

Honors Algebra II

Unit 3A – Polynomial Functions

October 14	PSAT Go over test (if graded)
October 15	<i>The Remainder & Factor Theorems</i> Warm-up Intro / Notes with SMART Finding Polynomial Roots Given Roots WS
October 16	<i>The Rational Root Theorem and Fundamental Theorem</i> Uses for Synthetic Substitution Warm-up / Graphic Organizer Notes with SMART Finding Polynomial Roots WS 1
October 19	<i>Polynomial Functions – Finding Zeros – Use Technology</i> Finding Polynomial Roots WS 2
October 20	<i>Polynomial Functions</i> Quiz 3
October 21	<i>Graphing Polynomial Functions</i> Sketching Polynomials PowerPoint w/ Outline Graphing Polynomials WS 1
October 22	<i>Graphing Polynomial Functions</i> Graphing Warm-up Graphing Polynomials WS 2
October 23	<i>Analyzing Polynomial Functions</i> Notes with SMART Analyzing Polynomial Graphs WS 1 Analyzing Polynomial Graphs WS 2
October 26	<i>Linear and Polynomial Systems</i> Quiz 4 Notes Linear and Polynomial Systems WS Linear and Polynomial Systems
October 27	<i>Polynomial Functions-CFA</i> Review WS 1
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October 29	<i>Polynomial Functions</i> Test 2

The Remainder and Factor Theorems

Warm-up intro ... Divide using synthetic division.

a. $(x^3 - 2x^2 - 2x + 8) \div (x - 1)$ b. $(x^3 - x^2 - 2x + 8) \div (x + 2)$

Mar 13-11:29 AM

The Remainder Theorem

The Factor Theorem

Mar 13-11:32 AM

Example 1:

Factor $f(x) = 3x^3 + 13x^2 + 2x - 8$ given that $f(-4) = 0$.

Mar 13-11:32 AM

Example 2:

One zero of $f(x) = x^3 + 6x^2 + 3x - 10$ is $x = -5$.

Find the other zeros of the function.

Mar 13-11:32 AM

Example 3:

One zero of $f(x) = 5x^3 - 27x^2 - 17x - 6$ is $x = 6$.

Find the other zeros of the function.

Mar 13-11:33 AM

Finding Polynomial Roots Given Roots WS
using the The Remainder and Factor Theorems

Name _____

Given one zero of the polynomial function, find the other zeros.

1. $f(x) = x^3 - 8x^2 + 5x + 14$; 2
2. $f(x) = 12x^3 + 8x^2 - 13x + 3$; $\frac{1}{2}$
3. $f(x) = x^3 + 3x^2 - 34x + 48$; 3
4. $f(x) = x^3 + 2x^2 - 20x + 24$; -6
5. $f(x) = 2x^3 + 3x^2 - 3x - 2$; -2
6. $f(x) = 2x^3 + 11x^2 + 9x + 2$; $-\frac{1}{2}$

Given two zeros of the polynomial function, find the other zeros.

7. $f(x) = x^4 + 6x^3 - 4x^2 - 54x - 45$; -3, 3
8. $f(x) = x^4 - 3x^3 + 3x - 1$; -1, 1
9. $f(x) = 2x^4 - 9x^3 + 4x^2 + 21x - 18$; 2, 3
10. $f(x) = 3x^4 - 2x^3 - 12x^2 + 6x + 9$; $-\sqrt{3}, \sqrt{3}$
11. $f(x) = x^4 + 2x^3 - 14x^2 - 32x - 32$; -4, 4
12. $f(x) = x^4 + 3x^3 + 7x^2 + 15x + 10$; -2, -1

Answers:

- | | |
|--------------------------------|---|
| 1. $x = -1, 2, 7$ | 2. $x = -\frac{3}{2}, \frac{1}{2}, \frac{1}{3}$ |
| 3. $x = -8, 2, 3$ | 4. $x = -6, 2$ |
| 5. $x = -2, -\frac{1}{2}, 1$ | 6. $x = -\frac{1}{2}, \frac{-5 \pm \sqrt{17}}{2}$ |
| 7. $x = \pm 3, -5, -1$ | 8. $x = \pm 1, \frac{3 \pm \sqrt{5}}{2}$ |
| 9. $x = -\frac{3}{2}, 1, 2, 3$ | 10. $x = \pm \sqrt{3}, \frac{1 \pm \sqrt{10}}{3}$ |
| 11. $x = \pm 4, -1 \pm i$ | 12. $x = -2, -1, \pm i\sqrt{5}$ |

Evaluating Polynomial Functions

Dividing Polynomials

Uses for
Synthetic Substitution/Division

Finding Zeros of a Function

Factoring a Polynomial

Copy and complete the following in the appropriate space on your graphic organizer. Use synthetic substitution!

1. Evaluate $f(x) = 4x^4 + 2x^3 - x + 7$ when $x = -2$.
2. Divide: $(2x^4 + 3x^3 + 3x + 17) \div (x + 2)$
3. Find the zeros of the function $f(x) = 12x^3 + 8x^2 - 13x + 3$ given that $\frac{1}{2}$ is one of the zeros.
4. Factor $f(x) = 3x^3 - 11x^2 - 6x + 8$ given that $(x - 4)$ is one factor.

The Rational Root Theorem and The Fundamental Theorem of Algebra

The Rational Root Theorem:

If $f(x) = a_n x^n + \dots + a_1 x + a_0$ has integer coefficients,
then every rational zero of $f(x)$ has the form:

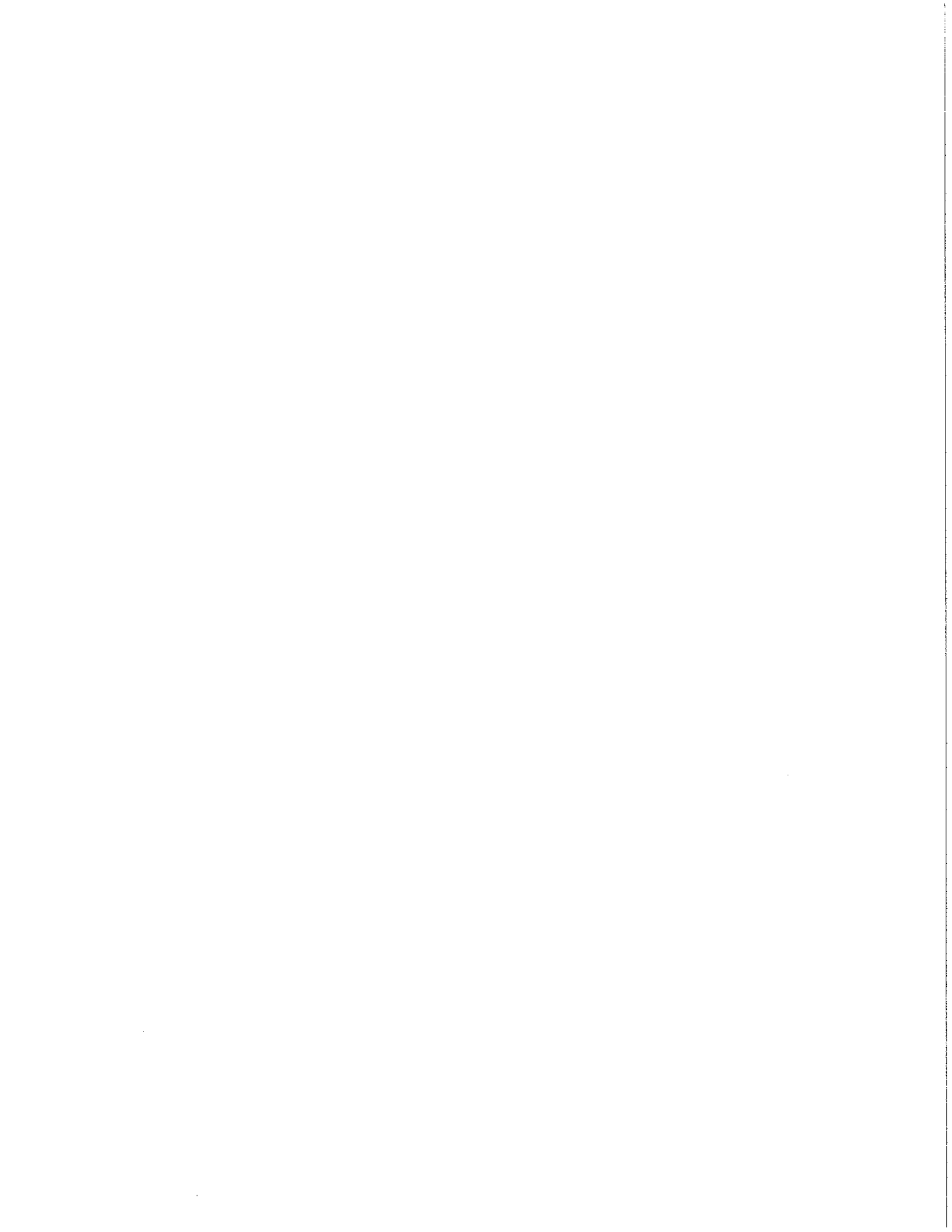
$$\frac{p}{q} = \pm \frac{\text{factors of } a_0}{\text{factors of } a_n}$$

Example 1:

List the possible rational zeros of $f(x) = x^3 - 4x^2 - 11x + 30$. Find the zeros.

Example 2:

List the possible rational zeros of $f(x) = 15x^4 - 68x^3 - 7x^2 + 24x - 4$. Find the zeros.



Find ALL roots of each of the following.

1. $P(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$

2. $P(x) = x^3 - 4x^2 + x - 4$

3. $P(x) = x^4 - 16$

4. $P(x) = x^4 - 1$

5. $P(x) = x^3 - x^2 - 7x + 15$

6. $P(x) = x^3 - 6x^2 + 13x - 20$

7. $P(x) = x^3 - 8$

8. $P(x) = x^3 + 8$

9. $P(x) = x^3 + 3x^2 - 2x - 6$

10. $P(x) = x^3 - x^2 - 3x + 3$

11. $P(x) = 5x^4 - 4x^3 + 19x^2 - 16x - 4$

12. $P(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$

13. $P(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

14. $P(x) = x^3 + 3x^2 - x - 3$

15. $P(x) = x^4 + 5x^3 - 27x^2 + 31x - 10$

16. $P(x) = x^3 + 5x^2 - x - 5$

17. $P(x) = 4x^3 - 3x^2 + 4x - 3$

18. $P(x) = 2x^3 - 3x^2 - x + 1$

19. $P(x) = x^5 - 5x^4 + 5x^3 + 15x^2 - 36x + 20$

20. $P(x) = x^5 - 3x^4 - 3x^3 + 9x^2 - 4x + 12$

Answers:

- 1) 2, 3, ±i 2) 4, ±i 3) ±2, ±2i 4) ±1, ±i 5) -3, 2±i 6) 4, 1±2i 7) 2, -1±i√3 8) -2, 1±i√3
 9) -3, ±√2 10) 1, ±√3 11) 1, -1/5, ±2i 12) -1, 1/3, 1±i 13) -2, -1, 3±√13 14) -3, ±1
 15) 1, 2, -4±√21 16) 5, ±1 17) ¾, ±i 18) ½, ½ ± ½√5 19) 1, ±2, 2±i 20) 3, ±2, ±i

Find all the roots of each function.

Function
1. $f(x) = x^3 - 11x^2 + 40x - 50$
2. $f(x) = x^3 + 2x^2 - x - 2$
3. $f(x) = x^3 - 4x^2 + 6x - 4$
4. $f(x) = x^3 + 2x^2 - 3x + 20$
5. $f(x) = 2x^3 - x^2 + 28x + 51$
6. $f(x) = 2x^3 - 17x^2 + 90x - 41$
7. $f(x) = x^3 - 3x^2 + 9x + 13$
8. $f(x) = x^3 - 10x^2 + 34x - 40$
9. $f(x) = x^4 - 6x^3 + 12x^2 + 6x - 13$
10. $f(x) = x^4 + 7x^3 + 13x^2 - 23x - 78$
11. $f(x) = x^3 - 7x^2 + 17x - 15$
12. $f(x) = 2x^4 - 9x^3 + 2x^2 + 21x - 10$

Answers:

- 1) $5, 3 \pm i$ 2) $1, -2, -1$ 3) $2, 1 \pm i$ 4) $-4, 1 \pm 2i$ 5) $-3/2, 1 \pm 4i$ 6) $1/2, 4 \pm 5i$
 7) $-1, 2 \pm 3i$ 8) $4, 3 \pm i$ 9) $\pm 1, 3 \pm 2i$ 10) $-3, 2, -3 \pm 2i$ 11) $3, 2 \pm i$ 12) $1/2, 2, 1 \pm \sqrt{6}$

Zeros and Multiplicity:

Multiplicity determines if a function crosses the x-axis or bounces off the x-axis.

Even multiplicity:

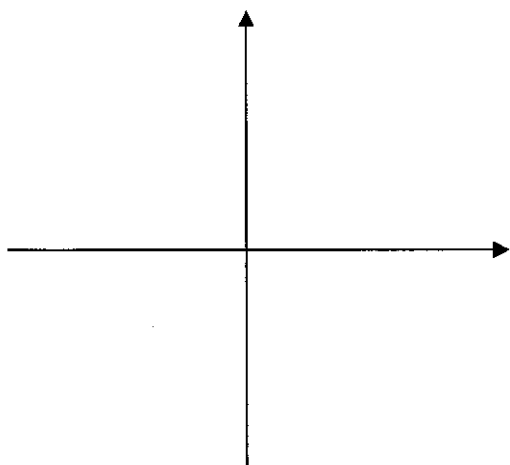
Odd multiplicity:

Degree and End-Behavior:

	Even degree	Odd Degree
Positive leading coefficient		
Negative leading coefficient		

Sketch each function, showing zeros, y-intercept, and end behavior. Identify the characteristics of each function.

1. $f(x) = (x-1)^2(x+1)$



Zeros multiplicity cross/bounce?

y-intercept? _____

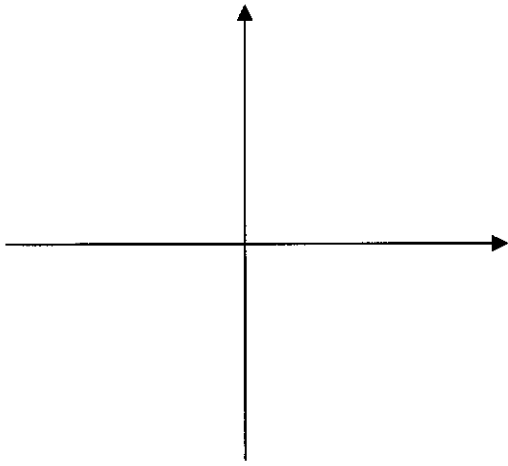
Degree of Polynomial Function? _____

Pos./Neg. Leading Coefficient? _____

Turning Points? _____

Describe the end behavior?

2. $f(x) = -\frac{1}{8}(x+1)(x-2)(x-4)$



Zeros multiplicity cross/bounce?

_____	_____	_____
_____	_____	_____
_____	_____	_____

y-intercept? _____

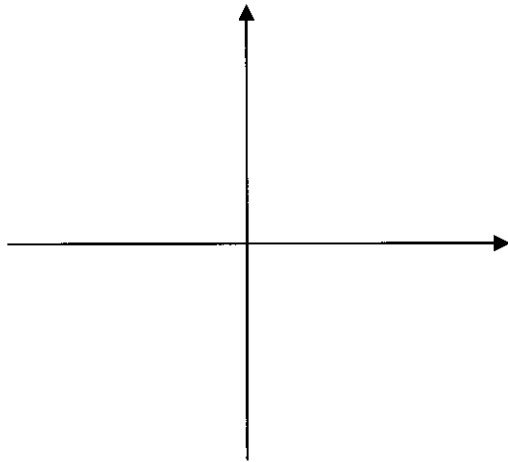
Degree of Polynomial Function? _____

Pos./Neg. Leading Coefficient? _____

Turning Points? _____

Describe the end behavior?

3. $f(x) = -\frac{1}{10}x(x-3)^2(x+2)$



Zeros multiplicity cross/bounce?

_____	_____	_____
_____	_____	_____
_____	_____	_____

y-intercept? _____

Degree of Polynomial Function? _____

Pos./Neg. Leading Coefficient? _____

Turning Points? _____

Describe the end behavior?

Graph the function. Identify x-intercepts (zeros), y-intercept, and correct end behavior on your graphs.

1. $f(x) = (x-1)(x+3)^2$

7. $f(x) = \frac{1}{8}(x+4)(x+2)(x-3)$

2. $f(x) = (x-1)(x+1)(x-3)$

8. $f(x) = 2(x+2)^2(x+4)^2$

3. $f(x) = \frac{1}{8}(x+1)(x-2)(x-4)$

9. $f(x) = 5(x-1)(x-2)(x-3)$

4. $f(x) = \frac{1}{5}(x-3)^2(x+1)^2$

10. $f(x) = \frac{1}{12}(x+4)(x-3)(x+1)^2$

5. $f(x) = (x-1)^3(x+1)$

11. $f(x) = (x+1)(x^2 - 3x + 3)$

6. $f(x) = \frac{1}{10}(x+3)(x+1)(x-4)$

12. $f(x) = (x+2)(2x^2 - 3x + 1)$

Graphing Warm-up:

For each, identify the degree and describe the end behavior. Sketch a graph of the function showing x-intercepts, y-intercepts, and approximate turning points.

1. $f(x) = \frac{1}{8}(x+4)(x+2)(x-3)$

2. $f(x) = 2(x+2)^2(x+4)^2$

Graphing Polynomials WS 2

Name _____

Graph the following polynomials.

1. $y = x^3 - 2x^2 - 9x + 18$

2. $y = 2x^5 - 14x^3 + 24x$

3. $y = 5x^3 + 6x^2 - 45x - 54$

4. $y = x^4 - 26x^2 + 25$

5. $f(x) = (x - 2)(x^2 + 2x + 1)$

6. $f(x) = (x - 3)(x^2 - 2x + 1)$

Analyzing Graphs of Polynomial Functions

Example 1: Find the following:

x-int:

y-int:

local maximum:

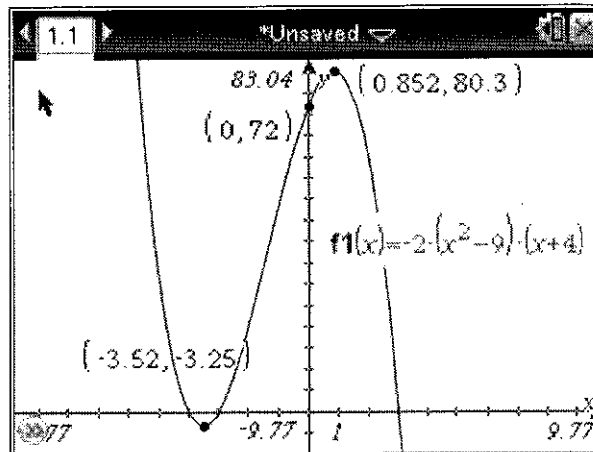
local minimum:

absolute maximum:

absolute minimum:

interval of increasing:

interval of decreasing:



Example 2: $f(x) = x^3 + 2x^2 - 5x + 1$. Find the following:

x-int:

y-int:

local maximum:

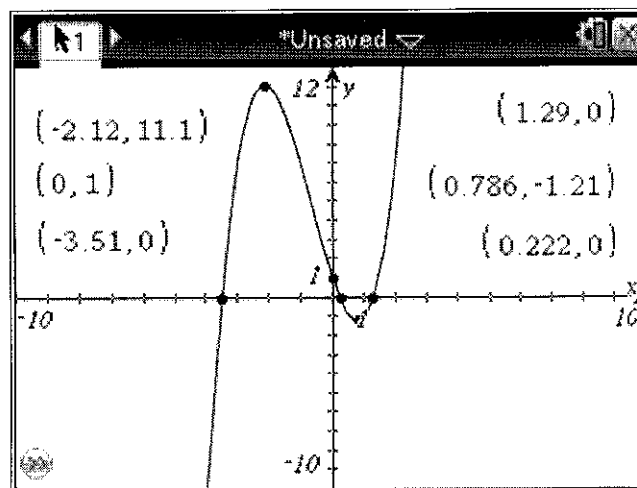
local minimum:

absolute maximum:

absolute minimum:

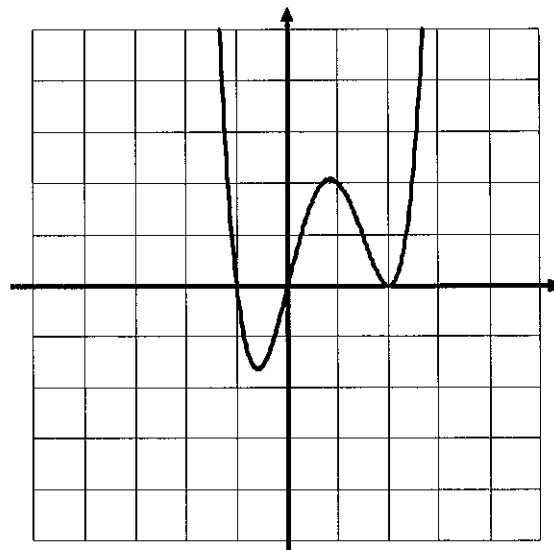
interval of increasing:

interval of decreasing:



Example 3:

List everything you know about the given function.



Using a graphing calculator, graph the following functions. For each function, determine intervals of increase and decrease, local maxima and minima, absolute maxima and minima, and zeros.

1. $f(x) = 3x^3 - 9x + 1$

interval of increase: _____

interval of decrease: _____

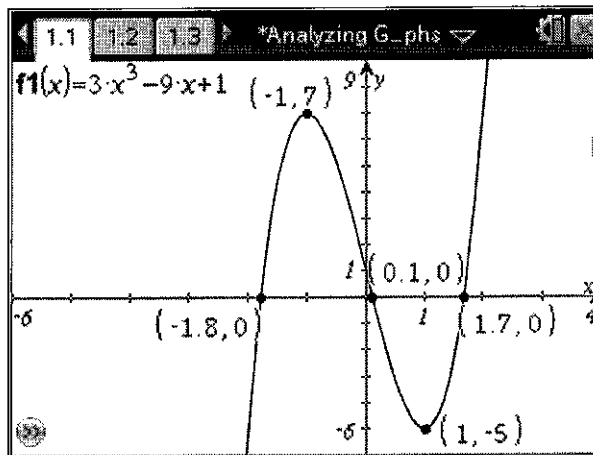
local maxima: _____

local minima: _____

absolute maxima: _____

absolute minima: _____

zeros: _____



2. $f(x) = x^4 - 2x^3 - 3x^2 + 5x + 2$

interval of increase: _____

interval of decrease: _____

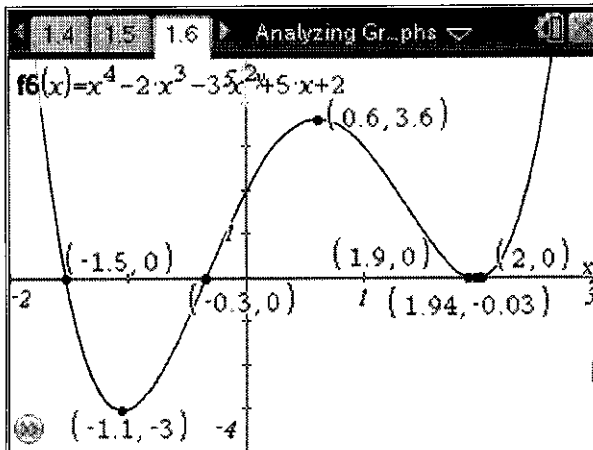
local maxima: _____

local minima: _____

absolute maxima: _____

absolute minima: _____

zeros: _____



3. $f(x) = x^5 - 6x^3 + 9x$

interval of increase: _____

interval of decrease: _____

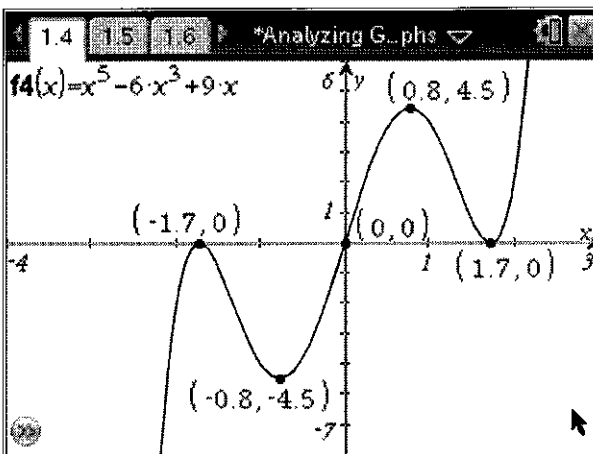
local maxima: _____

local minima: _____

absolute maxima: _____

absolute minima: _____

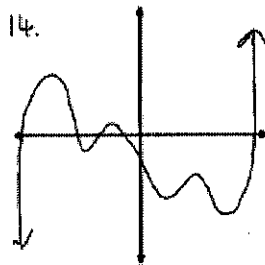
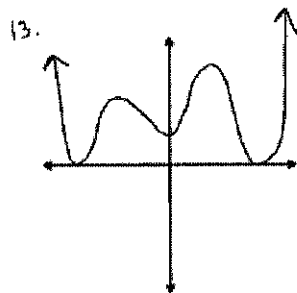
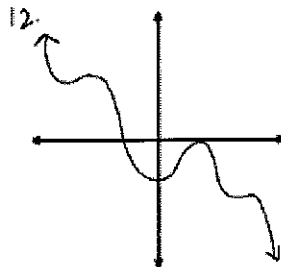
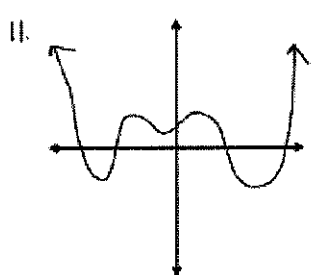
zeros: _____



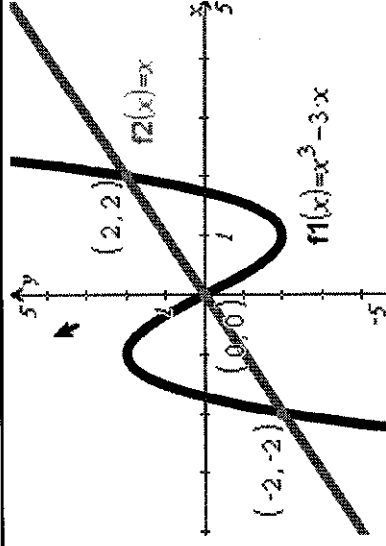
Sketch the graph of the polynomial function described or explain why no such function can exist.

1. Cubic function with two distinct negative zeros, one positive zero, and a positive y-intercept.
2. Cubic function with a negative double zero, a positive zero, and a negative leading coefficient.
3. Cubic function with one real zero, two complex zeros, and a positive leading coefficient.
4. Cubic function with no real zeros.
5. Quartic function with no real zeros.
6. Quartic function with two distinct positive zeros, two distinct negative zeros, and a negative y-intercept.
7. Quartic function with two double zeros.
8. Quartic function with two distinct real zeros and two complex zeros.
9. Quartic function with five distinct real zeros.
10. Quintic function with five distinct real zeros.

For the following polynomial functions, give the degree, the number of real zeros (counting double zeros as two), and the number of nonreal complex zeros the function could have.



Systems of Equations
Linear & Polynomial



Ex1: Solve algebraically.

$$\begin{cases} y = x^3 - 3x \\ y = x \end{cases}$$

Ex2: Solve algebraically.

$$\begin{cases} y = x^4 - 3x^3 \\ y = 10x^2 \end{cases}$$

Ex3: Solve algebraically.

$$\begin{cases} x^3 + y^3 = 7 \\ x + y = 1 \end{cases}$$

Systems of Equations WS
Linear & Polynomial

Name _____

Solve each system algebraically.

1.
$$\begin{cases} y = x^3 - 8x \\ y = x \end{cases}$$

2.
$$\begin{cases} y = x^3 \\ y = 2x \end{cases}$$

3.
$$\begin{cases} y = x^4 \\ y = x^2 + 2 \end{cases}$$

4.
$$\begin{cases} y = x^3 + x^2 \\ y = 3x^2 \end{cases}$$

5.
$$\begin{cases} y = x^4 - 5x^2 + 6 \\ y = 2 \end{cases}$$

6.
$$\begin{cases} x^3 + y^3 = -26 \\ x + y = -2 \end{cases}$$

7.
$$\begin{cases} x^3 - y^3 = -19 \\ x - y = -1 \end{cases}$$

8.
$$\begin{cases} x^3 + y^3 = 19 \\ x^3 - y^3 = -35 \end{cases}$$

Answers: 1. (0, 0), (3, 3), (-3, -3) 2. (0, 0), ($\sqrt{2}$, $2\sqrt{2}$), (-2, -2 $\sqrt{2}$) 3. ($\sqrt{2}$, 4), (- $\sqrt{2}$, 4)
4. (0, 0), (2, 12) 5. (± 2 , 2), (± 1 , 2) 6. (-3, 1), (1, -3) 7. (5, 6), (-2, -1) 8. (-2, 3)

Polynomial Functions
Review WS 1

Name _____

1. Graph $f(x) = 2(x + 2)^2(x + 4)^2$
2. One factor of $f(x) = x^3 - 8x^2 + 5x + 14$ is $(x - 2)$. Factor completely.
3. Find all the roots of $f(x) = x^4 - 3x^3 + 3x - 1$ if -1 and 1 are zeros.
4. Find all the roots: $f(x) = 2x^3 - 3x^2 - 11x + 6$
5. Find all the roots: $f(x) = x^5 - 3x^4 - 3x^3 + 9x^2 - 4x + 12$
6. Write the quadratic equation having roots $\frac{1}{2}$ and $\frac{3}{4}$.
7. Write the cubic function having roots 3 and $2 + i$

Answers:

- 1) x-intercepts @ $(-4, 0)$ bounce and $(-2, 0)$ bounce
y-intercept @ $(0, 128)$
relative maximum @ $(-3, 2)$
both ends go up
- 2) $f(x) = (x - 2)(x - 7)(x + 1)$
- 3) $x = \pm 1, \frac{3 \pm \sqrt{5}}{2}$
- 4) $x = -2, \frac{1}{2}, 3$
- 5) $x = 3, \pm 2, \pm i$
- 6) $y = 8x^2 - 10x + 3$
- 7) $y = x^3 - 7x^2 + 17x - 15$

Review WS 2

Find the zeros. If irrational leave in radical form. **Do not give decimal answers.**

1. $f(x) = x^4 + 4x^3 + 3x^2 - 4x - 4$

2. $f(x) = x^4 - 6x^3 + 12x^2 + 6x - 13$

3. $f(x) = x^3 - 3x^2 + 9x + 13$

4. Write the polynomial function with roots $2, \pm\sqrt{3}$.5. Write a polynomial function with the given roots: $-1, 3+i$

6. Given: $f(0) = 7, f(4) = 0, f\left(\frac{1}{3}\right) = 0, f(5) = 2$

a) name the factors of the function: _____

b) name the zeros of the function: _____

c) name the y-intercept of the function: _____

7. Given a function $h(x) = x^4 - 6x^3 + 6x^2 + 24x - 40$ and $3+i$ is a zero. Find the remaining zeros.8. Find the possible rational zeros of $f(x) = 2x^3 - x^2 - 7x + 6$.9. Find the remainder: $(x^3 - 2) \div (x + 1)$ 10. Solve the system of equations:
$$\begin{cases} x^3 - y^3 = -37 \\ x - y = -1 \end{cases}$$

Answers:

1) $-2, -1, 1$ 2) $-1, 1, 3 \pm 2i$ 3) $-1, 2 \pm 3i$ 4) $f(x) = x^3 - 2x^2 - 3x - 6$
 5) $f(x) = x^3 - 5x^2 + 4x + 10$ 6a) $(x - 4)(3x - 1)$ 6b) $4, 1/3$ 6c) $(0, 7)$
 7) $\pm 2, 3 \pm i$ 8) $\pm(1, 2, 3, 6, 1/2, 3/2)$ 9) $r = -3$ 10) $(3, 4), (-4, -3)$

