
range (-∞,-4]U[0,∞)

Sketch the graph: