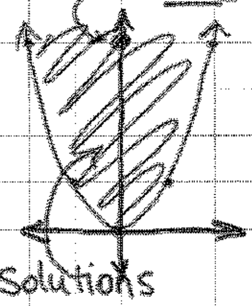
Quadratic Inequalities

Determine whether the given ordered pair is a solution of each inequality.

evaluate

a. $y \geq x^2$

(0, 4) Solution



Solutions

b. $y < -x^2 + 6x$

(6, −5)

$-5 < -(6)^2 + 6(6)$

$-5 < 0$ TRUE

Solution

c. $y \geq 2x^2 + 3x + 2$

(−3, 4)

$4 \geq 2(-3)^2 + 3(-3) + 2$

$\cancel{4 \geq} 11$ FALSE

NOT a Solution

Graph each inequality.

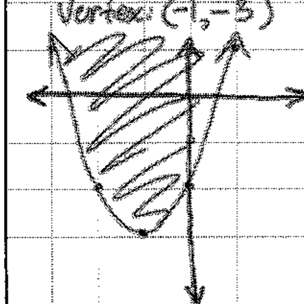
$x = \frac{-b}{2a}$

dotted

a. $y \geq x^2 + 2x - 2$

$x = -\frac{2}{2} = -1$

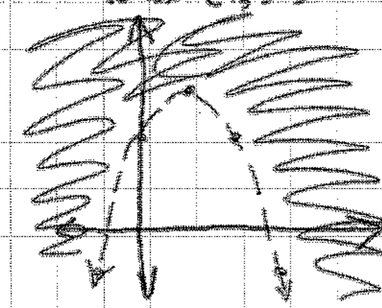
Vertex: (−1, −3)



b. $y > -x^2 + 2x + 2$

$x = \frac{-2}{-2} = 1$

Vertex: (1, 3)



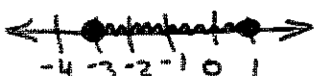
Solve each inequality algebraically.

a. $x^2 + 2x \leq 3$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3 \quad x = 1$



$x = 0$

$0 \leq 3$

SOLUTION: $[-3, 1]$

b. $2x^2 + 3x > 5$

open circles

$2x^2 + 3x - 5 = 0$

$(2x+5)(x-1) = 0$

$x = -2.5 \quad x = 1$

$\cancel{x = 0}$

$\cancel{0 > 5}$

SoL: $(-\infty, -2.5) \cup (1, \infty)$

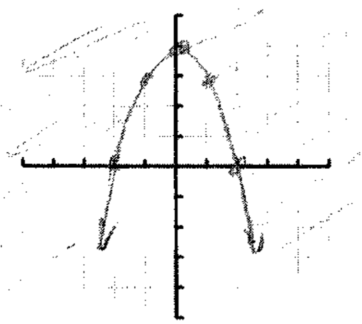
Pg 41-44

AC Math 1
Quadratic Inequalities WS

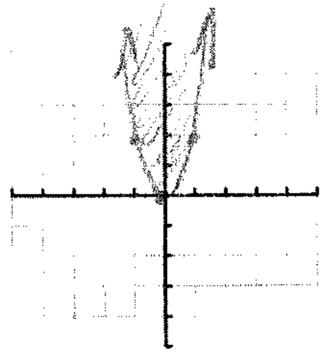
Name [illegible] Key

Graph each quadratic inequality.

1. $y \ge -x^2 + 4$



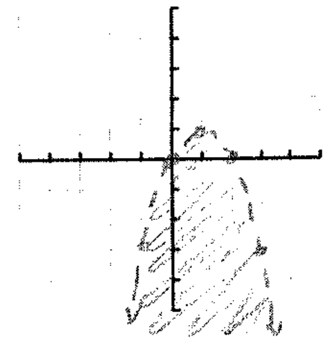
2. $y \ge 2x^2$



3. $y < -x^2 + 2x$

$x = -\frac{2}{-2} = 1$

$(1,1)$

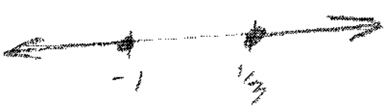


Graph each quadratic inequality algebraically (using a number line). State the solution set in interval notation.

4. $3x^2 + 2x - 1 \ge 0$

$(3x-1)(x+1)=0$

$x=\frac{1}{3}$ $\quad x=-1$



$(-\infty,-1] \cup [\frac{1}{3},\infty)$

5. $0 \ge 2x^2 + x - 3$

$(2x+3)(x-1)=0$

$x=-\frac{3}{2}$ $\quad x=1$

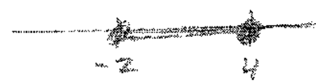
$[-\frac{3}{2}, 1]$

6. $0 \le -x^2 + 2x + 8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4$ $\quad x=-2$



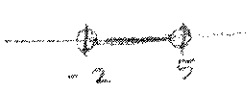
$[-2,4]$

7. $x^2 < 3x + 10$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5$ $\quad x=-2$



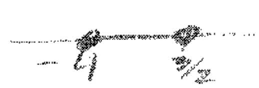
$(-2,5)$

8. $2x^2 + 5x \le 12$

$2x^2+5x-12=0$

$(2x-3)(x+4)=0$

$x=\frac{3}{2}$ $\quad x=-4$



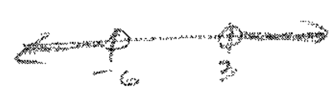
$[-4, \frac{3}{2}]$

9. $x^2 + 3x > 18$

$x^2+3x-18=0$

$(x+6)(x-3)=0$

$x=-6$ $\quad x=3$



$(-\infty,-6) \cup (3,\infty)$

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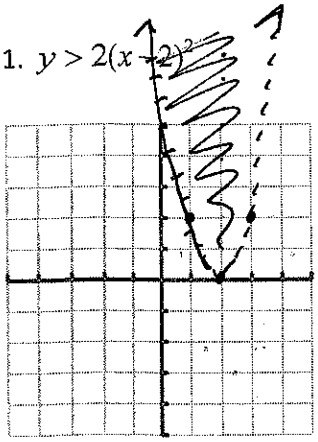
Quadratic Inequalities 2 WS Name Key

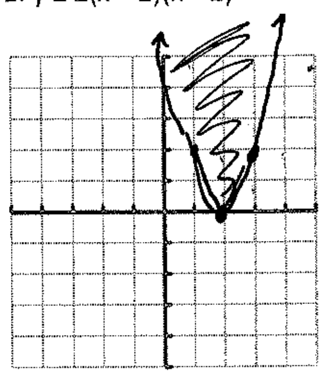
Graph each quadratic inequality.

$2(x^2-4x+4)$ $(x-2)(x-2)$

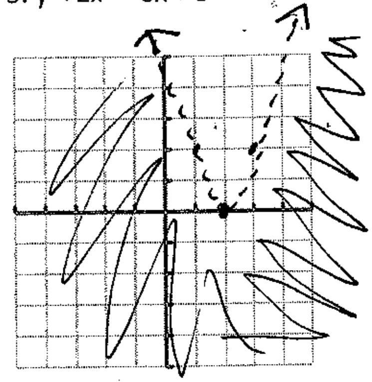
1. $y > 2(x-2)^2$



2. $y \geq 2(x-2)(x-2)$



3. $y < 2x^2 - 8x + 8$



4. What do graphs #1-3 have in common? all quadratics, all have same vertex / x-intercept

Solve each quadratic inequality algebraically (using a number line). State the solution set in interval notation.

5. $x^2 + 2x - 3 \geq 0$

$(x+3)(x-1) \geq 0$

$x = -3$ $x = 1$

-3 0 1

$(-\infty, -3] \cup [1, \infty)$

6. $9x^2 - 2 \leq -3x$

$9x^2 + 3x - 2 \leq 0$

$(3x+2)(3x-1) \leq 0$

$x = -2/3$ $x = 1/3$

$-2/3$ 0 $1/3$

$[-2/3, 1/3]$

7. $2x^2 - 8x > -6$

$2x^2 - 8x + 6 > 0$

$2(x^2 - 4x + 3) > 0$

$2(x-3)(x-1) > 0$

$x = 3$ $x = 1$

1 0 3

$(1, 3)$

8. $\frac{1}{2}x^2 + 3x \leq -6$

$\frac{1}{2}x^2 + 3x + 6 \leq 0$

no x-intercepts

9. $-2x^2 - 50 \geq -20x$

$0 \geq 2x^2 - 20x + 50$

$0 \geq 2(x^2 - 10x + 25)$

$0 \geq 2(x-5)(x-5)$

$x = 5$

0 5

$[5, \infty)$

10. $7x^2 - 8x > 0$

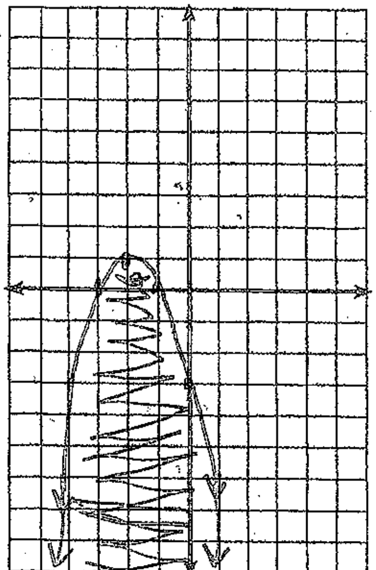
$x(7x-8) > 0$

$x = 0$ $x = 8/7$

-1 0 $8/7$ 2

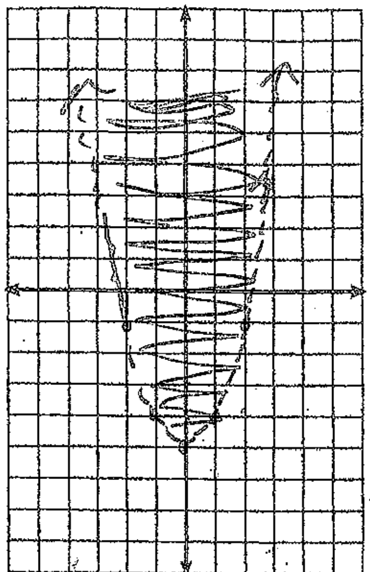
$(-\infty, 0) \cup (8/7, \infty)$

1.



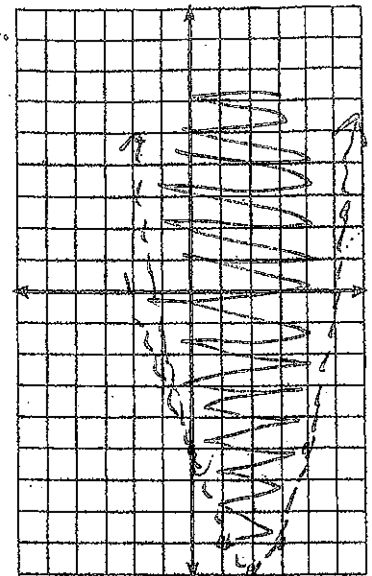
$y \le -(x+2)^2+1$

2.



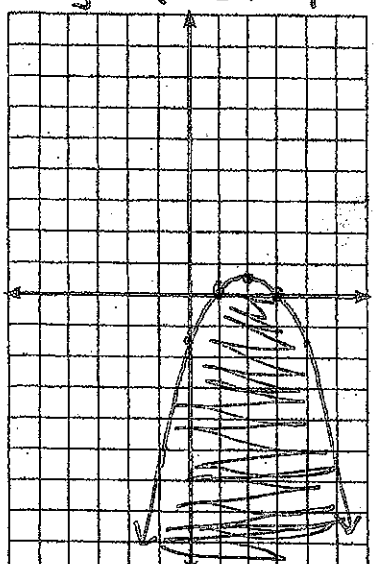
$y > x^2-5$

3.



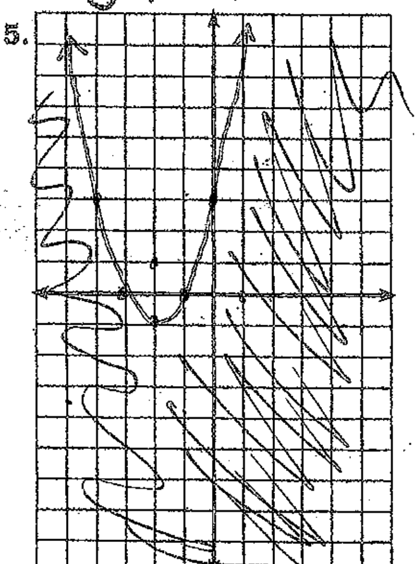
$y < (x-2)^2-9$

4.



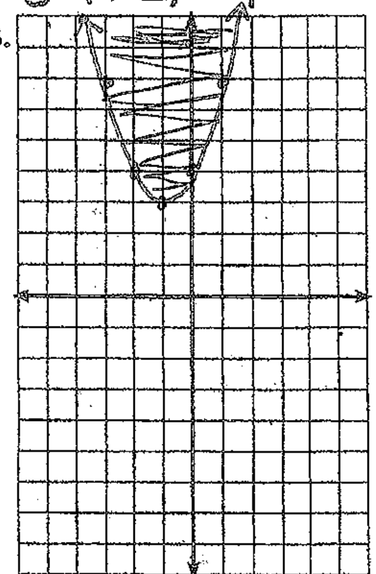
$y \le -\frac{1}{2}(x-1)(x-3)$

5.



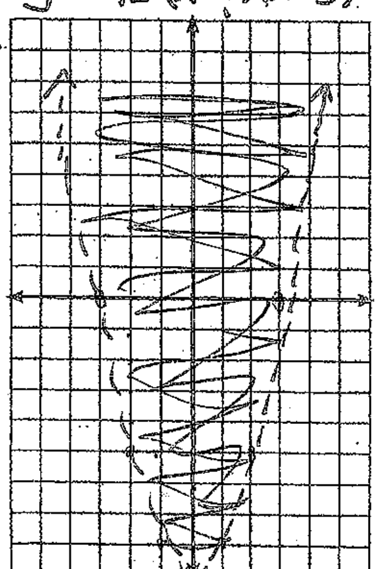
$y \le x^2+4x+3$ $(x+3)(x+1)$

6.



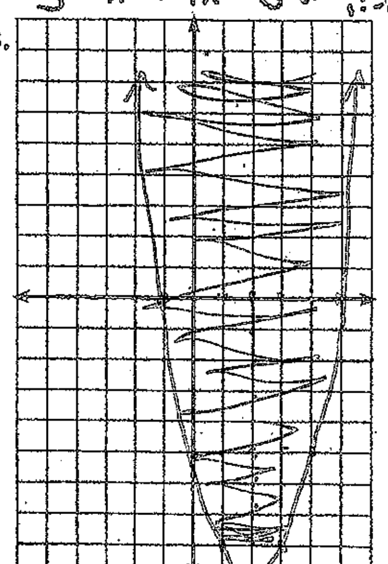
$y \ge (x+1)^2+3$

7.



$y > (x+3)(x-3)$

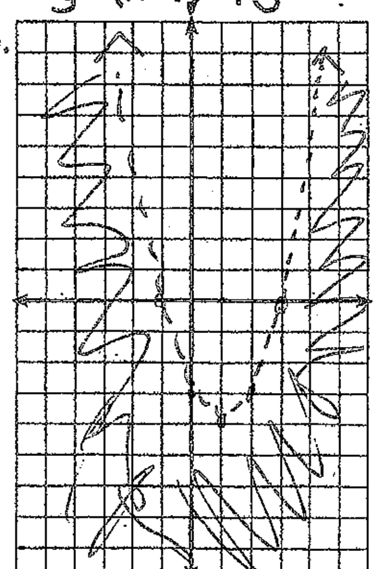
8.



$y \ge x^2-4x-5$

$(x-5)(x+1)$

9.



$y < x^2-2x-3$

$(x-3)(x+1)$

$(-3)(2)$

$-3\,(1)$

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Name: ______________________________ Date: ____________________

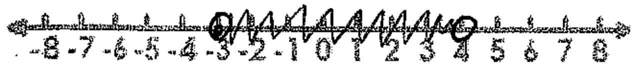
Solving Quadratic Inequalities

Find the solution set for each inequality:

1. $x^2 - x - 12 < 0$

$(x-4)(x+3)$

$x=4 \quad x=-3$



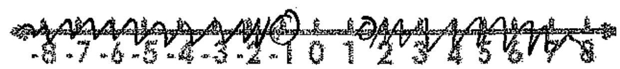
$(-3, 4)$

2. $3x^2 + 2x > 1$

$3x^2 + 2x - 1 > 0$

$(3x - 1)(x + 1) > 0$

$x = 1/3 \quad x = -1$

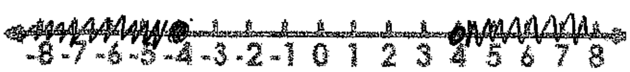


$(-\infty, -1) \cup (1/3, \infty)$

3. $x^2 - 16 \geq 0$

$(x-4)(x+4) \geq 0$

$x=4 \quad x=-4$



$(-\infty, -4] \cup [4, \infty)$

4. $5x^2 - 4x - 1 > 0$

$(5x+1)(x-1)$

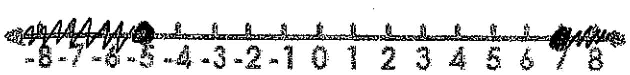
$x = -1/5 \quad x=1$

$(-\infty, -\frac{1}{5}) \cup (1, \infty)$

5. $x^2 - 2x - 35 \geq 0$

$(x-7)(x+5)$

$x=7 \quad x=-5$

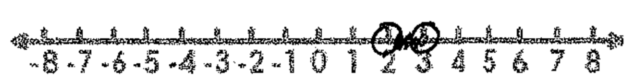


$(-\infty, -5] \cup [7, \infty)$

6. $x^2 - 5x + 6 < 0$

$(x-3)(x-2) < 0$

$x=3 \quad x=2$



$(2, 3)$

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7. $2x^2 + 5x \geq 3$

$2x^2 + 5x - 3 \geq 0$

$(2x - 1)(x + 3)$

$x = \frac{1}{2} \quad x = -3$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

$(-\infty, -3] \cup [\frac{1}{2}, \infty)$

8. $3x^2 > -14x + 5$

$3x^2 + 14x - 5 > 0$

$(3x - 1)(x + 5)$

$x = \frac{1}{3} \quad x = -5$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

$(-\infty, -5) \cup [\frac{1}{3}, \infty)$

9. $x^2 < -4x + 21$

$x^2 + 4x - 21 < 0$

$(x + 7)(x - 3)$

$x = -7 \quad x = 3$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

$(-7, 3)$

10. $6x^2 < -5x + 1$

$6x^2 + 5x - 1 < 0$

$(6x - 1)(x + 1)$

$x = \frac{1}{6} \quad x = -1$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

$(-1, \frac{1}{6})$

11. $2x^2 > -x + 15$

$2x^2 + x - 15 > 0$

$x = \frac{-1 \pm \sqrt{1 - 4(2)(-15)}}{2(2)} = \frac{-1 \pm \sqrt{61}}{4}$

$= 1.7 \text{ \& } -2.2$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

$(-\infty, \frac{-1 + \sqrt{61}}{4}) \cup (\frac{-1 - \sqrt{61}}{4}, \infty)$

12. $2x^2 + 11x + 5 < 0$

$(2x + 1)(x + 5)$

$x = -\frac{1}{2} \quad x = -5$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

$(-5, -\frac{1}{2})$

Graphing Quadratics with Characteristics

Vertex Form

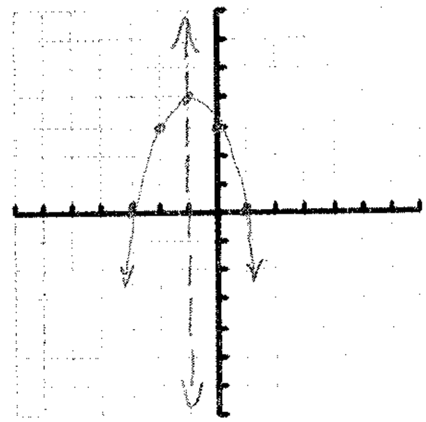
Steps:

1. **Vertex:**
   ordered pair: (opposite, same)
2. **Axis of Symmetry:**
   equation:
   x = x-coordinate of the VERTEX
3. **x-intercepts:**
   - x-intercept will either be your vertex
   - **or** graph your pattern from the vertex to find the x-intercepts
   - **or** there may not be any x-intercepts
   - **or** foil it out, add the constant and factor to solve
4. **y-intercept:**
   ordered pair: find f(0)

$f(x) = -(x+1)^2 + 4$

Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$
Vertex: $(-1, 4)$
Max or Min? max
Axis: $x = -1$
Zero(s): $x = -3$ $x = 1$
x-intercept(s): $(-3, 0)(1, 0)$
y-intercept: $(0, 3)$
End Behavior: as $x \to \pm\infty, f(x) \to -\infty$
Int. of Increase: $(-\infty, -1)$
Int. of Decrease: $(-1, \infty)$

Graph



Standard Form

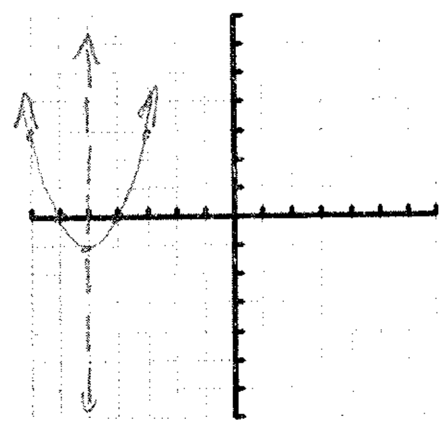
Steps:

1. **Axis of Symmetry:**
   $$x = \frac{-b}{2a}$$
2. **Vertex:**
   evaluate the function at the axis (plug in your x-value from the axis)
3. **x-intercepts:**
   - graph your pattern to see the x-untercepts
   - **or** factor (if you can) and solve for x to get the x-intercepts
4. **y-intercept:**
   find f(0)

$y = x^2 + 10x + 24$

Domain: $(-\infty, \infty)$
Range: $[-1, \infty)$
Vertex: $(-5, -1)$
Max or Min? min
Axis: $x = -5$
Zero(s): $x = -4$, $x = -6$
x-intercept(s): $(-4, 0)(-6, 0)$
y-intercept: $(0, 24)$
End Behavior: as $x \to \pm\infty, f(x) \to \infty$
Int. of Increase: $(-5, \infty)$
Int. of Decrease: $(-\infty, -5)$

Graph



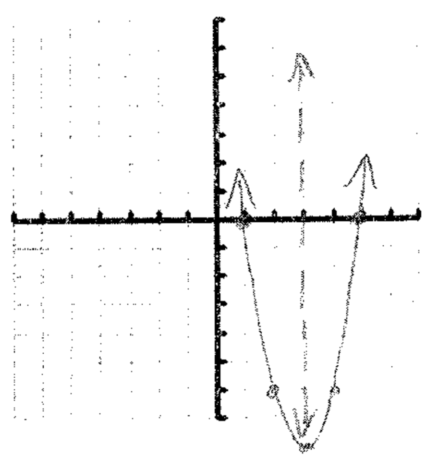
Intercept Form

Steps:

1. **x-intercepts:**
   set each factor equal to zero and solve for x
2. **Axis of Symmetry:**
   find the midpoint between the x-intercepts and draw a vertical line
3. **Vertex:**
   evaluate the function at the axis (plug in your x-value from the axis)
4. **y-intercept:**
   find f(0)

$y = 2(x-5)(x-1)$

Domain: $(-\infty, \infty)$
Range: $[-8, \infty)$
Vertex: $(3, -8)$
Max or Min? min
Axis: $x = 3$
Zero(s): $x = 5$ $x = 1$
x-intercept(s): $(5, 0)(1, 0)$
y-intercept: $(0, 10)$
End Behavior: as $x \to \pm\infty, f(x) \to \infty$
Int. of Increase: $(3, \infty)$
Int. of Decrease: $(-\infty, 3)$



Converting Quadratics

Vertex to Standard 1. square the binomial 2. distribute 3. combine like terms	$y = -(x+1)^2+3$ $y = -(x^2+2x+1)+3$ $y = -x^2-2x-1+3$ $y = -x^2-2x+2$
Intercept to Standard 1. multiply the binomials first 2. distribute	$y = 2(x-1)(x-5)$ $y = 2(x^2-6x+5)$ $y = 2x^2-12x+10$
Standard to Intercept 1. Factor out the GCF 2. Factor any other way	$y = 2x^2+20x+48$ $y = 2(x^2+10x+24)$ $y = 2(x+6)(x+4)$
Standard to Vertex 1. group the x^2 and x terms in parentheses 2. factor out the leading coefficient (from x^2 and x) 3. complete the square: $\left(\frac{b}{2}\right)^2$ in the parenthesis +____ 4. balance the equation: outside the parenthesis −____ 5. Factor the trinomial, it should be $(\quad)^2$ 6. combine like terms at the end	*ex 1:* $y = x^2+10x+24$ $y = (x^2+10x+25)+24-25$ $y = (x+5)^2-1$ *ex 2:* $y = 3x^2+12x+14$ $y = 3(x^2+4x+4)+14-12$ $y = 3(x+2)^2+2$

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Quadratic Inequalities

Graphing

Steps:

1. **Graph the Quadratic**
2. **Solid or Dotted**

   ≥ or ≤

   Then the parabola is a SOLID curve

   < or >

   Then the parabola is a DASHED curve

3. **Shade**

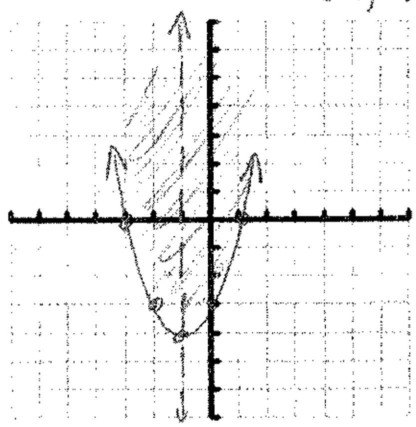
   If you have a **>** or **≥** symbol, shade where **y** is getting larger

   If you have a **<** or **≤** symbol, shade where **y** is getting smaller

1. $y \geq x^2 + 2x - 3$

$x = -1$

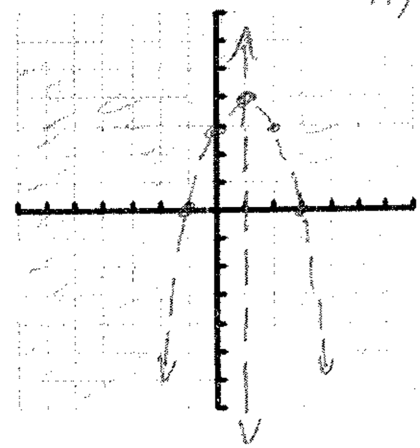
$(-1, -4)$



2. $y > -x^2 + 2x + 3$

$x = 1$

$(1, 4)$



Solving Algebraically

Steps:

1. **Equation** change to an equation
2. **Set equation = 0**
3. **Solve by factoring**
4. **Graph solutions on a number line**
5. **Test 3 numbers** to determine which interval(s) are solution(s)
6. **Write answer** in interval notation.

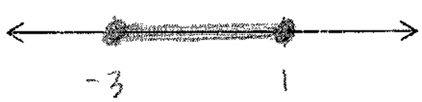
1. $\mathbf{x^2 + 2x \leq 3}$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

$8 \leq 3$ F　　$0 \leq 3$ T　　$4 \leq 3$ F



$[-3, 1]$

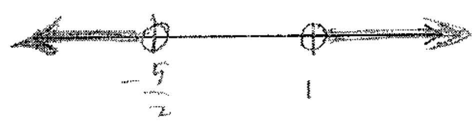
2. $2x^2 + 3x > 5$

$2x^2 + 3x - 5 = 0$

$(2x+5)(x-1) = 0$

$x = -\frac{5}{2} \quad x = 1$

$9 > 5$ T　　$0 > 5$ F　　$14 > 5$ T



$(-\infty, -\frac{5}{2}) \cup (1, \infty)$

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Modeling with Quadratics

Evaluating a Function **Hint**: Evaluate the function by finding $h(1)$.	A ball is hit into the air from a height of 4 feet. The function $g(t) = -16t^2 + 120t + 4$ can be used to model the height of the ball where t is the time in seconds after the ball is hit. Find the height of the ball after 1 second. $g(1) = -16 + 120 + 4 = 108$ ft.
Analyzing the Vertex **Hint**: 1. Find the axis of symmetry. (This is the time at which the ball reaches maximum height.) 2. Find the vertex by evaluating the function. The second coordinate in the vertex represents the maximum height.	A ball is hit into the air from a height of 4 feet. The function $g(t) = -16t^2 + 120t + 4$ can be used to model the height of the ball where t is the time in seconds after the ball is hit. What is the maximum height the ball reaches? $t = \frac{-120}{-32} = 3.75$ seconds $g(3.75) = 229$ ft.
Analyzing the Zeros **Hint**: 1. Set the quadratic equal to zero and solve. 2. Disregard solutions that are negative (since time does not go backwards!) 3. The remaining zero indicates the time at which the ball hits the ground.	A ball is hit into the air from a height of 4 feet. The function $g(t) = -16t^2 + 120t + 4$ can be used to model the height of the ball where t is the time in seconds after the ball is hit. How long is the ball in the air? $-16t^2 + 120t + 4 = 0$ $4t^2 - 30t - 1 = 0$ $t = 7.53$ seconds