Rewriting Exponential Functions For Equivalent Rates

Class Notes

At times, it is appropriate to rewrite an exponential function in a different form to highlight or find a different value. Today, we will be rewriting to identify an equivalent interest rate.

<u>In order to rewrite an exponential function to find an equivalent rate you will</u>

1. Write the function out using the given information
2. Determine which variable will be changing (the exponent)
3. Use properties of rational exponents to rewrite as necessary $(b^x)^y = b^{x \cdot y}$
4. Simplify your new function
5. Use the values in your new function to determine equivalent rates

Example:

The value of a brand new car is $32,000 and the value depreciates 25% every year. Write a function to represent the value of the car after t years, where the *quarterly* rate of change can be found from a constant in the function. Round all coefficients in the function to four decimal places. Also, determine the percentage rate of change per quarter, *to the nearest hundredth of a percent*.

Guess: 6.25%

$A = P(1-r)^t$

$32000(1-.25)^t$

$32000(.75)^t$

$32000(.75^{1/4})^{4t}$

$32000(.9306)^{4t}$

$1-.9306 = .0694 \rightarrow 6.94\%$

Example:

The number of users on a website is 6700 and is growing exponentially at a rate of 79% per year. Write a function to represent the number of users on the website after t years, where the *daily* rate of change can be found from a constant in the function. Round all coefficients in the function to four decimal places. Also, determine the percentage rate of change per day, *to the nearest hundredth of a percent*.

$A = P(1+r)^t$

$6700(1+.79)^t$

$6700(1.79)^t$

$6700(1.79^{1/365})^{365t}$

$6700(1.0016)^{365t}$

$1.0016-1 = .0016 \rightarrow .16\%$

Annual Equivalent Rates (AER)

Annual equivalent rates can be used to compare two accounts with different compounding periods.

Annual equivalent rates give us a "level playing field" to compare.

$$AER = \left(1 + \frac{r}{n}\right)^n - 1$$

If this looks familiar, it should! AER just calculates the interest earned in one year!

Example

Find the annual equivalent rate for an account that accrues 4.9% interest compounded weekly.

$AER: \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.049}{52}\right)^{52} - 1 = .0502$

5.02%

How does that compare to an account that earns 4.75% compounded daily?

$AER: \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.0475}{365}\right)^{365} - 1 = .0486$

4.86%

Difference: .16%