Geometric Sequences

Examples of Geometric Sequences:

a) 12, 6, 3, 1.5………

Geometric $r = 1/2$

b) 1, 3, 9, 27…..

$r = 3$

So what is a Geometric Sequence anyways?

A geometric sequence is one that has a common ratio to get from one term to the next.

Different type of equations:

Recursive:	Explicit:
$a_1 =$	$a_1 =$
$a_n = a_{n-1}(r)$	$a_n = a_1(r)^{n-1}$
*finds next few terms	*finds any term

Examples:

a) 1, 2, 4, 8….

Recursive:

$a_1 = 1$

$a_n = a_{n-1}(2)$

Explicit:

$a_1 = 1 \quad a_n = 1(2)^{n-1}$

b) 27, 9, 3, 1…..

Recursive:

$a_1 = 27$

$a_n = a_{n-1}(1/3)$

Explicit:

$a_1 = 27 \quad a_n = 27(1/3)^{n-1}$

c) 40, 10, 10/4…..

Recursive:

$a_1 = 40$

$a_n = a_{n-1}(1/4)$

Explicit:

$a_1 = 40 \quad a_n = 40(1/4)^{n-1}$

d) -1, -2, -4, -8…..

Recursive:

$a_1 = -1$

$a_n = a_{n-1}(2)$

Explicit:

$a_1 = -1$

$a_n = -1(2)^{n-1}$

AC Alg 1/Geo A

Name________________ ID: 1

Date________________ Period____

Geometric Sequences

Determine if the sequence is geometric. If it is, find the common ratio.

1) 2, 12, 72, 432, ...

$\boxed{r=6}$

2) −2, 10, −50, 250, ...

$\boxed{r=-5}$

3) 4, −20, 100, −500, ...

$\boxed{r=-5}$

4) 1, 4, 16, 64, ...

$\boxed{r=4}$

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

5) $a_n = 2 \cdot 6^{n-1}$

$a_1 = 2$, $a_2 = 12$, $a_3 = 72$, $a_4 = 432$, $a_5 = 864$

6) $a_n = 2 \cdot 4^{n-1}$

$a_1 = 2$, $a_2 = 8$, $a_3 = 32$, $a_4 = 128$, $a_5 = 512$

7) $a_n = 3 \cdot (-3)^{n-1}$

$a_1 = 3$, $a_2 = -9$, $a_3 = 27$, $a_4 = -81$, $a_5 = 243$

8) $a_n = 3 \cdot (-2)^{n-1}$

$a_1 = 3$, $a_2 = -6$, $a_3 = 12$, $a_4 = -24$, $a_5 = 48$

Given two terms in a geometric sequence find both the recursive and explicit formulas.

9) $a_1 = 4$ and $a_4 = -32$ $r=-2$

rec: $a_1 = 4$, $a_n = a_{n-1}(-2)$

expl: $a_n = 4(-2)^{n-1}$

10) $a_6 = -\frac{3}{64}$ and $a_5 = \frac{3}{16}$ $r = -1/4$

rec: $a_1 = 48$, $a_n = a_{n-1}(-1/4)$

expl: $a_n = 48(-1/4)^{n-1}$

11) $a_4 = -\frac{1}{4}$ and $a_3 = -\frac{1}{2}$ $r = \frac{1}{2}$

$a_1 = -2$, $a_n = a_{n-1}(1/2)$

expl: $a_n = -2(1/2)^{n-1}$

12) $a_4 = 432$ and $a_5 = -2592$

recur: $a_1 = -2$, $a_n = a_{n-1}(-6)$

expl: $a_n = -2(-6)^{n-1}$

Evaluate each geometric series described.

13) $-2 - 8 - 32 - 128...,\ n = 7$

$\boxed{-10922}$

14) $-1 - 5 - 25 - 125...,\ n = 8$

$\boxed{-97656}$

15) $4 + 12 + 36 + 108...,\ n = 9$

$\boxed{39364}$

16) $-1 - 2 - 4 - 8...,\ n = 9$

$\boxed{-511}$

Find the missing term or terms in each geometric sequence.

17) ..., −1, ___, ___, −27, ...

$\boxed{-3, -9}$

18) ..., −2, ___, ___, ___, −162, ...

$\boxed{-6, -18, -54}$

AC Algebra 1/Geo A

Unit 5-Comparing Functions
Even and Odd Functions

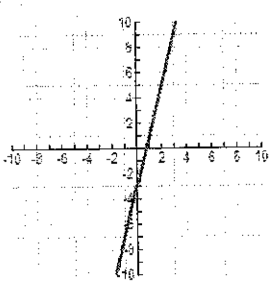
We can classify the graphs of functions as either ***even, odd, or neither.***

Even	Odd
Even functions are symmetric with respect to the y-axis. This means we could fold the graph on the axis, and it would line up perfectly on both sides! *The right side of the equation of an even function does NOT change if x is replaced with –x.	Odd functions are symmetric with respect to the X & y-axis. This means we can flip the image upside down and it will appear exactly the same! *Every term on the right side of the equation changes signs if x is replaced with –x.

If we cannot classify a function as even or odd, then we call it neither!

Directions: Determine graphically using possible symmetry, whether the following functions are even, odd, or neither.

1.



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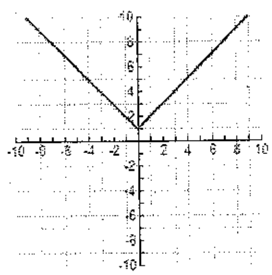
2.



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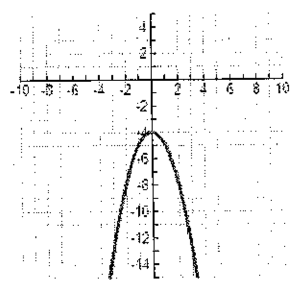
3.



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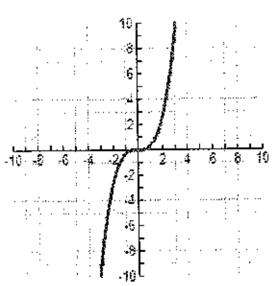
4.



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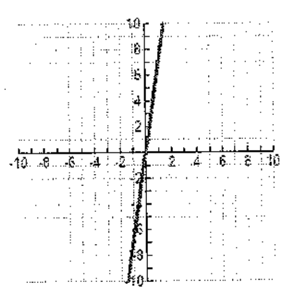
5.



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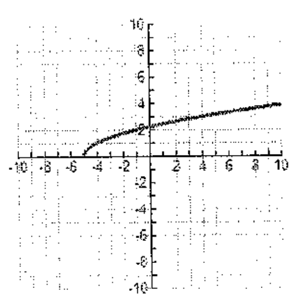
6.



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If neither of the above are true, we call the function neither!

What to do:	Example:
1) Pick a number such as 2 and -2 2) Evaluate the function at each value 3) If you get: -the same value out each time, the function is EVEN -if you get out opposite numbers the function is ODD -if you get two random numbers it's NEITHER	$f(x)=x^2 + 3x + 5$ $f(2)=(2)^2 + 3(2)+5$ $f(2) = 15$ $f(-2)=(-2)^2 + 3(-2) + 5$ $f(-2)=3$ NEITHER

Directions: Verify algebraically whether each function is even, odd, or neither!

1. $f(x) = x^3 - 6x$

$x = 2$

$f(2) = (2)^3 - 6(2)$

$= 8 - 12$

$f(2) = -4$

$x = -2$

$f(-2) = (-2)^3 - 6(-2)$

$= -8 + 12$

$= 4$

opposite = odd

2. $g(x) = x^4 - 2x^2$

$x = 2$

$g(2) = (2)^4 - 2(2)^2$

$= 16 - 8$

$= 8$

$x = -2$

$g(-2) = (-2)^4 - 2(-2)^2$

$= 16 - 8$

$= 8$

Same = Even

3. $h(x) = x^2 + 2x + 1$

$x = 2$

$h(2) = (2)^2 + 2(2) + 1$

$= 4 + 4 + 1$

$= 9$

$x = -2$

$h(-2) = (-2)^2 + 2(-2) + 1$

$= 4 - 4 + 1$

$= 1$

$\boxed{\text{Neither}}$

4. $f(x) = x^2 + 6$

$x = 2$

$f(2) = 2^2 + 6$

$= 10$

$x = -2$

$f(-2) = (-2)^2 + 6$

$= 10$

Same = $\boxed{\text{Even}}$

5. $g(x) = 7x^3 - x$

$x = 2$

$g(2) = 7(2)^3 - 2$

$= 7(8) - 2$

$= 54$

$x = -2$

$g(-2) = 7(-2)^3 - (-2)$

$= 7(-8) + 2$

$= -54$

$\boxed{\text{odd}}$

6. $h(x) = x^5 + 1$

$x = 2$

$h(2) = 2^5 + 1$

$= 33$

$x = -2$

$h(-2) = (-2)^5 + 1$

$= -32 + 1$

$= -31$

$\boxed{\text{Neither}}$

7. $f(x) = x\sqrt{4 - x^2}$

$x = 2$

$f(2) = 2\sqrt{4-(2)^2}$

$= 0$

$x = -2$

$f(-2) = 2\sqrt{4-(-2)^2}$

$= 0$

$\boxed{\text{Even}}$

8. $g(x) = x^4\sqrt{1 + x}$

$x = 2$

$g(2) = (2)^4\sqrt{1+2}$

$= 16\sqrt{3}$

$x = -2$

$g(-2) = (-2)^4\sqrt{1-2}$

$= 16\sqrt{-1}$

$\boxed{\text{Neither}}$

9. $h(x) = |x| - 1$

$x = 2$

$h(2) = |2| - 1$

$= 1$

$x = -2$

$h(-2) = |-2| - 1$

$= 1$

$\boxed{\text{Even}}$

10. $g(x) = \frac{1}{4}x^6 - 5x^2$

$x = 2$

$g(2) = \frac{1}{4}(2)^6 - 5(2)^2$

$= 16 - 20$

$= -4$

$x = -2$

$g(-2) = \frac{1}{4}(-2)^6 - 5(-2)^2$

$= 16 - 20$

$= -4$

$\boxed{\text{Even}}$

Name: Key Date: ______ Period: ______

Arithmetic and Geometric Sequences Practice

Directions: For each of the following tables:

a) Describe how to find the next term in the sequence.

b) Find the next term in the table.

c) Write a recursive rule for the function.

d) Write an explicit rule for the function.

e) Tell whether the function is linear, exponential, or neither.

1)

x	y
1	10
2	20
3	40
4	?
...	...
n	?

a) To find the next term, common ration of 2

b) Next term in table: 80, 160...

c) Recursive Rule: $a_1 = 10 \quad a_n = a_{n-1}(2)$

d) Explicit Rule: $a_1 = 10 \quad a_n = 10(2)^{n-1}$

e) Type of function: exponential

2)

x	y
1	40
2	200
3	1000
4	?
...	...
n	?

a) To find the next term, common ratio of 5

b) Next term in table: 5,000, 25,000...

c) Recursive Rule: $a_1 = 40 \quad a_n = a_{n-1}(5)$

d) Explicit Rule: $a_1 = 40 \quad a_n = 40(5)^{n-1}$

e) Type of function: exponential

3)

x	y
1	9
2	10
3	11
4	?
...	...
n	?

a) To find the next term, add 1 common difference

b) Next term in table: 12, 13, 14...

c) Recursive Rule: $a_1 = 1 \quad a_n = a_{n-1} + 1$

d) Explicit Rule: $a_n = 1 \quad a_n = 9 + (n-1)(1)$

e) Type of function: arithmetic

4)

x	y
1	16
2	64
3	256
4	?
…	…
n	?

a) To find the next term, common ratio of 4

b) Next term in table: 1024, 4096

c) Recursive Rule: $a_1 = 16 \quad a_n = a_{n-1}(4)$

d) Explicit Rule: $a_1 = 16 \quad a_n = 16(4)^{n-1}$

e) Type of function: exponential

5)

x	y
1	-2
2	-5
3	-8
4	?
…	…
n	?

a) To find the next term, common difference -3

b) Next term in table: -11, -14…

c) Recursive Rule: $a_1 = -2 \quad a_n = a_{n-1} - 3$

d) Explicit Rule: $a_n = -2 \quad a_n = -2 + -3(n-1)$

e) Type of function: arithmetic

6)

x	y
1	32
2	64
3	128
4	?
…	…
n	?

a) To find the next term, common ratio of 2

b) Next term in table: 256

c) Recursive Rule: $a_n = 32 \quad a_n = a_{n-1}(2)$

d) Explicit Rule: $a_1 = 32 \quad a_n = 32(2)^{n-1}$

e) Type of function: geometric

7)

x	y
1	3,125
2	625
3	125
4	?
…	…
n	?

a) To find the next term, common ratio of 1/5

b) Next term in table: 25, 5

c) Recursive Rule: $a_1 = 3,125 \quad a_n = a_{n-1}(1/5)$

d) Explicit Rule: $a_1 = 3125 \quad a_n = 3125(1/5)^{n-1}$

e) Type of function: geometric

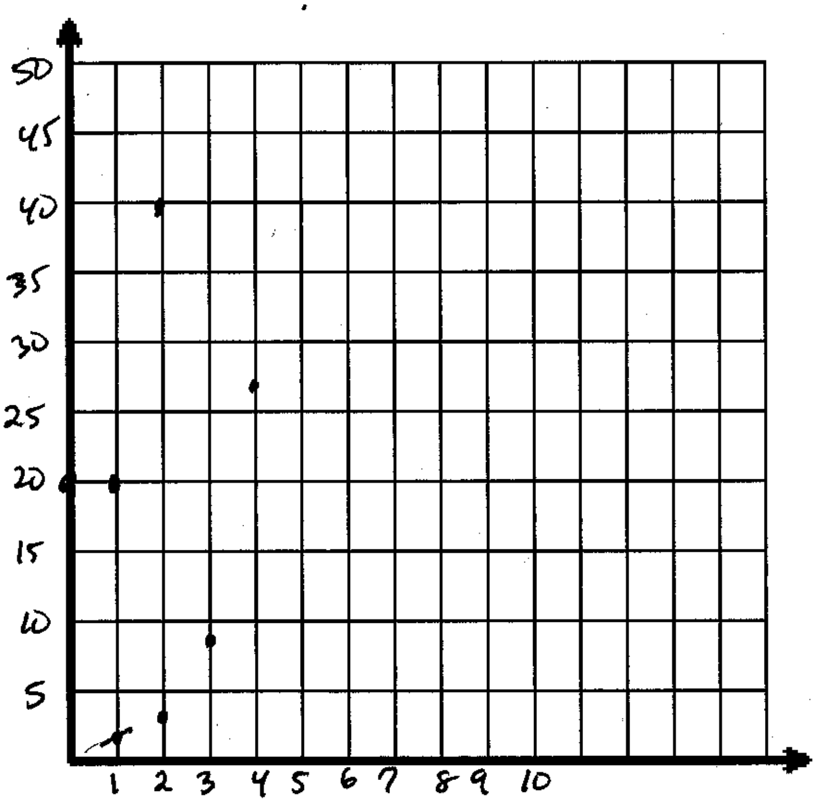
8) You won the lottery!!!! Don't forget about your favorite math teacher (that's me...Mrs. Kitchens ☺).

Option 1: You can receive your winnings $20,000 a year.
Option 2: Start out with $1,000 the first year, $3,000 the second year, $9,000 the third year and so on.

a) Write an explicit equation for each situation.

Option 1: $y = 20{,}000x$ **Option 2:** $y = 1000(3)^{n-1}$

b) Graph each function below (make a table of values and plot points to at least year 8):



option 1

x	y
1	20,000
2	40,000
3	60,000
4	80,000
5	100,000
6	120,000
7	140,000
8	160,000

option 2

x	y
1	1000
2	3000
3	9000
4	27000
5	81,000
6	243,000
7	729,000
8	2,187,000

c) Identify which option would be a better option for year two. What option would be a better option for year 10?

Year 2 → option 1 is better
Year 10 → option 2 is better

d) Option 1 is what type of sequence: ~~linear~~ arithmetic **and what type of function:** linear.

Option 2 is what type of sequence: Geometric **and what type of function:** exponential

9. Prove algebraically that the following functions are EVEN, ODD OR NEITHER.

a. $f(x) = x^2 + x^4 - x$

$f(2) = 2^2 + 2^4 - 2 = 18$

$f(-2) = (-2)^2 + (-2)^4 + 2 = 22$

Neither

b. $f(x) = -2x^2 + 1$

$f(-2) = -2(-2)^2 + 1$
$= -7$

$f(+2) = -2(2)^2 + 1$
$= -7$

Even

c. $f(x) = -2x + 3$

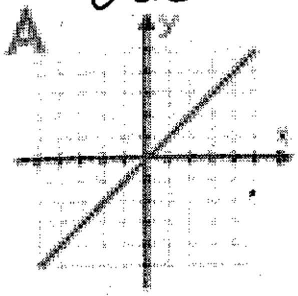
$f(2) = -2(2) + 3$
$= -1$

$f(-2) = -2(-2) + 3$
$= 7$

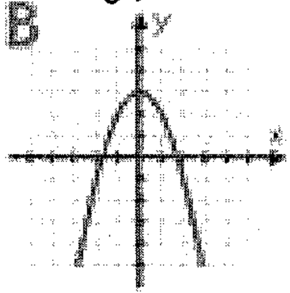
Neither

10. Determine whether each function is EVEN, ODD or NEITHER.

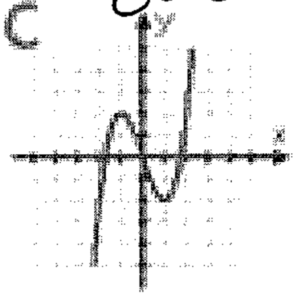
A odd



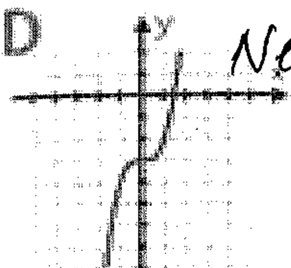
B even



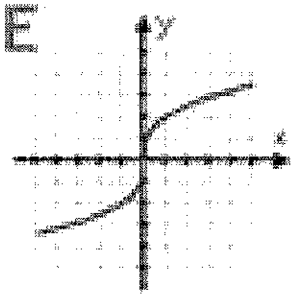
C odd



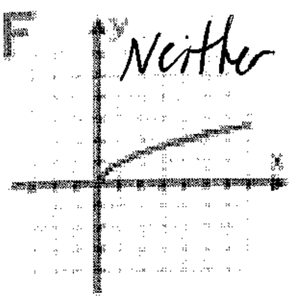
D Neither



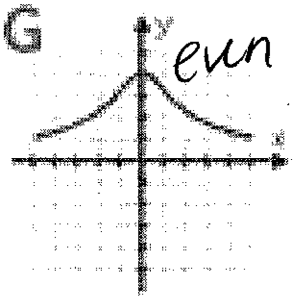
E odd



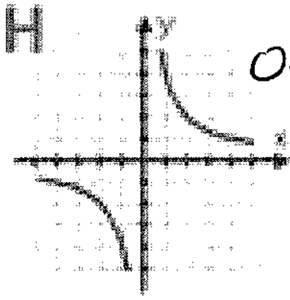
F Neither



G even



H odd



A___O___ B___E___ C___O___ D___N___ E___O___

F___N___ G___E___ H___O___

10

Name: ______________________________ Date: ____________ Per: _____

ARITHMETIC AND GEOMETRIC SEQUENCE WORD PROBLEMS PRACTICE
All final solutions MUST use the formula.

1. Edgar is getting better at math. On his first quiz he scored 57 points, then he scores 61 and 65 on his next two quizzes. If his scores continued to increase at the same rate, what will be his score on his 9th quiz? Show all work.

   a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula. arithmetic $a_1 = 57$

   $\boxed{a_n = 57 + 4(n-1)}$ $d = 4$

   b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

   $n = 9$

   $a_9 = 57 + 4(9-1) \rightarrow a_9 = 57 + 32$ $\boxed{a_9 = 89\%}$

   c. Write your final answer as a sentence.

   Edgar would score an 89% on his 9th quiz.

2. Suppose you drop a tennis ball from a height of 15 feet. After the ball hits the floor, it rebounds to 85% of its previous height. How high will the ball rebound after its third bounce? Round to the nearest tenth.

   a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula. $\boxed{a_n = 15(.85)^{n-1}}$ $a_1 = 15$

   $r = .85$

   b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

   $n =$ term value $a_3 = 15(.85)^{3-1}$

   $\boxed{a_3 \approx 10.84 \text{ ft}}$

   c. Write your final answer as a sentence.

   On the 3rd bounce the ball would get up to 10.84 ft.

3. Viola makes gift baskets for Valentine's Day. She has 13 baskets left over from last year, and she plans to make 12 more each day. If there are 15 work days until the day she begins to sell the baskets, how many baskets will she have to sell?

   a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula. $a_n = 13 + 12(n-1)$ $a_1 = 13$

   $r = 12$

   b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

   $n = 15$ (days) $a_{15} = 13 + 12(15-1)$

   $\boxed{a_{15} = 181}$

   c. Write your final answer as a sentence.

   On the 15th day Viola will have 181 baskets

4. In a certain region, the number of highway accidents increased by 20% over a four year period. How many accidents were there in 2006 if there were 5120 in 2002? Hint: When the percent increases, you want the original 100% plus the additional 20%.

a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula.

$y = 5120(1+.20)^{t/4}$ t = # of years from 2002

n=t

b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

n = # of years from 2002

$y=5120(1+.20)^{4/4} = 6144$ accidents

c. Write your final answer as a sentence.

There were 6144 accidents over the span of 2002-2006

5. A house worth \$350,000 when purchased was worth \$335,000 after the first year and \$320,000 after the second year. If the economy does not pick up and this trend continues, what will be the value of the house after 6 years.

a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula.

$y = 350,000(1- \quad)^t$ t=n = time in years

initial amount

b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

$y = 350,000(1-.086)^6$

y = \$204,054.05

c. Write your final answer as a sentence.

In 6 years the house will be worth \$204,054.05. eeekkk!!!!

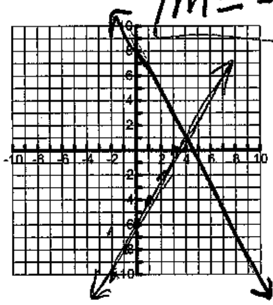
6. Write about anything that you need help with on the word problems we have done so far. Be specific so we can help you. Do you need to come for tutoring?

Name: Key Date: ______

Comparing Linear Functions

1. The functions f(x) ~~and g(x) are~~ described below. Compare the rate of change and intercepts of each.

$m = -2$



X-inter (5,0)
y-inter (0,8)

x	g(x)
-2	-10
-1	-8
0	-6
1	-4

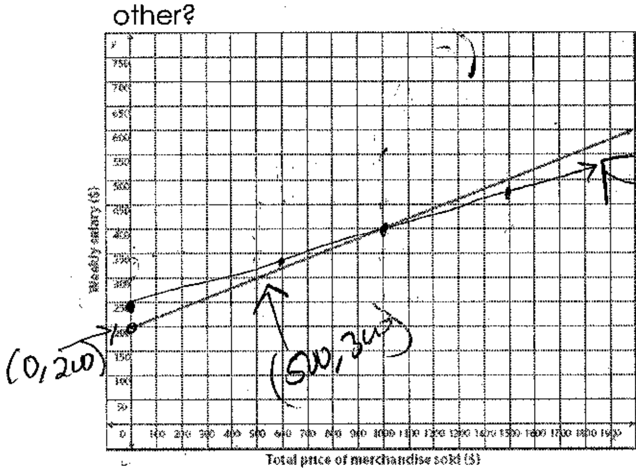
Opposite sign slopes

(-2,-10) (-1,-8)

$m = \frac{-8+10}{-1+2} = \frac{2}{1}$

$\boxed{m = 2}$ y-inter (0,-6) X-inter (3,0)

2. Your employer has offered two pay scales for you to choose from. The first option is to receive a base salary of $250 a week plus 15% of the price of any merchandise you sell. The second option is represented in the graph below. Compare the rate of change and intercepts of the functions. What does the rate of change tell you about the two scales? When would each scale be better than the other?



$y = 250 + .15X$ →

X	y
300	295
400	310
500	325
600	340
1000	400
1500	475

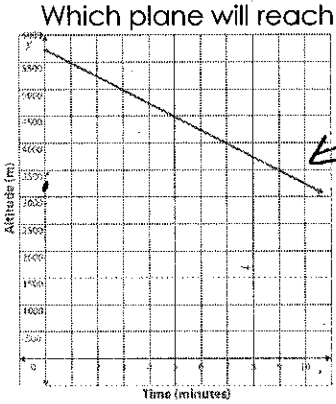
$y = 200 + .20X$

(0,200) (500,300)

$m = \frac{300-200}{500-0} = \frac{100}{500} = \frac{1}{5} = .20\%$

* y = 250 + .15X is better until you reach over 1,000. *

3. Two airplanes are in flight. The function f(x) = -100x + 3,350 represents the altitude, f(x), of one airplane after x minutes. The graph below represents the altitude of the second airplane. Compare the rate of change and intercepts of the functions. Would the two planes ever be at the ~~same altitudes?~~ Which plane will reach the ground first?



g(x) is decending faster

$f(x) = -100X + 3350$

$g(x) = \frac{-500 \text{ ft}}{2 \text{ min}} X + 5750$

$0 = -100X + 3350$

$-3350 = -100X$

$\boxed{X = 33.5 \text{ min to ground}}$

$0 = -250X + 5750$

$-5750 = -250X$

$\boxed{X = 23 \text{ min to ground}}$

4. Compare the rate of change of each function.

Function A

Number of beverages sold (x)	Profit (f(x))
0	0
25	29.25
50	58.50

Function B

For each hamburger sold, the restaurant makes $0.40.

* Function A has larger profit margin

(25, 29.25) (50, 58.50)

$m = \frac{58.50 - 29.25}{50 - 25} \approx 1.17$

Comparing Exponential Functions

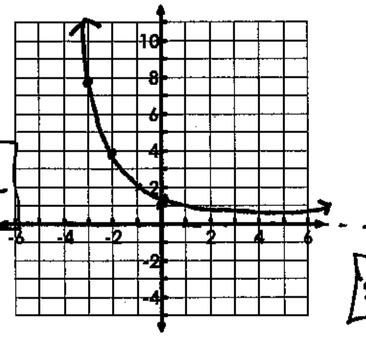
Key

5. Graph the two functions. Which function has a greater rate of change over the interval [0, 5]? How do you see that in the graph? Which function has the greater y-intercept?

$f(x)=\left(\frac{1}{2}\right)^x$

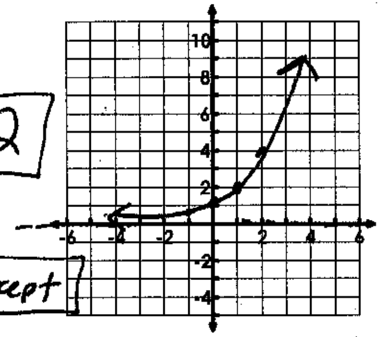
x	y
0	1
-3	8
-2	4
-1	2
5	.03125

$\boxed{m = -.19375}$



$f(x)=2^x$

$\boxed{m = 6.2}$



$\boxed{\text{same y intercept}}$

x	y
-1	.5
0	1
1	2
2	4
5	32

6. Compare the rate of change of each exponential function over the interval [1, 3]. Find the common ratio, r, for the two functions. Will Function B ever surpass the value of Function A?

Function A

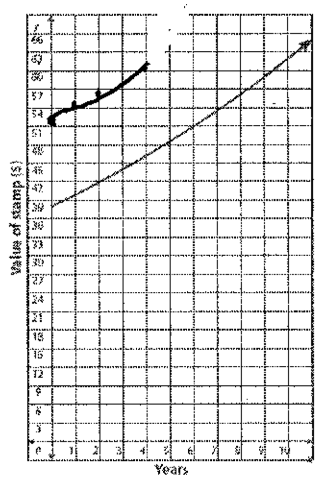
x	g(x)
0	52
1	54.08
2	56.24
3	58.49
4	60.83

(1, 54.08) (3, 58.24)

m = 2.08

r = .04

Function B



(1, 40) (3, 44)

$m = \frac{4}{2} = 2$

7. Jennifer has the choice of two bank accounts. She has $2,000 to invest. Compare the rate of change for the two banks over the first 10 years. Which account is better than the other? What part of the equation would indicate that this bank is better than the other?

Bank A: $A(x) = 2,000(1.05)^x$

Bank B: $B(x) = 2,000(1.08)^x$

Bank B is better b/c higher rate

A

$A(10) = 2,000(1.05)^{10}$

$A(10) = \$3257.79$

B

$B(10) = 2,000(1.08)^{10}$

$B(10) = \$4317.85$

Name: ______________________ Date: Key

Combining Functions Homework

Combining Functions

Given the functions $f(x) = 4x + 8$ and $g(x) = 2x - 12$

1. Find f(x) + g(x). $=$

$\boxed{6x-4}$

2. Find f(x) – g(x).

$(4x+8)-(2x-12)$
$4x+8-2x+12 = \boxed{2x+20}$

Given the functions $f(x) = 3x^2 + 5x - 8$ and $g(x) = 2x^2 - 9$

3. Find f(x) + g(x).

$\boxed{5x^2+5x-17}$

4. Find f(x) – g(x).

$(3x^2+5x-8)-(2x^2-9) = \boxed{x^2+5x+1}$

5. Find f(2) and g(2).

$f(2) = 3(2)^2+5(2)-8 = \boxed{14}$ $g(2) = 2(2)^2-9 = \boxed{-1}$

6. If e(x) = f(x) – g(x), find e(2).

$e(2) = (2)^2+5(2)+1 = \boxed{15}$

7. What do you notice about your answers to questions 5 and 6?

Given the functions $f(x) = 2x^2 + 3x$ and $g(x) = 5x + 1$

8. Find 2f(x) + 3g(x).

$4x^2+6x+15x+3 = \boxed{4x^2+21x+3}$

9. Find 5f(x) – 2g(x).

$10x^2+15x-(10x+2) = \boxed{10x^2+5x+2}$

10. Jill has a regular savings account that has $350 in it. She saves $55 each month in this account. She is also going on tour with her school choir next year. She opens up a new savings account just for tour. She deposits $25 to start the account and then, decides to save $40 each month from her paycheck into her tour savings account.

a. Write a function to represent the prices r(x) for Jill's regular savings account. $r(x) = 350+55x$

b. Write a function t(x) to represent Jill's tour savings account. $t(x) = 25+40x$

c. Combine the two functions into one function s(x) = r(x) + t(x). $= 95x+375$

d. Calculate her totals savings after 3 months, 6 months, and 10 months.

3 months $= 95(3)+375 = 660$ 6 months $= 95(6)+375 = 945$ 10 m $= 95(10)+375 = 1325$

11. Joseph's Plumbing Company employs 3 workers. They employ out at the following rates.

- Joseph (owner): $75 (flat fee) + $65 per hour
- Sam (an apprentice): $10 (flat fee) + $25 per hour
- Sally: $50 (flat fee) + $45 per hour

a. Write 3 functions, one for each employee.

$y = 75+65x$ $z = 10+25x$ $K = 50+45x$

b. Write a new function to show the total amount of money coming in for the company in terms of hours worked? $f(x) = 135x+135$

c. How much money will the company make if all the employees work an 8 hour day?

$f(8) = 135(8)+135$
$f(8) = 1215$

15