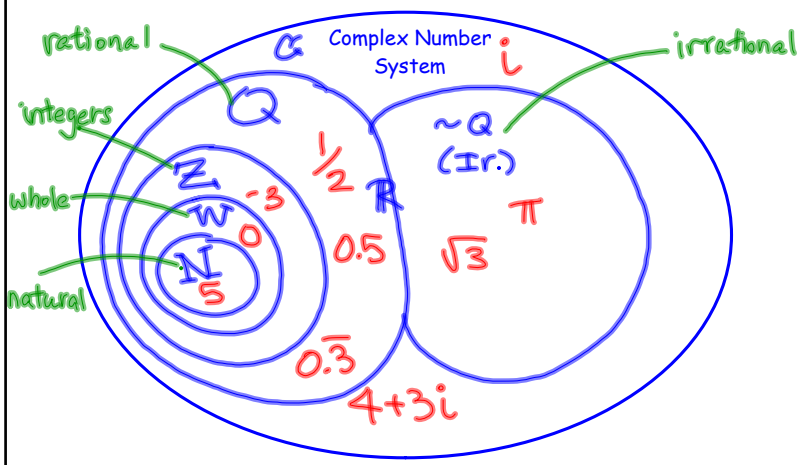
Complex Numbers

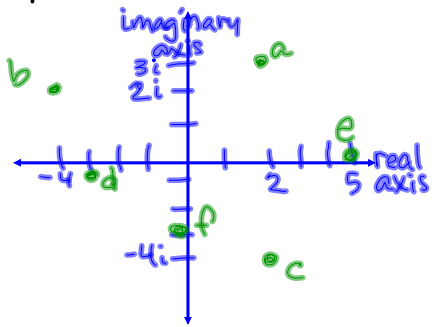
$i = \sqrt{-1}$



complex number: (standard form) $a + bi$ (a: real #, b: imaginary #)

Ex.1 Plot each complex number.

a. 2 + 3i
b. -4 + 2i
c. 2 - 4i
d. -3 - i
e. 5
f. -3i



$\sqrt{-1} = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

} memorize!

Ex.2 Simplify

a. $i^7 = -i$ ($7 \div 4 = 1$ R 3, i^3)

b. $i^{15} = -i$ ($15 \div 4 = 3$ R 3, i^3)

c. $i^{54} = -1$ ($54 \div 4 = 13$ R 2, i^2)

Ex.3 Simplify

a. $\sqrt{-4} = 2i$

b. $\sqrt{-25} = 5i$

c. $3\sqrt{-16} = 3 \cdot 4i = 12i$

d. $\sqrt{-\frac{4}{9}} = \frac{2}{3}i$

e. $\sqrt{-18} = \sqrt{-9} \cdot \sqrt{2} = 3i\sqrt{2}$

f. $\sqrt{64} + \sqrt{-63} = 8 + \sqrt{-9} \cdot \sqrt{7} = 8 + 3i\sqrt{7}$

g. $-\sqrt{25} + 7 = -5 + 7 = 2$

Ex.4 Add or Subtract

C.L.T. (Combine like terms)

a. $(3 + 5i) + (-2 + 3i) = 5 + 8i$

b. $(-1 - 4i) - (3 + 2i) = -4 - 6i$
$-1 - 4i - 3 - 2i$

c. $(4 + i) + (-2 + 3i) - (-5i) = 2 + 9i$
$4 + i - 2 + 3i + 5i$

d. $10 - (-4 - 2i) - 5 = 9 + 2i$
$10 + 4 + 2i - 5$

Ex.5 Multiply and Simplify

$i^2 = -1$

a. $2i(3 - 4i) = 6i - 8i^2 = 6i - 8(-1) = 8 + 6i$ (real + imag.)

b. $-i(2 + 3i)$
$-2i - 3i^2 = -2i - 3(-1) = 3 - 2i$

c. $(2 + 6i)(-3 + 2i)$
$-6 + 4i - 18i + 12i^2 = -18 - 14i$

d. $(5 + 2i)(5 - 2i) = 25 - 10i + 10i - 4i^2$
$= 29$ real #!

Conjugates

Ex.6 Divide and Simplify

$i^2 = -1$
$a + bi$

a. $\frac{(2 + i)}{(4 - 5i)} \cdot \frac{(4+5i)}{(4+5i)} = \frac{8 + 10i + 4i + 5i^2}{16 + 20i - 20i - 25i^2} = \frac{3 + 14i}{41}$
$= \frac{3}{41} + \frac{14}{41}i$

Conjugates

b. $\frac{(5 - 2i)}{(3 + 3i)} \cdot \frac{(3-3i)}{(3-3i)} = \frac{15 - 15i - 6i + 6i^2}{9 - 9i + 9i - 9i^2} = \frac{9 - 21i}{18}$
$= \frac{9}{18} - \frac{21}{18}i$
$= \frac{1}{2} - \frac{7}{6}i$

Multiply top & bottom by conjugate of denom.